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O·R Operational Research Quarterly

Issued by
the Operational
Research Society
London

Volume 9 [redacted]
March 1958

Operational Research Quarterly

VOLUME 9 NUMBER 1 MARCH 1958

Joint Editors for the O.R. Society: MAX DAVIES AND R. T. EDDISON
11 Park Lane, London, W.1 (*Grosvenor* 4751)

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The Operational Research Quarterly is published for the Operational Research Society London, in March, June, September and December, at these subscription rates (all post free):

	British Commonwealth	*Other Countries*
Surface mail	20*s.*	30*s.*
Air mail	35*s.*	45*s.*
Single copies (surface mail)	6*s.*	8*s.*

Overseas copies will be sent surface mail unless airmail instructions are received. North American subscribers may send orders and cheques to the honorary subscription agent in the U.S.A.:

DR. PETER B. MYERS,
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Operational Research Quarterly
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Note on a Stock Model

P. D. FINCH
Sunvic Controls Limited

IN a recent paper by R. R. P. Jackson[1] an interesting model of a certain class of stock problems is constructed. In effect, Jackson considers that the distribution of issues from stock is random with a mean value μ units per unit of time. The purpose of this note is to generalize the results of Jackson to the case when the time interval between demands is a random variable with arbitrary distribution function and the number of units issued per demand is also a random variable with arbitrary distribution function.

The Model

The class of problems considered by Jackson is described by supposing that initially N units of stock are held, demands for issues are made and when the stock is exhausted it is replenished at once by a further N units.

Let t_{r+1} be the time interval between the rth and the $(r+1)$th demands for stock, it is supposed that:

Postulate 1

The t_r are identically and independently distributed with finite mean $E(t_r)$.

Let n_r be the number of units requested at the rth demand, it is supposed that:

Postulate 2

The n_r are identically and independently distributed with finite mean $E(n)$.

If N_r is the amount in stock just before the rth demand and if I_r is the amount of the rth issue, then

$$\begin{aligned} I_r &= n_r \quad \text{if} \quad 0 < n_r \leqslant N_r \\ &= N_r \quad \text{if} \quad N_r < n_r. \end{aligned} \tag{1}$$

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These equations are equivalent to the assumption that an unsatisfied demand is lost and is not satisfied by drawing on the N units which replenish the stock. In the case $n_r = 1$ with probability 1 this assumption is no restriction; when it is not the case $n_r = 1$ with probability 1 it must be ascertained whether or not the assumption gives a reasonable model of the process being investigated. Equations (1) imply that only that part of the last demand which cannot be met (if any) is lost, e.g. a quantity $(n_r - N_r)$. There remain several other assumptions each of which could serve as an alternative to the one chosen in this note. These other assumptions will be discussed in a further note[2] where the methods given below are generalized to the case when the moment of replenishment of stock is also a random variable. When the demand rate is high so that the time interval between demands is small, the assumption (1) made above will not give results significantly different from the case when the balance of the demand is satisfied from the replenishment stock.

If we consider one cycle of stockholding from the moment an initial stock of N units is held until this stock is exhausted, we can write down the following inequality

$$I_1 + I_2 + \ldots + I_{m-1} < N = I_1 + \ldots + I_m, \tag{2}$$

where m is the number of demands which occur until the stock is exhausted and

$$\begin{aligned} I_j &= n_j, \qquad 1 \leqslant j \leqslant m-1 \\ I_m &= N - (n_1 + \ldots + n_{m-1}). \end{aligned}$$

The Cost of Holding Stock

Consider a single cycle of stockholding starting at the time $t = t_0$ when the stock level is N units. If the stock is exhausted just after m demands, then exhaustion occurs at the time $t = t_0 + t_1 + \ldots + t_m$. During the interval of time $(t_0 + t_1 + \ldots + t_r, t_0 + t_1 + \ldots + t_{r+1})$ of duration t_{r+1} the number of units of stock held is $N - n_1 - n_2 - \ldots - n_r$. Following Jackson, place the cost of holding a unit of stock for a unit of time equal to $c = sb$ where b is the basic value of one unit of stock.

$C_h(N)$ the cost of holding stock during a single cycle is given by equation (3)

$$C_h(N) = c[Nt_1 + (N-n_1)\,t_2 + \ldots + (N-n_1-\ldots-n_{m-1})\,t_m]. \tag{3}$$

Consider a sequence of k cycles and denote the cost of stockholding in the jth cycle by $C_h^j(N)$, then the random variables $C_h^j(N)$ are identically and independently distributed with a distribution function which can be obtained from equation (3). If T^j is the duration of the jth cycle, then

$$C^k(N) = C_h^1(N) + \ldots + C_h^k(N)/T^1 + \ldots + T^k$$

is the cost of stockholding per unit of time for a sequence of k cycles. As $k \to \infty$, then the probability 1 by the strong law of large numbers,

$$C(N) = \lim_{k\to\infty} C^k(N) = E[C_h(N)]/E[T]. \tag{4}$$

If $m(N)$ is the random variable which specifies the number of demands which just exhausts the store when the initial level is N units, then it is shown below that

$$E[C_h(N)] = cE(t)\{E[m(1)] + \ldots + E[m(N)]\}. \tag{5}$$

Since $$T^j = t_1 + \ldots + t_{m(N)}$$

and $$E[T] = E(t)\,E[m(N)],$$

it follows from (4) and (5) that

$$C(N) = c\{E[m(1)] + \ldots + E[m(N)]\}/E[m(N)]. \tag{6}$$

In the case when demands are for one unit of stock only it is clear that $E[m(N)] = N$. In this case from equations (5) and (6)

$$E[C_h(N)] = cE(t)\,N(N+1)/2,$$

$$C(N) = c(N+1)/2.$$

These equations include the results of Jackson, who considers, in effect, the case of unit demands when the time interval between successive demands has a negative exponential distribution with mean $E(t) = 1/\mu$.

Equation (6) shows that the total cost per unit of time in holding an initial-replacement stock of N units for a long time is independent of the distribution of the time interval between successive demands and, apart from a costing factor, depends only upon the distribution of the number of stock units per demand (and, of course, N). This result is plausible intuitively since the greater the rate at which issues are made the more cycles take place in a given time interval (for fixed N) and a particular unit of stock is held for a shorter time interval.

Jackson[3] has shown that the cases of uniform and random stock withdrawals are equivalent costwise. Equation (6) generalizes this result. Jackson's method of associating the cost of manufacture with the cost of holding stock can be carried over to the more general case considered in this note. In any particular case, however, it is not easy to calculate the exact value of $E[m(N)]$. The next section gives an approximate evaluation which may be used in conjunction with equations (5) and (6). In subsequent sections the theoretical determination of $E[m(N)]$ is given and a particular case for which the exact value can be obtained easily; this exact value is then compared with the approximate formula.

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Approximate Evaluation of the Number of Issues per Cycle

Consider a sequence of k cycles and let the number of issues in the jth cycle be $m_j(N)$, or simply m_j when there is no ambiguity in the initial stock level. Let these demands be in amount $n_1^j, n_2^j, \ldots, n_{m_j}^j$, respectively.

Then
$$n_1^j + n_2^j + \ldots + n_{m_j}^j = N + d^j$$
where
$$0 \leqslant d^j < n_{m_j}^j. \tag{7}$$

Consider the identity

$$\left\{\frac{m_1 + m_2 + \ldots + m_k}{k}\right\}\left\{\frac{(n_1^1 + \ldots + n_{m_1}^1) + \ldots + (n_1^k + \ldots + n_{m_k}^k)}{m_1 + \ldots + m_k}\right\}$$
$$\equiv N + \frac{d^1 + \ldots + d^k}{k}. \tag{8}$$

The expression on the right-hand side of (8) is derived from that on the left-hand side by substitution from (7) into the second factor. Taking the limit as $k \to \infty$,

$$E[m(N)]\,E[n] = N + E[d] \tag{9}$$

where $E(d)$ is the expected value of the identically and independently distributed random variables d^j.

Since $0 \leqslant E(d) < E(n)$, it follows from (9) that

$$\frac{N}{E[n]} \leqslant E[m(N)] < 1 + \frac{N}{E(n)}. \tag{10}$$

If N is large compared to $E(n)$, which is usually the case in practice, the inequalities (10) enable a good approximation to the value of $E[m(N)]$ to be obtained.

From equations (5) and (10)

$$cE(t)\left\{\frac{N(N+1)}{2E(n)}\right\} \leqslant E[C_h(N)] \leqslant \left\{N+\frac{N(N+1)}{2E(n)}\right\} cE(t), \tag{11}$$

and from equations (6) and (10)

$$c\left\{\frac{N+1}{2(1+E(n)/N)}\right\} \leqslant C(N) < c\left\{\frac{E(n)}{N}+\frac{N+1}{2}\right\}. \tag{12}$$

As an example of the use of the formulae, consider the case

$$N = 100E(n) = 3.$$

If $E(t)$ is taken as the unit of time and following Jackson we place $s = 0{\cdot}25\%$, it will be found that

$$\begin{aligned} 0{\cdot}12256b &\leqslant C(N) &&< 0{\cdot}12625b, \\ 4{\cdot}208b &\leqslant E[C_h(N)] &&< 4{\cdot}458b, \\ 33\tfrac{1}{3} &\leqslant E[m(N)] &&< 34\tfrac{1}{3}. \end{aligned}$$

Thus, for example, if $E(t) = 90$ minutes, the average length of a cycle lies between 50 hours and 51 hours 30 minutes. This example shows that the inequalities given above give reasonable approximations for practical applications.

Exact Evaluation of E[m(N)]

Consider a sequence of demands for stock, the amount of the jth demand being n_j. According to postulate 2, the n_j are identically and independently distributed. Place

$$a_n^1 = P(n_j = n) \qquad n = 1, 2, \ldots$$

where $P(A)$ denotes the probability of occurrence of the event denoted by "A", and so a_n^1 is the probability that in a single demand the demand is for n units of stock. Write

$$a^1(x) = a_1^1 x + a_2^1 x^2 + \ldots + a_p^1 x^p + \ldots,$$

then $a^1(x)$ is the generating function of the distribution for the number of units of stock contained in a demand.

The probability distribution for the total amount of stock requested in k demands has the generating function

$$a^k(x) = a_k^k x^k + a_{k+1}^k x^{k+1} + \ldots + a_{k+p}^k x^{k+p} + \ldots$$

where

$$a^k(x) = [a^1(x)]^k.$$

Define $m(N)$ as the random variable which specifies the number of demands until it first happens that

$$n_1 + n_2 + \ldots + n_{m(N)} \geqslant N.$$

The expected value $E[m(N)]$ of the random variable is to be determined. Since each $n_j > 0$,

$$\begin{aligned} m(N) &= m(N-1) \quad \text{if} \quad n_1 + \ldots + n_{m(N-1)} \neq N-1, \\ &= m(N-1)+1 \quad \text{if} \quad n_1 + \ldots + n_{m(N-1)} = N-1. \end{aligned}$$

If $\delta(N-1)$ denote a random variable which is zero unless

$$n_1 + \ldots + n_{m(N-1)} = N-1,$$

in which case it is equal to unity, then define

$$Q(N-1) = E[\delta(N-1)]$$

where

$$Q(N-1) = a_{N-1}^1 + a_{N-1}^2 + \ldots + a_{N-1}^{N-1}$$

is the probability that $n_1 + \ldots + n_k = N-1$ for some k. Further, it follows from the above that

$$m(N) = m(N-1) + \delta(N-1),$$

hence, taking expected values on both sides of this equation,

$$E[m(N)] = E[m(N-1)] + Q(N-1).$$

Since $E[m(1)] = 1$, by an iteration of the above equation for N and summing it can be seen that, where $Q(0) = 1$,

$$\begin{aligned} E[m(N)] &= Q(0) + Q(1) + \ldots + Q(N-1) \quad \text{for} \quad N \geqslant 1 \\ &= 1 + a_1^1 + (a_2^1 + a_2^2) + \ldots + (a_{N-1}^1 + \ldots + a_{N-1}^{N-1}). \end{aligned}$$

Thus

$$\sum_{r=1}^{\infty} E[m(r)]\, x^{r-1} = \frac{1}{(1-x)\,[1-a^1(x)]}. \tag{13}$$

This result may be stated as follows: $E[m(N)]$ is the coefficient of x^{N-1} in the power series expansion of $\{[1-x]\,[1-a^1(x)]\}^{-1}$. In like manner it may be shown that $Q(N-1)$ is the coefficient of x^{N-1} in the expansion of $[1-a^1(x)]^{-1}$.

The evaluation of $E[m(N)]$ by this method does not seem to lead to easily computable expressions except in the case where the distribution for the n_j is geometric. It is for this reason that the approximate methods of the previous section have been discussed.

Before proceeding to the case of the geometric distribution equation (5) which gives the expected cost of stockholding for a single cycle will be

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proved to be the case. To do this I start from equation (3), which gives explicitly the cost of holding stock due to one cycle:

$$C_h(N) = c[Nt_1 + (N-n_1)\,t_2 + \ldots + (N-n_1-\ldots-n_{m(N)-1})\,t_{m(N)}]. \qquad (3)$$

In order to determine a relation iterative in N, I suppose that initially the stock level had been $(N+1)$ units instead of N, and consider the effect of the same history of demands on the cost of stockholding. Instead of equation (3) the following equations hold:

$$C_h^*(N+1) = c[(N+1)\,t_1 + (N+1-n_1)\,t_2 + \ldots + ((N+1)-n_1-\ldots-n_{m(N)-1})\,t_{m(N)}]$$

when it happens that

$$n_1 + n_2 + \ldots + n_{m(N)} \neq N,$$

and in the case

$$n_1 + n_2 + \ldots + n_{m(N)} = N,$$

$$C_h^*(N+1) = c[(N+1)\,t_1 + (N+1-n_1)\,t_2 + \ldots + (N+1-n_1-\ldots-n_{m(N)-1})\,t_{m(N)}] + ct_{m(N)+1}.$$

Hence

$$C_h^*(N+1) - C_h(N) = c[t_1 + \ldots + t_{m(N)}] + ct_{m(N+1)}\,\delta(N),$$

where $\delta(N)$ is the random variable defined earlier in this section.

Taking expected values of each side of this last equation gives

$$E[C_h^*(N+1) - C_h(N)] = c\{E[m(N)] + Q(N)\}\,E(t) = cE(t)\,E[m(N+1)].$$

The value of $E[C_h^*(N+1)]$ is independent of the particular history of the cycle considered so that $E[C_h^*(N+1)] = E[C_h(N+1)]$ and hence

$$E[C_h(N+1)] = E[C_h(N)] + cE(t)\,E[m(N+1)].$$

Since $E[C_h(1)] = cE(t)$, it follows that

$$E[C_h(N)] = cE(t)\,\{E[m(1)] + \ldots + E[m(N)]\}, \qquad (5)$$

as stated.

The Geometric Distribution

When $a_n^1 = P(n_j = n) = \alpha^{n-1}\beta$, $n = 1, 2$, $\alpha + \beta = 1$, a very simple formula for $E[m(N)]$ can be obtained from the power series expansion (13).

In this case $a^1(x) = \beta x(1-\alpha x)^{-1}$ and it will be found that

$$E[m(N)] = 1 + N - 1/E(n) \qquad (14)$$

with

$$E(n) = 1/\beta.$$

Conversely, it can be shown that the geometric distribution is the only distribution for which (14) is the case. From (14), (5) and (6) there follow the following expressions:

$$E[C_h(N)] = cNE(t)\,[1+\beta(N-1)/2],$$
$$C(N) = cN(N+2\beta-1)/2(\beta+N-1).$$

It is interesting to compare the value of these expressions with the approximate evaluation for the example given above. Computation of the above expressions for this example gives

$$E[m(100)] = 34,$$
$$E[C_h(N)] = 4{\cdot}375b,$$
$$C(N) = 0{\cdot}12542b.$$

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Simulation Techniques in Operational Research

Presented at London School of Economics seminar on Operational Research, 13 June 1957

JOHN HARLING
ORbit (Operational Research) Ltd.

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THE purpose of this paper is to give an introductory account of the techniques of simulation, to present a few of the leading ideas which have been developed, and to draw attention to what is in fact a very open and somewhat ill-defined subject.

Considerable confusion exists over the best terminology to use. The term "Monte Carlo" is presently somewhat fashionable. The term "Simulation" is to be preferred, because it does not suggest that the technique is limited to what is familiar to statisticians as a sampling experiment. By simulation is meant the technique of first setting up a stochastic model of a real situation, and then performing sampling experiments upon the model. The feature which distinguishes a simulation from a mere sampling experiment in the classical sense is that of the stochastic model. Whereas a classical sampling experiment in statistics is most often performed directly upon raw data, a simulation entails first of all the construction of an abstract model of the system to be studied.

In using a simulation to investigate the properties of a real system, the following stages may be distinguished:

1. Reduction of the raw data of the problem to an appropriate form.
2. Construction of a model of the real system which neither over-simplifies the system to the point where the model becomes trivial, nor carries over so many features from the real system that the model becomes intractable and prohibitively clumsy.
3. Bringing the data and the model together in a sampling experiment, whose purpose is to discover how the real system behaves under a variety of prescribed conditions.

At each of these stages, there are several precautions to be taken, and many useful techniques exist for extracting the maximum amount of information from the data available in a reasonable period of time and at a reasonable cost.

Partly because of the confusion of terminology, there is little published literature on simulation, as compared with Monte Carlo techniques. Most

writers have approached the problem either as pure mathematicians who use a sampling experiment to solve a deterministic problem, or as statisticians who properly enough have been concerned with reducing the variance of their sampling experiments. Little has been written about the operational research use of simulation, and such practical applications as have been published seem to make little use of the very valuable work that the theoreticians have done. The tendency seems to have been either to perform "brute force" simulations lacking any sort of stochastic ingenuity, or else to be exclusively preoccupied with the formal problems involved.

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The Components of a Simulation

Although the detailed structure of a simulation obviously depends upon the real system under study, there are a few basic operations which occur irrespectively of the particular system. These are:

1. The supply of random or pseudo-random numbers.
2. The supply of random variates.
3. The need to reduce the variance of the answer as far as possible.

Generation of Random or Pseudo-Random Numbers

The advisability of using true random digits or pseudo-random digits, generated by determinate sequences, has been fully discussed elsewhere.[1]

In the writer's view, the arguments in favour of pseudo-random numbers are decisive, especially where a computer is being used. First of all, pseudo-random numbers can be generated on a machine without placing a heavy demand upon the machine's main store. Secondly, a sequence is always reproducible, and its properties can be thoroughly mapped before it is used in the context of a particular problem.

There is some force, however, in the argument that deterministic sequences may not be wholly random in character, though not for the rather metaphysical reasons which are sometimes given. It may happen, for instance in a queueing problem, that one uses a sequence which has passed the conventional test for randomness but which gives an answer which is known to be wrong. This draws attention to the possible inadequacy of many of the classical statistical tests when applied to random numbers which are to be used in a dynamic simulation. It is therefore always desirable to test the sequence to be used in some sample model whose theoretical properties are known, the "test model" being as similar

as possible to the actual model of the real system. It is always sound to test the components of a model in their operating context.

Various methods exist for the generation of pseudo-random digits, and there are two schools of thought as to how they should be used. The first, as represented by Tocher,[1] maintains that almost any process may be used to generate a crude sequence, and that the resulting sequence may then be refined by a pairing process, or in some other suitable way. The other point of view holds that reliable sequences of long period can be found, and that these sequences can then be used directly, without refinement.

The methods fall under three headings: mid square method; congruential methods, both additive and multiplicative; and miscellaneous methods modulo 2.

All these methods are reviewed in a paper by Taussky and Todd.[2]

The mid square method has been proved to be faulty by Tocher[1] and should probably not be used without refinement. The congruential methods are either of the type:

$$x_{n+1} = kx_n (\text{mod } M)$$

or:

$$F_{n+2} = F_{n+1} + F_n (\text{mod } M).$$

Depending upon the values of k and M, sequences of varying periods may be generated.

The addition, modulo 2, of repeated sequences of different binary numbers has also been used, because it is suitable to the order codes of most binary machines. Not all these sequences have been found to be reliable.

F. G. Foster[1] mentions a method due to Teichroew (the Residue Class method). A random number r is chosen, and then some number x. Retain the k least significant bits of r, rx, rx^2, etc. It can be shown that with a suitable choice of x, the sequence generates all the 2^{k-2} digits of length $k-2$. By this method, a sequence of period of the order of 10^{10} can be generated.

Various transcendental functions have also been tried as sources of random digits, such as the exponential function, but results have not been very satisfactory in terms of the conventional tests for randomness.

Generation of Random Variates

Given a sequence of random digits, the problem here is to use the sequence to gain access to one or more prescribed cumulative density functions; i.e. to use them to make a draw from a known distribution.

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Methods for most of the common distributions are given by Tocher.[1] The way chosen depends upon the particular features of the distribution and the way in which it is related to other distributions. For example, Normal variates may be generated by summing short (5 to 10) sequences of uniformly distributed variables, and Chi-squared variates with n degrees of freedom by summing squares of n normal variates.

For some purposes, a polynomial approximation to the distribution can be used, although this usually entails a longer programme than a method based on the properties of the distribution.

The method chosen in any particular case depends mainly upon balancing machine storage (and access time to store) against the complexity of the programme. If, at one extreme, the programme does not make heavy demands on storage, then it may be advisable to store the desired distribution and use the random digits in a table look-up routine without generating the variates at all. If, on the other hand, storage space is short, then it is usually preferable (and faster) to generate the desired variates, perhaps in blocks.

Variance Reducing Techniques

In any sampling experiment, the importance of obtaining as much information as possible from a limited amount of experimentation is considerable. However, when a large machine is being used, the need to reduce the variance of the answers may become crucial. This problem has attracted a good deal of attention lately, both in this country and in the United States. A useful general summary of the work done is given by Kahn,[2] and the principal contributors have been Tukey[2] in the United States, and Hammersley, Morton and Mauldon in the United Kingdom.[3, 4, 5]

Many of the techniques will be familiar to statisticians in connexion with survey sampling procedures, but perhaps the most important innovation created directly by simulation work is the Antithetic Variate technique of Hammersley, Morton and Mauldon. The basic idea of this technique is that given some estimator t of an unknown parameter y, a second estimator t'' is sought with the following properties:

1. t'' has a negative correlation with t.
2. $E(t'') = y$.

t and t'' are then sampled simultaneously, and $\frac{1}{2}t + \frac{1}{2}t''$ is used as the estimator of y. It can be shown that the sampling variance of this estimator is very considerably less than that of the crude estimator t. The method

has been applied by Hammersley and Morton to simulation studies of the critical size of atomic piles.[5]

It would be interesting to apply this technique to a simulation involving several different variates, such as may be met in queueing problems. How can one secure, for example, that the variance of the total waiting time generated by the system is reduced? In general, this waiting time is the result of repeated draws from an arrival time distribution and a service time distribution, which are statistically independent of one another.

The application of variance reducing techniques to operational research simulations could be extremely valuable; in the writer's experience most of these simulations have been of the crude type and carried out without benefit of economical design.

Types of Simulation

Following a suggestion of Dr. I. J. Good,[2] simulation problems may be roughly classified as in Table 1.

TABLE I

Class	Problem		Model	Type
1	M	$\rightarrow$	S	Ulam, v. Neumann
2	S	$\rightarrow$	M	
3	S	$\rightarrow$	S	"Crude"
4	S	$\rightarrow$	SM	"Sophisticated"

S denotes a stochastic problem or model, and M a mathematical one. Thus, $M \rightarrow S$ means the solution of a determinate mathematical problem by means of a stochastic model.

So far as operational research is concerned, we are interested exclusively in Classes 3 and 4, although much of the impetus was given to the study of simulation by problems of Class 1.

Before 1950, it had been established that many mathematical problems, especially those involving the solution of Integral Equations, could be approximated numerically by means of a sampling experiment of the Monte Carlo type. It is often argued that better numerical methods exist for the solution of this type of problem, but the historical importance of the Class 1 problems was considerable in furthering simulation work generally. In fact, Class 1 problems may not always be as distinct from the operational research problems of Classes 3 and 4 as is sometimes supposed; for instance, it is often possible to set up a queueing problem in terms of

an integral equation of the Fredholm type, which may then be solved either by a standard numerical method, or by a sampling experiment.

Strictly speaking, Class 2 does not entail a simulation at all. It represents the class of problems, many of them operational research ones, which may be solved by analytical means. The work of D. G. Kendall,[6] R. R. P. Jackson[7] and other theoreticians falls into this class.

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Classes 3 and 4 represent the extreme points of a range of simulations. A pure Class 3 simulation does no more than transfer the stochastic features of a real situation into the model, without recourse to any of the techniques discussed above. Class 4, on the other hand, entails a critical study of the real system, and the production of a model which makes use of these techniques, and which incorporates only those features of the real system which affect the solution of the problem.

Anyone who has worked in this field will know that the use of the techniques described earlier can make all the difference between a successful simulation and an incoherent attempt to programme one's own ignorance. The tendency has too often been, among operational research workers, to transfer all the detailed difficulties in the real situation into the model, to pay scant attention to the reliability or precision of the answer, and to hope that a large computer will get one out of his difficulties. Quite apart from the unsatisfactory nature of this approach, it is also likely to cost a lot of money, since machine time is not to be had for nothing.

The difference between the Class 3 and Class 4 approaches is illustrated in Table 2.

TABLE 2

Peature	Crude	Sophisticated
Random Numbers	Store	Generate Pseudo-Random Numbers
Random Variate	Store Distribution	Generate Polynomial
Variance?	Replicate	Variance reducing techniques

Although these differences may be trivial in the case of a small-scale desk calculator simulation, they become of dominant importance on a large machine. Not only does the crude approach make much heavier demands upon the machine's storage capacity, but, generally speaking, the drawing process is prodigal of machine time as compared with the process of function generating. The slow convergence of the sample mean on the population mean has already been noted, but when a large-scale machine simulation is involved, the money saved by the use of a variance reducing technique may be considerable.

It is sometimes forgotten that not every simulation requires an electronic computer. In any case, whether it be decided to run the final simulation on a large machine or not, it is worthwhile to test out the model on a desk calculator before writing a final machine programme. Apart from the information these small programmes provide, it is sometimes possible to discover faults in the model which would be disastrous were they only revealed after the machine programme had been written. As Dr. Howlett[1] and many others have pointed out, it is not always an advantage to have easy access to a large machine.

In most machine simulations, there is a high ratio of organization to arithmetic orders, and the machine spends most of its time deciding rather than calculating. Probably the main machine requirement in a simulation is a large fast-access store. Simulations always require a new programming effort, as opposed to the use of library routines; also, the operational research worker may not be as familiar with the machine as are those in continuous contact with it, so that he requires to concentrate his attention on the programme rather than on the idiosyncrasies of the machine. For these reasons it is most important that the machine be easy to programme, although it may be true that optimally programmed machines are faster than those not requiring optimal programming.

Simulation and Operational Research

Nearly all operational research simulations fall into the last two classes of Table 1, and the rest of the paper will be devoted to these types of simulation.

Although a considerable volume of work has been done by operational research groups both in this country and the U.S.A. on the subject of simulation, there is a dearth of published case histories. The reason for this is probably that a simulation undertaken by an industrial company contains the kind of data that a company does not wish to disclose. Further, the structure of the simulation will often give important clues to the company's future plans. This is much truer of a simulation than of many other operational research studies, because, after all, a simulation is a model of some part of the company's operations and will usually carry over unchanged many of the features of the company's business. The effect of this lack of published information on operational research simulation is unfortunate: the impression is given that the use of simulation is restricted to solving mathematical problems, or else problems which arise in nuclear physics or in some other purely technological context.

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Conditions to be Satisfied by the Problem

It has been often said that a simulation is a last resort. It is certainly true that whenever an operational research problem can be reduced to a simple model and solved analytically, there is no need for a simulation. It is therefore extremely important in examining any particular problem to satisfy oneself that a simple analytical model will not be adequate before considering the possibility of a simulation. Because large machines are so often involved, the cost of a simulation is nearly always high compared with that of solving a small analytical model, and the probable cost and time of the simulation must always be weighed against the amount of information it is likely to produce.

Generally speaking, a simulation should only be used when one is satisfied that a simple theoretical model is wholly inadequate to solve the problem. Even if this is the case, it cannot be too strongly emphasized that every effort should be made to simplify the model before trying to programme it to a machine.

Design of the Model

In some cases, the steps to be carried out are as follows:

1. Reduction of the real system to a logical flow diagram.
2. Identification of the data needed by the model, and their reduction to an appropriate form.
3. Piece by piece test of the model on a desk calculator.
4. Reduction of the flow diagram to the appropriate computer code.
5. Running the model with sufficient replication to ensure the required degree of precision.
6. Interpretation of the results.

The danger with the above approach—that is, starting with a flow diagram of the real operation—is that the model may tend to be too detailed, and that components of the real process will be included which contribute nothing of importance to the stochastic model.

For this reason, it is sometimes more advisable to design the model around the questions to be answered rather than directly as an imitation of the real system. This procedure will tend to keep the purpose of the simulation in perspective, and prevent the model's degenerating into a mass of meandering detail. The tendency is nearly always to simulate too much rather than too little, and to write a machine programme which does not distinguish important factors from trivial ones.

The design of a stochastic model always involves a choice between using frequency or probability distribution of raw data, and the best theoretical fit that can be obtained to these distributions. This question is really a fundamental one, because the use of raw data in a stochastic model implies that all one is doing is simulating the past. For example, if in a queueing problem one were to use observed arrival and service time distributions for 1956, the model would effectively be replicating only that year, and delivering no information about the expected future performance of the system. It is a much stronger assumption that the basic form of the distributions will remain unchanged with time than it is to assume that the idiosyncrasies of a particular year will always be reproduced.

It would be desirable to have *a priori* some measure of a model's sensitivity to the precise form of the distributions it contains. However, this is rarely the case; and in fact if such a measure were available, a simulation of the system might not be needed. This is far from being an academic point, and emphasizes the importance of testing the model stage by stage before assembling the stages into a machine programme. In one queueing simulation, the total annual waiting time generated by the system was extremely sensitive to the precise form of the arrival distribution, but insensitive to that of the service time. In particular, the presence or absence of a long positive tail on the service time distribution made almost no difference to the total waiting time generated, in spite of the fact that the system was critically loaded.

The importance of good model design is often overlooked, especially by those who believe that an operational research problem can be trivially translated into a machine programme by anyone familiar with a particular machine.

Typical Problems

In principle, any system may be simulated provided that a stochastic model of the system can be realistically set up.

The relevance of this to industrial problems is immediate, since the requisite uncertainty is never in short supply. Arrivals, whether of ships at a port, customers in a store, or goods in a warehouse, nearly always have a stochastic character. Similarly, service time, sales and production throughput time are also distributed quantities. It follows that most industrial simulations will have the same underlying structure, whether they are designed to represent queues, stocks, or the flow of materials through a production process.

In practice, the systems which have been most frequently studied by operational research workers have been of three kinds: Waiting line systems with multiple service points; Finished stock systems with complex constraints on the input to stock; Production line problems involving feedback.

Under the first heading, mention must be made of the work done by various companies on the provision of port facilities with the purpose of

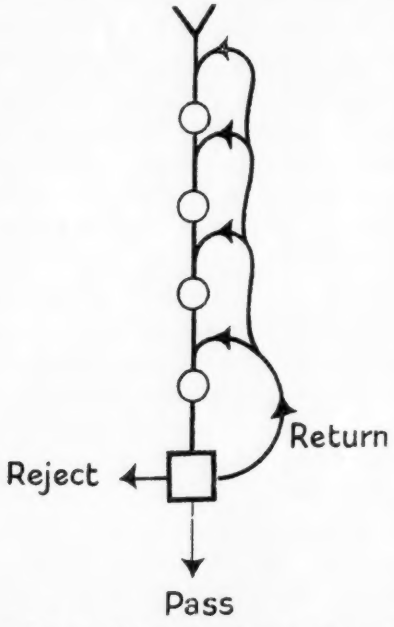

FIGURE 1. *A production line of which the behaviour can be simulated. Circles indicate production stages, the square indicates inspection.*

minimizing the total cost of these facilities, plus waiting time incurred by ships. The main reason for resorting to a simulation in this case has been the complicated nature of the berthing constraints, which would be prohibitively difficult to incorporate into a model to be solved analytically.

Simulation methods have been applied to a wide variety of Stock Control problems. In one case, it was required to determine the most economic level of refinery finished tankage in order to support recurring throughput on the one hand and stochastic demand for products by tankers on the other. Again, the constraints on the input side were complicated enough to

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rule out an analytical solution to the problem. In this case, the least cost balance was sought between the cost of tankage and refined products on the one hand, and ship waiting on the other.

A variety of problems associated with production lines have also been studied. For example, it is possible to simulate the behaviour of a production line of the type shown in Figure 1.

Because of the presence of the feedback loops from inspection to the various process stages, the problem is completely intractable to analytical methods. If appropriate assumptions are made with regard to the input rate per unit time, the distribution of machine throughput rates and down time, and the inspection criteria for pass, reject, and return, then the response of the line to various levels of loading can be evaluated. Hence, the effect of different inspection criteria upon the quantity of work in progress in the line can be found.

An Example of an Industrial Simulation

The purpose of this simulation was to discover the most economic level of pier wharfage facilities to provide for barges bringing grain and removing finished milled products from the same group of wharves. Clearly, if insufficient wharfage is provided, then barges will have to wait and incur demurrage. On the other hand, if an over-investment in wharfage capacity is made, then there will be long periods during which the piers are idle. Any calculation based upon the mean arrival rate of barges will inevitably fail to take account of the bunching which takes place in the arrivals. Although the correct average arrival density may be known, wharfage capacity based upon this alone will lead to periods of time during which barges must wait for a wharf, and also periods during which wharves are waiting for barges. Both of these eventualities cost money, and it was therefore required to determine what the most economic level of investment in wharfage facilities might be.

In addition, because of restricted loading and unloading facilities, not every type of cargo could be handled at every wharf.

This problem is clearly of the waiting line type, and requires two distributions for its solution: Distribution of arrivals per unit time; distribution of alongside time.

It is required to find the response of the system in total waiting hours per year, when the system is subjected to a plausible range of future arrival densities of barges. The control variable is the number of wharves.

A year's data on arrival times and alongside times were examined, and although some scheduling of the barges was attempted, the arrival distribution was found, according to the χ^2 test, to be Poisson, and the alongside time to be almost Normal. A first model was set up which ignored the berthing constraints with the purpose of testing *in situ* whether the actual arrival and service time distributions generated the same number of waiting hours per year as the theoretical distributions.

A small experiment was designed as shown in Table 3:

TABLE 3

Experiment	Arrival	Service Time
1	*T*	*A*
2	*T*	*T*
3	*A*	*T*
4	*A*	*A*

where *A* denotes the actual arrival or service time distribution, and *T* the corresponding theoretical one.

Each experiment was run and replicated five times. No significant difference was observed between Experiments 3 and 4, nor between Experiments 1 and 2. From this, it was concluded that the Normal distribution was an adequate representation of the facts. On the other hand, there was a 1·5 to 1 discrepancy in waiting time between Experiment 2 and Experiment 3, and also between Experiments 1 and 4. This indicated that although the arrival distribution appeared to be indistinguishable from the Poisson one in terms of the χ^2 test, it none the less failed in the context of the problem to represent the true state of affairs. This last discrepancy underlines the need for a careful examination of the components of the model before putting it together on the machine.

The waiting time generated by the theoretical arrival distribution in Experiment 1 was consistently larger than that of Experiment 4. The inference to be drawn from this was that some vestige of the scheduling process from the docks up and down stream remained in the pattern of arrivals, which were in some sense "less than random".

It was therefore decided to apply controls to the Poisson arrival distribution in order to generate the same amount of waiting time in Experiment 2 as had been produced by the actual arrival distribution in Experiment 3. The details of this control mechanism need not concern us any further here, but it was so arranged that the Experiment 2 waiting time lined up with that of Experiment 3 over the necessary range of arrival densities.

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A forecast of future arrival densities was obtained from the Company and the barge operators, and the completed model, with berthing constraints built into it, was run for various arrival densities and various numbers of wharves.

By this means, it was possible to relate the cost of investment in wharves to the expected total waiting time of the barges at various arrival densities. Provided that the unit cost of waiting and the unit cost of a new wharf are both known, then the most economical level of investment in wharfage at any given level of arrival density can be calculated.

This example is admittedly of the Class 3 or "Crude" type. The only defence that can be offered is that a reasonable answer was called for in a not quite so reasonable time.

In addition to experiments of Table 3, an investigation was made of the effect on total annual waiting time of the standard deviation of the service time distribution, keeping the same arrival distribution. Although, as expected, the total waiting time depended sensitively upon the mean of the service time distribution, it was quite insensitive to changes in the standard deviation over a 2 to 1 range. This result was somewhat surprising, since one feels intuitively that the occurrence of a small number of very long service times should affect the total waiting time considerably. The Traffic Intensity of the system was close to unity, and so this result becomes even more surprising.

It is felt that small deviations from true randomness in the arrival distribution may be fairly common in industrial problems, because there is usually some scheduling of arrivals attempted. This is a question which should be of some interest to theoreticians.

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Cyclic Queues

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The usual queue or interference problem is stated, examined and resolved for closed loop operations; i.e., a unit which has completed service in a number of series-parallel service stages rejoins the queue at the initial stage. The important results can be stated in terms of a function Z_M^N,

$$Z_M^N = \sum_{\text{All partitions of } N} x_1^{n_1} x_2^{n_2} \dots x_M^{n_M}, \quad \sum_{i=1}^{M} n_i = N$$

$$Z_M^N = \sum_{i=1}^{M} \frac{x_i^{N+M-1}}{\prod_{\substack{j=1 \\ j \neq i}}^{M} (x_i - x_j)}.$$

The problem arises in a study of conventional mechanized coal mining operations and examples based on actual data are cited. The results can be applied to such problems as bunching in public transport systems and the overhaul-spare parts problem in fleet operations.

WHILE research in queueing theory has been extensive, a small and relatively simple problem appears to have been overlooked. Most queueing studies are based on the assumption that the arrival rate (and distribution) is independent of the service rate.[1] Burke[2] has shown that in the case of multiple series service, the arrival rate is the same for all stages. Jackson[3, 4] has examined multiple service systems (series and parallel combinations) for a finite number of customers and evaluated service standards which have been applied to the aircraft—repair base—stores problem.

This paper deals with service standards and output for a closed loop queueing system which services a finite number of customers. A customer who has completed service at the Mth stage rejoins the queue at the first stage. The calculations are based on the assumption of an exponential service time distribution. The results for such a "cyclic queue" can be compared with those of Jackson.[3] The two studies differ only in that for cyclic queues one assumes no fixed arrival rate.

This work is the result of a preliminary examination of "conventional" mechanized deep mining operations. In these operations a "section", which consists of a group of specialized machines and their complement,

works on a number of faces in succession. The operations are cyclic; for example, at Mine No. 22 of the Old Ben Coal Corporation, Benton, Illinois, a face is worked in sequence by a cutting machine, a drilling machine, a blasting crew, a loading machine group (one loader and one or more shuttlebuggies), and a timbering or roof bolting machine. Each machine proceeds to the next face after completing its task. The time to complete a particular task is not constant but is subject to some random variation. Machine breakdowns and transit time also contribute to a random service time distribution.

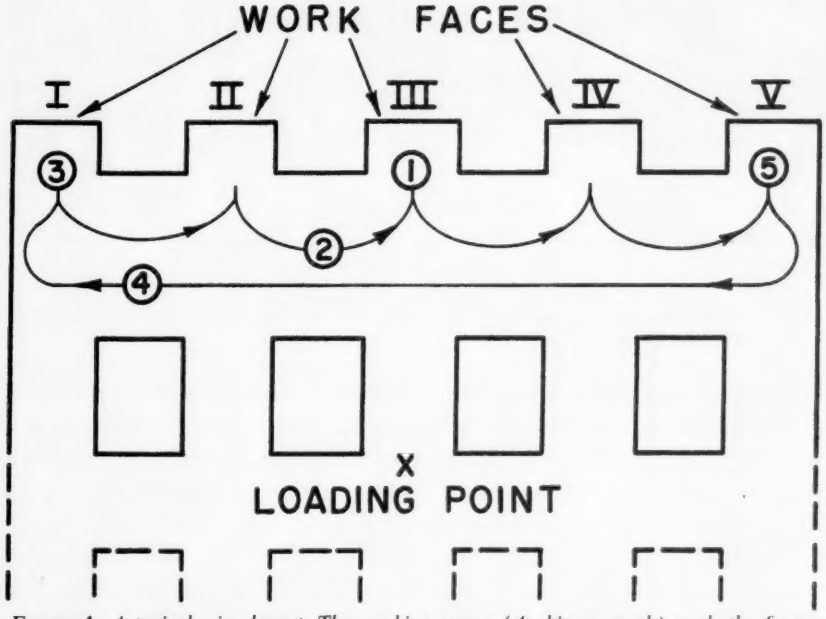

FIGURE 1. *A typical mine layout. The machine groups (Arabic numerals) work the faces (Roman numerals) in cyclic order. 1 indicates machine location at an arbitrary instant of time.*

It is apparent that the system has the aspects of a queue system with a particular queue discipline. One could attempt to approach the problem directly, i.e., machines queueing up to serve faces with the machines in proper sequence. An alternative approach is to consider the machines as fixed in sequence and position and the faces queueing for service in "first come—first served" order. It is the latter method which has been used, since it allows certain simplifications in formulation.

Figure 1 is a "map" of the working area in a typical coal mine. Five faces (Roman numerals) are being worked on by five machines (Arabic

numerals). The machines operate in a prescribed sequence in cyclic order. Figure 2 is the analogue used in this study. The machines are fixed and the faces pass through the production system in cyclic order. The numerals represent the system at the same instant of time in both figures.

The operations can be represented as a closed queue with N customers. The customers receive order of arrival service from M machines. Upon completion of service, the customer rejoins the queue at stage 1. Since the system is cyclic it can be considered as beginning at any stage and ending at the stage immediately preceding the first.

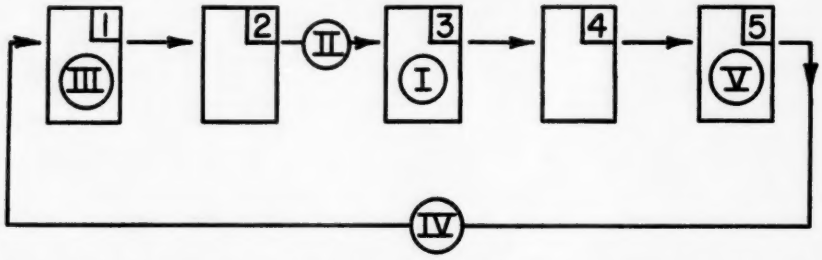

FIGURE 2. *The conceptual model of mining operations. The machine groups are considered fixed and the faces move through the "production line" in cyclic order. The system is in the same state as is shown in Figure* 1, *and* II *indicates position of faces at same instant of time.*

Although the problem equations and solutions have been framed in terms of conventional coal mining practice, they have applications in other fields. Among these, one might mention the overhaul/spare parts problem in fleet operations, certain transportation problems, and bunching in public transportation systems.

Analysis of the Cyclic Queue

We consider a system with M sequential stages in a loop: each stage acts as a single server. The system serves N units, each of which goes through all stages in succession and continuously repeats the process.

A state is indicated by $(n_1, n_2 \ldots n_M)$ where $\sum_{i=1}^{M} n_i = N$ and n_i indicates the number of units being served or waiting for service at the ith stage. A waiting unit has completed service at the $(i-1)$st stage. The probability of being in a state $(n_1, n_2, \ldots n_M)$ is written as $P(n_1, n_2 \ldots n_M)$.

Transitions between states occur when a unit enters or leaves a stage. We shall assume that the service rate in each stage is a random variable

which can be described by a mean value μ_i and a distribution $f(\mu_i)dt$, which is exponential; i.e.

$\mu_i \Delta t$ = probability that a unit being serviced has completed in the time interval between t and $t+\Delta t$ (μ_i = mean service rate).

The probability that the system is in the state $(n_1, n_2, \ldots n_M)$ at $t = t+\Delta t$ is expressed as the sum of probability terms. Since we assume that transitions only occur from nearby states, they can occur when the system was in the states $(n_1, n_2, n_{i-1}+1, n_i-1, \ldots n_M)$ and $(n_1, n_2, \ldots n_i+1, n_{i+1}-1, \ldots n_M)$, at the time t. Then

$$P(n_1, n_2, \ldots n_M)(t+\Delta t) = P(n_1, n_2, \ldots n_M)[1-(\mu_1+\mu_2 \ldots \mu_M)\Delta t] + P(n_1+1, n_2-1, n_3, \ldots n_M)\mu_1 \Delta t + P(n_1, n_2+1, n_3-1, \ldots n_M)\mu_2 \Delta t + \ldots + P(n_1-1, n_2, n_3, \ldots n_M+1)\mu_M \Delta t. \quad (1)$$

This can be written[5] as

$$\frac{P(n_1, n_2, \ldots n_M)(t+\Delta t) - P(n_1, \ldots n_M)t}{\Delta t} = -\sum_{i=1}^{M} \mu_i P(n_1, n_2, \ldots n_M) + \mu_1 P(n_1+1, n_2-1, \ldots n_M) + \mu_2 P(n_1, n_2+1, n_3-1, \ldots n_M) + \ldots + \mu_M P(n_1-1, n_2, n_3, \ldots n_M+1). \quad (2)$$

The left-hand side is the defined derivative of $P(n_1, n_2, \ldots n_M)$, so one can write

$$\frac{dP(n_1, n_2, \ldots n_M)}{dt} = -\sum_{i=1}^{M} \mu_i P(n_1, n_2, \ldots n_M) + \sum_{i=1}^{M} \mu_i P(n_1, n_2, \ldots n_i+1, n_{i+1}-1, n_{i+2}, \ldots n_M), \quad (3)$$

subject to the following restrictions

(a) $\sum_i n_i = N$,

(b) if $n_i = 0$ the μ_i term = 0 {term does not enter the equation},

(c) if $n_{i+1}-1 < 0$ the μ_i term = 0,

(d) the Mth stage is linked back to the first stage; i.e., transitions occur from $(n_1-1, n_2, \ldots n_M+1)$ to $(n_1, n_2, \ldots n_M)$.

We are interested in the steady-state condition; i.e.,

$$\frac{d}{dt} P(n_1, n_2, \ldots n_M) \equiv 0.$$

Thus there are $\left(\frac{N+M-1!}{(M-1)!\,N!}\right)$ equations in $\left(\frac{(N+M-1)!}{(M-1)!\,N!}\right)$ unknowns.

$\left[\frac{(N+M-1)!}{(M-1)!\,N!}\right.$ = number of ways of putting N things in M boxes any number to a box (see Mayer and Mayer,[6] p. 438).$\Big]$

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The solutions are of the form

$$P(n_1, n_2, \ldots n_M) = \frac{\mu_1^{N-n_1}}{\mu_2^{n_2}\mu_3^{n_3}\ldots\mu_M^{n_M}} P(N, 0, 0, \ldots), \tag{4}$$

where
$$\sum_{\text{All partitions of } N} P(n_1, n_2, \ldots n_M) = 1. \tag{5}$$

Then writing

$$P(N, 0, 0, 0, \ldots 0) = \frac{1}{Z_M^N}, \tag{6}$$

$$Z_M^N = \sum_{\text{All partitions of } N} \frac{\mu_1^{N-n_1}}{\mu_2^{n_2}\mu_3^{n_3}\ldots\mu_M^{n_M}},$$

$$Z_M^N = \sum_{\text{All partitions of } N} \left(\frac{\mu_1}{\mu_2}\right)^{n_2}\left(\frac{\mu_1}{\mu_3}\right)^{n_3}\ldots\left(\frac{\mu_1}{\mu_M}\right)^{n_M}. \tag{7}$$

$$Z_M^N = \sum_{\text{All partitions of } N} x_1^{n_1} x_2^{n_2} x_3^{n_3} \ldots x_M^{n_M} \tag{7a}$$

where

$$x_i = \frac{\mu_1}{\mu_i}; \quad x_1 = 1 \text{ (added for symmetry and completeness).}$$

Note that μ_1 can be selected arbitrarily for convenience in computations.

Now, given a particular machine, say i, it is working whenever $n_i \geq 1$. The fraction of time it does not work is given by

$$D_i = \sum_{\Sigma n_i = N} P(n_1, n_2, \ldots n_{i-1}, 0, n_{i+1}, \ldots n_M)$$

$$D_i = \frac{1}{Z_M^N} \sum_{\text{All partitions of } N} x_1^{n_1} x_2^{n_2} \ldots x_{i-1}^{n_{i-1}} x_{i+1}^{n_{i+1}} \ldots x_M^{n_M} = \frac{{}_iZ_{M-1}^N}{Z_M^N} \tag{8}$$

where the i in ${}_iZ_{M-1}^N$ indicates that x_i is omitted from the summation.

The probability that a stage i is working is then given by $1 - D_i = U_i$. The output of the stage in number of faces per unit time is

$$\theta_i = (1 - D_i)\,\mu_i = U_i\,\mu_i \tag{9}$$

and since each stage must produce the same output

$$\theta_i = \theta_j \text{ (for all } i \text{ and } j). \tag{10}$$

The mean number of units at the ith stage is given by

$$\bar{n}_i = \sum_{\text{All partitions of } N} n_i P(n_1, n_2, \ldots n_i, \ldots n_M)$$

$$\bar{n}_i = \frac{1}{Z_M^N} \sum_{\text{All partitions of } N} n_i x_1^{n_1} x_2^{n_2} \ldots x_i^{n_i} \ldots x_M^{n_M}$$

$$\bar{n}_i = \frac{x_i}{Z_M^N} \frac{dZ_M^N}{dx_i}. \tag{11}$$

The mean number of units awaiting service at the ith stage is given by

$$\bar{w}_i = \sum_{\substack{\text{All partitions of } N \\ n_i \geq 1}} (n_i - 1) P(n_1, n_2, \ldots n_M)$$

$$= \frac{1}{Z_M^N} [\bar{n}_i Z_M^N - Z_M^N + {}_iZ_{M-1}^N]$$

$$= [\bar{n}_i - 1 + D_i] = \bar{n}_i - U_i. \tag{12}$$

The utilization factor is $1 - D_i = U_i$ since this represents the ratio of working time to total process time. This is different from the definition in the usual queue case, but has the same basic meaning. The utilization factor of a cyclic queue is not constant, but a function of the service rates of the various stages.

It is to be noted that the state probabilities as well as the output, etc., are independent of the relative positions of the machines; i.e., for a given set of service rates there is a solution regardless of the work sequence. Similar results have been obtained by Finch[7] in an analysis of the role of working stores in an assembly line subject to random breakdowns.

Balanced Machines

In the special case of balanced machines; i.e., where

$$\mu_1 = \mu_2 = \ldots = \mu_M = \mu$$

$$x_1 = x_2 = \ldots = x_M = x = 1$$

$$Z_M^N = \sum_{\text{All partitions of } N} x_1^{n_1} x_2^{n_2} \ldots x_M^{n_M} = \frac{(N+M-1)!}{(M-1)!\,N!}. \tag{13}$$

Now, a machine or stage i is idle when $n_i = 0$. The probability that stage i is idle (the same for all stages) is from Eq. (8)

$$D_i = \frac{1}{Z_M^N} Z_{M-1}^N = \frac{(N+M-2)!}{(M-2)!\,N!} \cdot \frac{N!(M-1)!}{(N+M-1)!}$$

$$D = D_i = \frac{M-1}{N+M-1}. \tag{14}$$

The mean number of units at the ith stage (same for all stages) is by definition,

$$\bar{n} = \bar{n}_i = \frac{1}{Z_M^N} \sum_{n_{i-1}}^{N} n_i x_i^{n_i} \sum_{\text{All partitions of } N} x_1^{n_1} x_2^{n_2} \dots x_M^{n_M}.$$

Using Eq. (13),

$$\bar{n} = \bar{n}_i = \frac{1}{Z_M^N} \sum_{n_{i-1}}^{N} n_i \frac{(N+M-n_{i-2})!}{(M-2)!(N-n_i)!}$$

$$= \frac{1}{Z_M^N} \cdot \frac{(M+N-1)!}{M!(N-1)!}$$

$$\bar{n} = \bar{n}_i = \frac{N}{M}. \tag{15}$$

This result is obvious from the statement of the problem. Since on the average each stage will have the same number of units being served or awaiting service. The number of units waiting at the ith stage (again, the same for all stages) is, by definition

$$\bar{w}_i = \frac{1}{Z_M^N} \sum_{\substack{\text{All partitions of } N \\ n_i > 1}} (n_i - 1) x_1^{n_1} x_2^{n_2} \dots x_M^{n_M}$$

$$\bar{w}_i = \bar{n}_i - 1 + \frac{M-1}{N+M-1}$$

$$\bar{w}_i = \frac{1}{Z_M^N} (\bar{n}_i Z_M^N - Z_M^N + Z_{M-1}^N)$$

$$\bar{w} = \bar{w}_i = \frac{N(N-1)}{M(N+M-1)}. \tag{16}$$

Then the delay at the ith stage is

$$\frac{w_i}{\mu_i} = \frac{w}{\mu}. \tag{17}$$

The mean cycle time (the time to complete service at all M stages) is then given by

$$T = \frac{M}{\mu} + \frac{M\bar{w}}{\mu}$$

$$T = \frac{1}{\mu}\left(M + \frac{N(N-1)}{(N+M-1)}\right). \tag{18}$$

D_i is defined as the probability that a stage is idle. Then the probability that a stage is working (or the fraction of time that it works) is $(1 - D_i)$. The work output rate is then

$$(1 - D_i)\mu_i = (1 - D)\mu = \frac{N}{N+M-1}\mu. \tag{19}$$

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Then if there are H work hours, and μ is the work rate per hour (in units), the daily or shift output is

$$\theta_1 = H\left(\frac{N}{N+M-1}\right)\mu. \tag{20}$$

Table I shows the results for balanced machines with $M = 3$, 4, 5, and 6. The case $M = 3$ can be compared with those calculated by Jackson[3] for the finite customer multi-stage queue with $M = 3$ (Jackson does not include the only case for which the utilization factors are equal; $N = 4$, $U = 0{\cdot}667$).

Solutions in the General Case

The important factors can be determined, in the general case ($\mu_i \neq \mu_j$ for all i and j), once the Z_M^N are determined;

$$Z_M^N = \sum_{\text{All partitions of } N} x_1^{n_1} x_2^{n_2} \dots x_M^{n_M}$$

$$Z_M^N = \sum_{i=1}^{M} \frac{x_i^{N+M-1}}{\prod_{\substack{j=1 \\ j\neq i}}^{M} (x_i - x_j)}.$$

The values for $M = 1$ to 5 are given below.

$$Z_1^N = x_1^N, \tag{21}$$

$$Z_2^N = (x_1^{N+1} - x_2^{N+1})/(x_1 - x_2), \tag{22}$$

$$Z_3^N = \frac{x_1^{N+2}}{(x_1-x_2)(x_1-x_3)} - \frac{x_2^{N+2}}{(x_1-x_2)(x_2-x_3)} + \frac{x_3^{N+2}}{(x_1-x_3)(x_2-x_3)}, \tag{23}$$

$$Z_4^N = \frac{x_1^{N+3}}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} - \frac{x_2^{N+3}}{(x_1-x_2)(x_2-x_3)(x_2-x_4)} + \frac{x_3^{N+3}}{(x_1-x_3)(x_2-x_3)(x_3-x_4)} - \frac{x_4^{N+3}}{(x_1-x_4)(x_2-x_4)(x_3-x_4)}, \tag{24}$$

$$Z_5^N = \frac{x_1^{N+4}}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)} - \frac{x_2^{N+4}}{(x_1-x_2)(x_2-x_3)(x_2-x_4)(x_2-x_5)} + \frac{x_3^{N+4}}{(x_1-x_3)(x_2-x_3)(x_3-x_4)(x_3-x_5)} - \frac{x_4^{N+4}}{(x_1-x_4)(x_2-x_4)(x_3-x_4)(x_4-x_5)} + \frac{x_5^{N+4}}{(x_1-x_5)(x_2-x_5)(x_3-x_5)(x_4-x_5)}. \tag{25}$$

One should remember that the subscript notation can be selected arbitrarily but one must be consistent in notation; e.g., to determine ${}_1Z_3^N$, x_2 replaces x_1, x_3 replaces x_2, x_4 replaces x_3 in Eq. (23).

TABLE 1

RESULTS FOR 3 TO 6 MACHINE SYSTEM FOR VARIOUS NUMBERS OF CUSTOMERS

Legend:

N = Number of units in system.
$\bar{n}$ = Mean number of units at each stage (waiting or being serviced).
$\bar{w}$ = Mean number of units waiting at each stage.
T = Time for a unit to complete M stages.
U = Utilization factor (fraction of time each stage is operating).
$\bar{S}$ = Average number of customers being served in the system.

N	$\bar{n}$	$\bar{w}$	T	$U = 1-D$	$\bar{S}$
			$M = 3$		
3	1	0·4	4·2	0·6	1·8
4	1·33	0·67	5·0	0·67	2·0
5	1·67	0·95	5·85	0·71	2·13
6	2·00	1·25	6·75	0·75	2·25
11	3·67	2·83	11·49	0·85	2·55
16	5·33	4·45	16·35	0·89	2·67
21	7·00	6·35	22·05	0·91	2·73
			$M = 4$		
3	0·75	0·25	5·0	0·5	2·00
4	1·0	0·43	5·72	0·571	2·28
5	1·25	0·625	6·50	0·625	2·50
6	1·50	0·83	7·32	0·67	2·66
10	2·50	1·73	10·92	0·77	3·08
11	2·75	1·97	11·88	0·787	3·13
15	3·75	2·91	15·64	0·835	3·34
16	4·00	3·15	16·60	0·845	3·38
20	5·00	4·11	20·44	0·870	3·48
21	5·25	4·37	21·48	0·875	3·50
			$M = 5$		
3	0·6	0·17	5·85	0·43	2·15
4	0·8	0·30	6·50	0·50	2·50
5	1·0	0·445	7·22	0·555	2·775
6	1·2	0·60	8·00	0·60	3·00
9	1·8	1·10	10·50	0·70	3·50
10	2·0	1·28	11·40	0·72	3·60
11	2·2	1·47	12·35	0·73	3·65
15	3·0	2·21	16·05	0·79	3·95
16	3·2	2·40	17·00	0·80	4·00
20	4·0	3·16	20·80	0·84	4·20
21	4·2	3·36	21·80	0·84+	
			$M = 6$		
3	0·5	0·125	6·75	0·375	2·25
4	0·667	0·223	7·34	0·444	2·66
5	0·833	0·333	8·00	0·500	3·00
6	1·0	0·455	8·73	0·545	3·27
10	1·667	1·00	12·00	0·67	4·00
11	1·833	1·146	12·86	0·687	4·12
15	2·50	1·750	16·50	0·750	4·50
16	2·667	1·905	17·45	0·762	4·57
20	3·333	2·533	21·20	0·800	4·80
21	3·500	2·692	22·10	0·808	4·85

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The D_i are obtained, as defined in Eq. (8), by ratios of the Z_M^N's. $\bar{n}_i$ is obtained from the derivatives with respect to x_i, and w_i includes both $\bar{n}_i$ and D_i.

In mining operations D_i is probably the most important factor since it determines the output of the system. $\bar{w}_i$ can be important if space is at a premium, as may be the case when N is small (N is generally greater than M in conventional mining).

The Degenerate Case

Equations (21–25) are singular when $x_i = x_j (\mu_i = \mu_j)$. This will be referred to as the degenerate case. When all machines are balanced, the system can be called completely degenerate; but as has already been shown, the solutions can be found quite readily. In the case of partial degeneracy a general form of the solution is not so apparent but the algebraic manipulation required is not formidable.

As an example, consider a 4-stage system ($M = 4$) in which three of the stages are balanced ($x_1 = x_2 = x_3$) and the fourth has a higher service rate; i.e., $x_4 = 0{\cdot}8x_1$ or ($\mu_4 = 1{\cdot}25\mu_1$). As before

$$Z_M^N = \sum_{\text{All partitions of } N} x_1^{n_1} x_2^{n_2} x_3^{n_3} x_4^{n_4}. \tag{26}$$

Using the results developed for balanced machines this becomes

$$Z_M^N = \sum_{n_4=0}^{N} x_4^{n_4} \frac{(N+M-n_4-2)!}{(M-2)!\,(N-n_4)!}. \tag{27}$$

In a similar manner,

$${}_iZ_{M-1}^N = \sum_{n_4=0}^{N} x_4^{n_4} \frac{(N+M-n_4-3)!}{(M-3)!\,(N-n_4)!}, \quad (i = 1, 2, 3) \tag{28}$$

and ${}_4Z_{M-1}^N$ is given by

$${}_4Z_{M-1}^N = \frac{(N+M-2)!}{(M-2)!\,N!}. \tag{29}$$

When $N = 6$,

$$\begin{aligned} (1 - D_4) &= 0{\cdot}564 \\ (1 - D_i) &= 0{\cdot}701 \quad (i = 1, 2, 3). \end{aligned} \tag{30}$$

Application of the Results

Standard time data have been obtained from two "conventional" mines. Both of these are mined in the manner described as "Room and Pillar" in mining texts. Of the two mines under consideration here, the first (Deerfield Mine of the Pochahontas Fuel Corporation, Pochahontas,

West Virginia) works a 58-in. seam of metallurgical coal. The second (Mine No. 22, Old Ben Coal Corporation, Benton, Illinois) works a 7–12 ft. seam of Illinois No. 6 coal. (The time data in the latter case are based on a 100-in. seam.)

At Deerfield Mine, the section under study works six faces in four stages (cutting and drilling are a single operation). Each face is cut, 18 ft. wide by 8·0 ft. deep by 4·8 ft. high, and the average cut is 28·8 tons. The standard times (t_i), service rate ($\mu_i = 60/t_i$), and the utilization of each stage [$U_i = (1 - D_i)$] are listed in Table II for 5, 6, and 7 face operation. The output of the system increases from 45·8 tons/hr. to 50·6 tons/hr. in going from 5 to 7 face operations.

TABLE II

CALCULATED RESULTS BASED ON DATA FROM DEERFIELD MINE

Job	Time per Face (min.)	Service Rate (faces/hr.)	% Working Time or % Utilization $N = 5$	$N = 6$	$N = 7$
Loading	25·24	2·3772	67·0	71·3	74·3
Cutting	24·50	2·4490	65·0	69·1	72·5
Bolting	23·83	2·5178	63·1	67·2	70·2
Shooting	20·29	2·9571	53·8	57·2	59·9
	Output (tons/hr.)		45·8	49·0	50·6
	Actual Output (tons/hr.)		—	48·5	—

Section No. 6 at Mine No. 22, Old Ben Coal Corporation, works 6 faces in 5 stages (considerably more drilling is required). The average cut is about 50 tons. The results are tabulated in Table III for 6, 7, and 9 face operations.

TABLE III

CALCULATED RESULTS BASED ON DATA FROM OLD BEN NO. 22

Job	Time per Face (min.)	Service Rate (faces/hr.)	% Working Time or % Utilization $N = 6$	$N = 7$	$N = 9$
Blasting	28·1	2·135	72·3	76·4	84·3
Loading	25·6	2·344	65·9	69·6	76·8
Cutting	23·0	2·609	59·2	62·5	69·0
Drilling	19·8	3·030	51·0	53·8	59·4
Bolting	17·4	3·448	44·8	47·3	52·2
	Output (tons/hr.)		77·25	81·54	90·04
	Actual Output (tons/hr.)		80·0	—	—

In all cases (general, partial degeneracy, complete degeneracy) the output increases with increasing N. The rate of change of increase decreases with increasing N and is limited by the service rate of the slowest machine.

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Results of this type are characteristic of queueing problems. It is important to remember that all calculations have been based on the assumption that the service rates are independent of N. There are other factors which must, of course, be considered in applying the results to a particular operation. These will be discussed in a later section.

One can, however, compare the output at Deerfield Mine under the assumptions of complete degeneracy, partial degeneracy, and the general case. The standard times and service rates are listed in Table II which indicates an output of 49 tons/hr. in the general case.

If one assumes that all stages operate at the service rate of the slowest machine ($\mu_1 = 2{\cdot}3772$ faces/hr.), Table I indicates that $M = 4$, $N = 6$, the utilization ($U = 1 - D$) is 0·67, and the output is

$$\theta = U \times \mu_i = 1{\cdot}585 \text{ faces/hr.}$$

equivalent to 45·65 tons/hr.

Under the assumption of equal service times for loading, cutting, and bolting ($\mu_1 = \mu_2 = \mu_3 = 2{\cdot}377$) and $\mu_4 = 1{\cdot}25\mu_1$, the results have been obtained, in part, in the previous example of partial degeneracy (Eq. 30)

$$\theta = U \times \mu_1 = 1{\cdot}667 \text{ faces/hr.}$$

equivalent to 48·0 tons/hr.

The simplifying assumption of partial degeneracy results in an error of about 2 per cent; the assumption of total degeneracy produces an error of about 7 per cent. Looking at the results in yet another way, one can evaluate the value of increasing the service rate of a machine. In going from the balanced machine case to the partial degeneracy case (speeding up the service rate of one machine) increases the output from 45·65 tons/hr. to 48·0 tons/hr. Provided that all service rates are fixed and costs are not increased, an unbalanced system can be more productive than a balanced one.

Parallel Service at One Stage

The solutions for parallel service can be developed in a manner similar to that described previously. The basic equations differ slightly. In a case of immediate interest where the jth stage has two service points μ_j is replaced by $2\mu_j$ when $n_j \geq 2$. Otherwise the equations are unchanged.

For two service points at the jth stage, the state probabilities are given by

$$P(n_1, n_2, \ldots, n_M) = x_1^{n_1} x_2^{n_2} \ldots x_M^{n_M} P(N, 0, \ldots, 0), \quad (n_j \leq 1) \tag{31}$$

$$P(n_1, n_2, \ldots, n_M) = x_1^{n_1} x_2^{n_2} \ldots, \frac{x_j^{n_j}}{2!\, 2^{n_j-2}} \ldots, x_M^{n_M} P(N, 0, \ldots, 0), (n_j \geq 2). \tag{32}$$

These equations are similar to those of Jackson.[4]

In this case

$$P(N, O, \ldots, O) = 1/Z_M^N, \tag{33}$$

and Z_M^N can be shown to be given by

$$Z_M^N = 2Z_M^N - {}_jZ_{M-1}^N \tag{34}$$

where Z_M^N is evaluated with $x_j/2$ replacing x_j. The probability the machines j are not working is then given by D_j°, where

$$D_j^\circ = \frac{{}_jZ_{M-1}^N}{Z_M^N} = \frac{{}_jZ_{M-1}^N}{2Z_M^N - {}_jZ_{M-1}^N}. \tag{35}$$

The probability that only one machine is working at stage j is given by D_j', where

$$D_j' = \frac{x_j\,{}_jZ_{M-1}^{N-1}}{Z_M^N} = \frac{x_j\,{}_jZ_{M-1}^{N-1}}{2Z_M^N - {}_jZ_{M-1}^N}. \tag{36}$$

Then the output of stage j is

$$\begin{aligned}\theta_j &= 2\mu_j(1 - D_j^\circ - D_j') + \mu_j D_j' \\ &= 2\mu_j(1 - D_j^\circ - D_j'/2),\end{aligned} \tag{37}$$

and $\theta_i = \theta_j$ as before.

Table IV shows the results for Mine No. 22, Old Ben Coal Corporation, for $N = 6$ and 9 under the assumption that the slowest stage has two service stations. For 6 face operation, doubling up on blasting increases the output from 77·25 to 85·50 tons/hr.; i.e., by 8·25 tons/hr.

Work is in progress on an extension of the treatment outlined in this paper to multiple series-parallel systems.

TABLE IV

DATA FROM OLD BEN NO. 22

(Doubling on slowest job (two blasters))

Job	Time per Face (min.)	Service Rate (faces/hr.)	% Working Time or % Utilization $N = 6$	$N = 9$
Blasting	28·1	2·135	40·0*	45·9*
Loading	25·6	2·344	72·9	83·6
Cutting	23·0	2·6092	65·5	75·1
Drilling	19·8	3·030	56·4	64·6
Bolting	17·4	3·448	49·6	56·8
	Total Output (tons/hr.)		85·50	97·95

* Each blasting "crew" works 40 (or 45·9) per cent of the time. Since there are two crews, the output is 80 (or 91·8) per cent of 2·135 faces/hr.

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Additional Considerations

All the results presented thus far indicate that increasing the number of faces worked increases the output of the system. Under the assumption that the service rate of all stages is independent of the number of faces worked this is certainly true.

In many cases, however, increasing the number of faces worked will decrease the service rate of some (or all) of the stages. The service time includes "tramming" or transport time. Increasing the number of faces worked will increase the tramming time (particularly in the loading stage) and thus decrease the service rate. The widening of the working area will make it necessary to provide additional power take-off points and thus increase "manœuvring" time and again decrease the service rate.

Cost factors must also be considered. Widening the working area may require additional track for loading mine cars or (in mines working a low seam) the blasting of additional dumping points. In considering doubling up of slow stages the cost of equipment and man power must be weighed against the value of increased output.

In so far as mine operations are concerned, analysis of the type described in this paper is the first phase of a more complete operations research study.

ACKNOWLEDGEMENTS

The author is indebted to Gerald Von Stroh, Director of Mining Development Committee, Bituminous Coal Research, Inc., and James E. Brown, Pochahontas Fuel Company, for an introduction to coal mining and the problem discussed here. Thanks are also accorded to Messrs. W. Campbell, O. D. McDaniel, and F. Padovic of the Old Ben Coal Corporation.

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A Monte Carlo Problem in Underground Communications

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THE problem presented in this paper was a part of a larger survey into the study of the adequacy of underground communications in British collieries. The results of this were summarized in a paper to the International Conference on Operational Research in September 1957. The particular aspect considered here is a method by which the times to warn each man and clear the colliery may be calculated taking into account the various probabilities of obtaining an answer from some of the telephone points.

When the Field Investigation Group of the National Coal Board were studying the problem, they devised a method of carrying out repeated warning trials by pencil and paper methods which proved to be laborious. They therefore approached the Mathematics Division of the National Physical Laboratory for advice on how this could be speeded up by the use of machines. It was found that the best method was to use punched card machines in the initial stages, to perform the mathematical calculations in the middle on the ACE pilot model, and to use punched card machines at the end. It is felt that a description of the method of using these machines to solve a particular problem may be of interest to a wider audience and this paper has therefore been prepared.

In this paper, the method of approaching the problem is given first. We then derive the mathematical formulation of the problem, illustrating it with a numerical example for a particular simple telephone layout. The method of obtaining the results for repeated trials using the punched card and ACE pilot model equipment is then given. Since the need for the mathematical formulation adopted may not be apparent and since a more detailed worked example may be of interest, the Appendix gives a full statement of the solution for one colliery. The final section of the body of the report quotes the results obtained in this instance and in two others where the method was used.

Method of Approach

At each colliery, a plan was obtained showing the positions of each man relative to the telephone system of the colliery. The aim was to run through several warning trials as paper and pencil exercises to calculate how long it would take to warn the men and clear the colliery, starting with a message from the surface.

The spread of the warning was taken in stages. First, the times were calculated when each telephone point would receive the message. Some telephone points might not answer or only after a long time. Warning would then be passed to these by runners from the answering telephones. As a second stage, the other workers in the vicinity would have to be warned from each telephone point. The men would then return to the pit bottom.

There was no special system of priority calling. Studies were therefore made of the telephone systems in everyday use to determine the distribution of answering times. It was found that although the chances of an answer varied from point to point, the distributions of answering times of answered calls were much the same. There was an exception to this when, along main roads, calling could be effected by special bells and shorter answering times were then more frequent. But the important factor affecting the warning time was whether or not an answer was achieved at all; beyond this, the actual answering time was a second order factor.

Hence for any required warning trial, it was possible to select two random numbers for each telephone point. The first determined whether or not there would be an answer and, if there were, the second would determine what the answering time would be. The next step in the problem was to calculate on the basis of these answering times t the corresponding times w when each telephone point would be warned. From these, the times to warn the men and clear the colliery would be easily deducible.

Mathematical Formulation of the Problem

In this Section we consider the equations which determine the times w at which each telephone point would be warned, given the value of the answering time t (from zero to infinity inclusive) at each telephone. We illustrate the use of the equations, as they are derived by reference to a simple example. Figure 1 shows a layout where O, the origin of the warning, is on the surface and A is an exchange at pit bottom. From here, three telephones B, C, D, are on a party line. When A calls on this party line,

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B, C, and D should all answer, but may not. The numbers in circles represent the number of men at each point and the figures between the points represent the times (in arbitrary units) which it would take a runner to travel the distance. We shall consider the particular instance when A answers in 1 unit of time, C in 8 units of time when called by A and when B and D fail to answer.

We shall need some definitions and notation as follows:

$t(X)$ represents the time elapsing between the bell at the telephone starting to ring to call X and X answering.

$w(X)$ represents the time when X is warned, measured from the start of the warning routine.

$u(X)$, $v(X)$ represent intermediate warning times, similar to $w(X)$ but when for the time being certain possible routes of warning have been left out of consideration.

We also require a special notation for the smaller of two numbers which we shall represent as

$$a, b/$$

This is to be considered as a single quantity in any equation in which it occurs. Thus

$$a, b/ + c, d/$$

equals by definition the smaller of $e+c$ and d, where e is the smaller of a and b.

When the minimum of three quantities is required, the symbol is used twice, thus:

$$a, b/c/$$

The following examples will illustrate its use:

$$8, 10/ = 8$$
$$4, 2/6/7/ = 2$$
$$4, 2/ + 6, 3/ = 3$$

The reason for this notation is that it is convenient for use with the ACE pilot model, as will be seen later.

Warning Each Telephone Point

Assumptions. We have to make a number of assumptions about the times to pass messages and about the behaviour of men and their willingness to act as runners in an emergency. These may be somewhat idealistic and if

anything they err on the side of reducing the time to warn the men. The assumptions made are that:

1. Six units of time are taken to pass a message to a point that answers. If on a party line, two telephone points answer at different times, e.g., after 1 and 5 units of time respectively, each is supposed warned 6 units of time after it has answered.
2. When an exchange has to call points on a party line, the man at the exchange will also instruct a runner as soon as he starts calling. In this way, a runner will be ready in 6 units of time to go in to warn points which have not answered.
3. A runner may pass a message to men whom he meets without any delay. He can talk to them as he travels along the roadway.
4. Runners know where the men to be warned are to be found and are both willing and able to take the shortest path to them.
5. The emergency does not affect the communication system or its performance.

Algebraic Statement of the Warning Times

The method of deriving equations to determine when each telephone point is warned is discussed in the paper to the International Conference. For the present purpose it will be sufficient to state the warning equations.

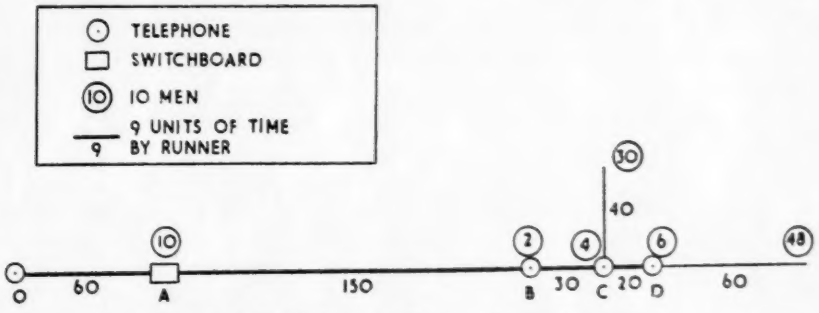

FIGURE 1. *Simple layout to illustrate the warning routine.*

In these equations $u(X)$ means the "left hand warning time at X", i.e., the time X is warned either by telephone or by runner from the left of the diagram in Figure 1.

Proceeding step by step in from O to D and returning to B, the equations are:

	Value in the simple example
1. 60, $t(A)/+6 = w(A)$	7
2. 150, $t(B)/+w(A)+6 = u(B)$	163

	Value in the simple example
3. $u(B)+30,\ w(A)+t(C)+6 = u(C)$	21
4. $u(C)+20,\ w(A)+t(D)+6 = w(D)$	41

Note that the right-hand side of the last equation has $w(D)$ instead of $u(D)$. For $u(D) = w(D)$, since there is no other way of warning D but from the left-hand direction. We may retrace our steps to C since C is either warned from the left-hand side or by runner from D. Similarly for B. Hence:

5. $w(D)+20,\ u(C)/ = w(C)$	21
6. $w(C)+30,\ u(B)/ = w(B)$	51

It will be observed that equation 1 converts $t(A)$ into $w(A)$ and that $t(A)$ is not needed in the later equations. Similarly equation 2 converts $t(B)$ to $u(B)$, $t(B)$ not being needed again, and equation 6 later converts $u(B)$ to $w(B)$. This makes the method convenient for programming on a machine. It also enables it to be extended fairly readily to more complex layouts.

Warning and Pit Bottom Arrival Times

Having now obtained the warning time at each telephone point, we have to extend it to each group of men and then calculate when they reach pit bottom. If there were groups of men between the telephone points, it would again be necessary to resort to a minimizing procedure for these. However, in collieries, the bulk of the men are either at telephone points or farther in beyond the last telephone and this procedure is not needed.

The method obtaining the times when each group will be warned will be seen from applying it to the example, where there are six groups to be warned (Table I).

TABLE I

Position of Group	No. of Men	Warning Time		Pit Bottom Arrival Time	
		Formula	Value for the example	Formula	Value for the example
A	10	$w(A)$	7	$w(A)$	7
B	2	$w(B)$	51	$w(B)+150$	201
C	4	$w(C)$	21	$w(C)+180$	201
In from *C*	30	$w(C)+40$	61	$w(C)+260$	321
D	6	$w(D)$	41	$w(D)+200$	241
In from *D*	48	$u(D)+60$	101	$w(D)+320$	421

When the warning time and pit bottom arrival time of each group have been found as in the table, the cumulative number of men warned by any particular time may be calculated. These are shown in Figure 2 for this example. The minimum possible time may be obtained by putting all values of t to zero and the maximum by putting all of them infinite.

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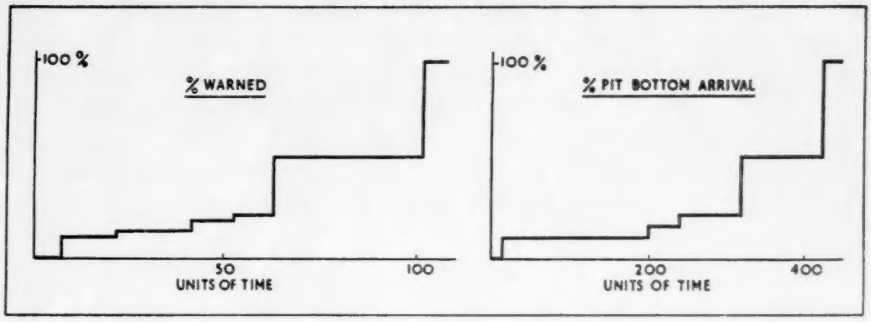

FIGURE 2. *Proportion warned and proportion who have arrived at pit bottom with one particular set of answering times in the layout of figure* 1.

Machine Calculation

It was decided to carry out 100 warning trials to study the possible variations due to unanswered calls. The machine calculation to enable these repeated trials to be made may be divided conveniently into three parts. In the first stage, the required values of t for each telephone point were obtained on cards by the use of a pack of random number cards and a punched card sorter and reproducer. These cards were then used with the ACE pilot model to derive the warning times w from the given values of t. Punched card equipment was then used again to calculate when each group of men would be warned and reach pit bottom.

Deriving the Values of the Answering Times

One hundred cards were prepared for each telephone point and a four-figure random number was punched onto each by using a pack of random number cards with the reproducer. Each card bore a number from 00 to 99 to identify it with a particular warning trial. Each telephone point was given a code from which its chance of answer would be known.

The cards were first sorted according to their chance of answer. Suppose that the chance of answer at points of a certain type was 85 per cent. Then if the first pair of digits of the random number were greater than or equal to 85, the call would be deemed unanswered and a value of $t = 9999$ was emitted. If it were less than 85, the value of the answering time would be determined from the second pair of digits of the random number. A set of master cards was prepared so that if this random number were for example from 01 to 24 inclusive, t would be punched as 0·5, thus corresponding to the probability of occurrence of this delay.

In this way, 100 values of t were put onto cards for each telephone point. For use with ACE pilot model, one card was required for each warning trial, although through lack of room, the information might have to be continued onto two or three extra cards. Hence corresponding to each warning trial 00, 01, ..., the values of t for each telephone point were reproduced on to one card or extension card, and from this they would be fed into the desired storage positions in the machine.

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ACE Pilot Model Calculations to Convert Answering Times to Warning Times

The part of the computation carried out by the ACE pilot model was the calculation of the warning times $w(X)$ from the answering times $t(X)$ using the warning equations.

To make the programming of the computer easy when new sets of warning equations are to be investigated, an "interpretative" mode of operation was employed. This means that the computer, by means of a special programme, examines instructions given to it in a special code and carries them out. It interprets each instruction into its own language as it uses it.

The "foreign language code" used in this case is devised so that the particular notation used for the warning equations can be coded very simply. It is a "single address" code, with 8 different possible operations.

We write an instruction "f, n". f is the operation performed. Depending on the value of f, n can be either:

(i) The integer n, or

(ii) The address from which a number is to be taken.

The addresses refer to a special part of the machine store reserved as working space and numbered from 0 to 31.

The machine simulated by this interpretative scheme is to be conceived as having two special registers, X and Y. The possible operations are:

0 n Add n to register X.

1 n Add the number in address n to register X.

2 n Transfer the contents of X to Y and put n into X.

3 n Transfer the contents of X to Y and put the number in address n into X.

Codes 4 to 7 now repeat codes 0 to 3 with the subsequent addition that if the number in register Y is less than that in X, the number in Y is to be transferred to X.

Finally, there is the instruction:

8 *n* Transfer the number in *X* to address *n*.

An additional instruction 9 for a special purpose is defined in the appendix.

Working a computer in an interpretative mode is always slow, but in this case the loss of speed is more than compensated by the ease of programming. This arises because the warning equations can be translated almost symbol by symbol into instructions. The correspondence is as follows:

Equations	Instruction
+ *n*	0 *n*
+ variable	1 address of variable
, *n*	2 *n*
, variable	3 address of variable
+ *n*/	4 *n*
+ variable/	5 address of variable
, *n*/	6 *n*
, variable/	7 address of variable
= variable	8 address of variable

For example, in the equation 2 which was:

$$,150, t(B)/ + w(A) + 6 = u(B)$$

suppose $w(A)$ is in storage position 1 and $t(B)$ in position 2. Then this equation may be coded as:

2	150
7	2
1	1
0	6
8	2

The first instruction puts 150 in *X*. The second takes 150 to *Y*, puts $t(B)$ in *X* and then puts 150, $t(B)/$ in *X*. The third instruction adds $w(A)$ to it and the fourth adds 6. Lastly the whole answer 150, $t(B)/ + w(A) + 6$ is storage position 2 where $t(B)$ was stored but is no longer required.

In this way, the values of *w* will be obtained and can normally be placed in the same storage positions as the answering times *t*. Hence the values of *w* at each point for a particular warning trial may be taken out of the machine on cards identical in their layout to the cards on which the values of *t* were fed in.

Deriving the Cumulative Warning and Pit Bottom Arrival Times

One card was prepared for each group of men for each warning trial. On this card were entered the additional time, a, taken to warn the group after their nearest telephone point had been warned (at time w) and also the additional time, b, it would take them to reach pit bottom. The value of w corresponding to any particular warning trial was reproduced from the cards from the ACE pilot model.

For each warning trial, therefore, there was one card for each group of men on which the number of men, w, a and b were given. The cards were then arranged in ascending order of $w+a$ and put through the tabulator. The numbers of men at each value of $w+a$ occurring were tabulated and so was the cumulative number from 0 up to $w+a$. Similar calculations were made on $w+a+b$ to arrive at the distribution of pit bottom arrival times.

Normal desk machine calculations were made to determine the mean time and the range to warn each successive 10 per cent of the men and for them to arrive at pit bottom.

Results

Figure 3 shows the average and maximum times of arrival at pit bottom for Colliery 1, the calculations being considered in detail in the Appendix. If the minimum possible time is taken as 100, the men would be out on average at 138 although in the worst of a hundred trials it could take 151.

Table II shows the comparable results for the other collieries where the Monte Carlo method was used. It shows that on average the time to arrive at pit bottom would be 30 per cent above the minimum possible; abnormal delays could increase this to about 60 per cent.

TABLE II

PIT BOTTOM ARRIVAL TIMES OF THE LAST MAN IN AN EMERGENCY

Colliery	Minimum possible	Average	Maximum of the trials
1	100	138	151
2	100	124	172
3	100	138	162

At the time these studies were made, there were no alarm systems suitable for use in collieries. One has now been developed and is installed in two collieries. The effect of these installed at faces should be to bring the times down almost to the minimum possible.

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FIGURE 3. *Percentage of men arriving at pit bottom by any given time—Colliery* 1.

The effects of reorganizing colliery telephone systems and improving the chances of answer at key points were also calculated. In these instances the important improvement was in reducing the chance of a high delay, the effect on the average not being so marked.

ACKNOWLEDGEMENT

The author wishes to acknowledge the help of D. W. Davies of the National Physical Laboratory in suggesting the interpretative code code and programming the ACE Pilot Model Computer.

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APPENDIX

Detailed Analysis for One Colliery

In this appendix, the detailed analysis is shown for one colliery. The layout is briefly described. The chances of answer and the distribution of answering times are given. These are followed by the warning equations. The method of locating the information in storage positions in ACE is then outlined and the coding of the warning equations is stated. The distribution of the groups of men and their travelling times from the nearest telephone point and from pit bottom are given so that the method of deriving the cumulative graphs, such as Figure 3, may be seen. The results for this colliery have already been given.

Colliery Layout

Figure 4 shows the colliery layout. The exchange A can call telephone points B, $G1$, $L1$ and $T1$ simultaneously. From G, L, and T the warning can pass to C, D and E respectively. B, C, D or E may telephone to Q although not to each other. When Q has been warned it can pass the message back to any of C, D or E who have not yet been warned.

There is a man-riding train available from F (near B) to Q, C or D, and from D to C and Q which reduces travelling times between these places. It would not be practicable to use it when starting from C or Q.

Answering Times

The telephone points were coded as shown in Table A1.

Where the first two digits were 00, the chances of answer were 81 per cent. With 01 and 02, the chances were 14 per cent and with 10, 93 per cent. Hence if the appropriate two digits of the random number were greater than or equal to 81, 14 or 93 respectively, $t = 9999$ was emitted to represent an unanswered call.

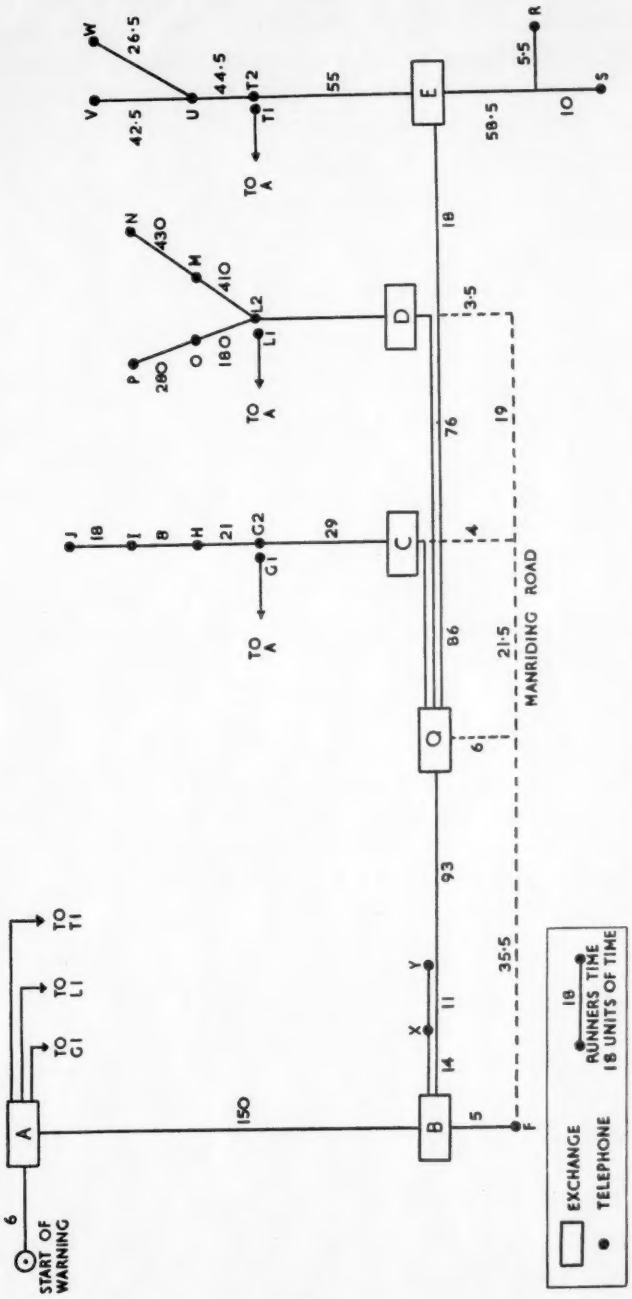

FIGURE 4. *Diagrammatic layout of the telephone system at Colliery* 1.

There were two distributions of answering times, one for the telephone points whose codes began with 0, and the other for those beginning with 1. These led to master card readings at the values of the random number shown in Table A2.

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TABLE A1

CODES FOR TELEPHONE POINTS

A	001	*H*	011	*C*	101
B	002	*I*	012	*D*	102
F	003	*J*	013	*E*	103
G1	004	*M*	014	*G2*	104
K	005	*N*	015	*L2*	105
L1	006	*O*	016	*T2*	106
T1	007	*P*	017	*X*	107
		R	018	*Y*	108
		S	019	*Q*	109
		U	020		
		V	021		
		W	022		

TABLE A2

SELECTION OF ANSWERING TIMES

Random Number	Answering Time $t \times 10$	Random Number	Answering Time $t \times 10$
00	0000	00	0000
01	0005	05	0005
25	0010	45	0010
46	0015	60	0015
58	0020	67	0020
65	0025	73	0025
71	0030	77	0030
75	0035	81	0035
78	0040	83	0040
80	0045	85	0045
81	0050	87	0050
83	0055	88	0055
84	0060	90	0060
86	0065	91	0065
87	0070	92	0070
88	0080	93	0075
89	0090	94	0080
90	0100	95	0100
91	0120	96	0120
92	0140	97	0140
93	0160	98	0160
94	0180	99	0180
95	0200		
96	0250		
97	0350		
98	0450		
99	0600		

Warning Equations

There are 40 warning equations shown in Table A3 but they fall into several logical groups and may be followed more simply than might appear at first glance.

The first two equations are straightforward. In equations 3 to 8, u is used to denote the warning by telephone direct from the surface exchange A to G, L and T. v is then the transfer of the message from these points to C, D and E respectively. Equations 9 and 10 allow for the message spreading from the direction of E towards C, this being denoted by x. It is then possible to calculate $w(Q)$ as in equation 11. Equations 12 to 14 now calculate w for C, D and E. The warning along the party lines from C, D and E may be considered separately.

(GC) in equation 15 is the time when the message first reaches the party line from C, and equation 16 calculates $w(G)$. The rule laid down for the man at G is that if he is warned from the surface, he must go to the other telephone and ring to the man at C and the man further in. He will simultaneously be instructing a runner who can leave 6 units of time later. However if C has already been warned and is already calling on the party line he is free to act as a runner and go in straightaway. This is the criterion expressed in equation 17. $1(H)$ and $1(I)$ are the "left-hand warning times" at these points and equations 18 to 21 follow the standard pattern.

Equations 22 to 28 repeat the procedure for the party line from L and equations 29 to 34 deal with the line from E through T. In these equations, y denotes the time when a runner is available to go in from L or T. Equations 35 to 40 now obtain w for the isolated points not yet considered, i.e., R and S, F, X and Y, and K.

Coding for the ACE pilot model

Table A4 shows the key to the storage positions used for the calculation on ACE pilot model. It will be seen for example that in storage position 9 $t(J)$ is entered initially and becomes changed subsequently to $w(J)$. There are no warning times for $G1$, $L1$ and $T1$, the men being allotted to $G2$, $L2$ and $T2$ for these points. It is not possible to replace $t(C)$ by $v(C)$ when v has been calculated in equation 4, since $t(C)$ is required again in equation 14. $v(c)$ and later $x(C)$ have therefore to be put on a separate storage position, 28, and similarly this is done for D and E—29 and 30. Storage position 31 is used successively for $w(GC)$, $w(LD)$ and $w(TE)$.

Using the storage positions in this table, the warning equations given in Table A3 can be translated into machine instructions, using the code given

in the paper. It will be noticed that some of the equations of Table A3 start with a comma and some do not. This corresponds to a machine instruction. For at the end of one equation, it may be necessary to remove the answer and start on the next equation; in these instances a comma is shown. However, it sometimes happens that the first value required in the calculation is already on the machine. For example, equation 9 begins with $v(E)$, which has just been calculated and is already available. Hence the first instruction can begin directly with $+24$.

Table A5 shows the set of instructions in coded form, the equation numbers being given for reference opposite the instruction 8 which prints them in their required storage position.

One additional instruction 9 was necessary to deal with the selection involved in equations 17, 24 and 31. The instruction 9—x means that the number on the register is to be compared with source x and if it is greater than or equal to x, the next instruction is to be jumped. Hence in equation 17, we have 9—3 followed by 3—22 and 0—21. This means that s, which is in the register, is to be compared with source 3, which from Table A4 is at this stage $u(G)$. If $s > u(G)$ we jump one instruction, that is to 0—21, so that we add 21, obtaining $s+21$ as required in the equation. If, however, $s < u(G)$, we carry on normally with the next instruction 3—22. This replaces s by source 22, that is $w(G)$, and then the next instruction adds 21. We therefore have $w(G)+21$ as required.

Warning and Pit Bottom Arrival Times

The values of w for each telephone were taken from the appropriate storage positions and put onto a card (if necessary with continuations onto others due to lack of room) for each warning trial. A card was then prepared for each group of men for each warning trial and the value of w for their nearest telephone point was reproduced onto it. In addition, the value "a" of the time to reach them from their nearest telephone and the value "b" to be added to $w+a$ to determine the time the men would reach pit bottom were also punched on the cards.

The nearest telephone point and the values of a and b for each group of men are given in Table A6. The cards for each trial were then arranged in ascending order of $w+a$ and master cards inserted at 006, 012, etc. The number warned in each interval could then be tabulated and the number to date cumulated.

Desk calculating machines were used from these points to continue the calculations necessary to produce Figure 3. These have assumed no

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waiting time for the man-rider. This would have to be added into the pit bottom arrival times with all methods.

TABLE A3

WARNING EQUATIONS

1. $, 6, t(A)/+6 = w(A)$
2. $, 15, t(B)/+6+w(A) = w(B)$
3. $, 6+t_1(G)+w(A) = u(G)$
4. $, 29, t(C)/+6+u(G), 72+w(B)/ = v(C)$
5. $, 6+t_1(L)+w(A) = u(L)$
6. $, 70, t(D)/+6+u(L), 82+v(C)/, 90{\cdot}5+w(B)/ = v(D)$
7. $, 6+t_1(T)+w(A) = u(T)$
8. $, 55, t(E)/+6+u(T), 24+v(D)/ = v(E)$
9. $v(E)+24, v(D)/ = x(D)$
10. $x(D)+32{\cdot}5, v(C)/ = x(C)$
11. $x(C), x(D)/, v(E)/, w(B)/+6+t(Q), 56+x(D)/, 92+x(C)/, 52{\cdot}5+w(B)/ = w(Q)$
12. $, 18+t(D), t(E)/+6+w(Q), v(E)/ = w(E)$
13. $w(E)+24, 6+t(D)+w(Q)/, x(D)/ = w(D)$
14. $w(D)+32{\cdot}5, 6+t(C)+w(Q)/, x(C)/ = w(C)$
15. $w(C), u(G)/ = w(GC)$
16. $, 29, 21+t(H)/, t_2(G)/+6+w(GC), u(G)/ = w(G)$
17. $, 6+u(G), w(C)/ = s(G)$ If $s < u(G)$, $w(G)+21, 6+t(H)+w(GC)/ = 1(H)$
    If $s \geqslant u(G)$, $s+21, 6+t(H)+w(GC)/ = 1(H)$
18. $1(H)+8, 6+t(I)+w(GC)/ = 1(I)$
19. $1(I)+18, 6+t(J)+w(GC)/ = w(J)$
20. $w(J)+18, 1(I)/ = w(I)$
21. $w(I)+8, 1(H)/ = w(H)$
22. $, w(D), u(L)/ = w(LD)$
23. $, 70, 46+t(P)/, 18+t(O)/, 41+t(M)/, t_2(L)/+6+w(LD), u(L)/ = w(L)$
24. $, 6+u(L), w(D)/ = s(L)$ If $s < u(L)$, $w(L) = y(L)$
    If $s \geqslant u(L)$, $s = y(L)$
25. $, 28+t(P), t(O)/+6+w(LD), 18+y(L)/ = w(O)$
26. $w(O)+28, 6+t(P)+w(LD)/ = w(P)$
27. $, 43+t(N), t(M)/+6+w(LD), 41+y(L)/ = w(M)$
28. $w(M)+43, 6+t(N)+w(LD)/ = w(N)$
29. $, w(E), u(T)/ = w(TE)$
30. $, 55, 44{\cdot}5+t(U)/, t_2(T)/+6+w(TE), u(T)/ = w(T)$
31. $, 6+u(T), w(E)/ = s(T)$ If $s < u(T)$, $w(T) = y(T)$
    If $s \geqslant u(T)$, $s = y(T)$
32. $, 26{\cdot}5+t(W), 42{\cdot}5+t(V)/, t(U)/+6+w(TE), 44{\cdot}5+y(T)/ = w(U)$
33. $w(U)+42{\cdot}5, 6+t(V)+w(TE)/ = w(V)$
34. $, 26{\cdot}5+w(U), 6+t(W)+w(TE)/ = w(W)$
35. $, 64, 15{\cdot}5+t(S)/, t(R)/+6+w(E) = w(R)$
36. $, 68{\cdot}5, t(S)/+6+w(E), 15{\cdot}5+w(R)/ = w(S)$
37. $, 5, t(F)/+6+w(B) = w(F)$
38. $, 14, 11+t(Y)/, t(X)/+6+w(B) = w(X)$
39. $w(X)+11, 6+t(Y)+w(B)/ = w(Y)$
40. $, 15, t(K)/+6+w(C) = w(K)$

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TABLE A4

KEY TO ACE PILOT MODEL STORAGE POSITIONS

Storage Position	Corresponding Telephone Point	Initial Entry	Subsequent Entries	
0	*A*	*t*	*w*	
1	*B*	*t*	*w*	
2	*F*	*t*	*w*	
3	*G*1	*t*	*u*	
4	*K*	*t*	*w*	
5	*L*1	*t*	*u*	*y*
6	*T*1	*t*	*u*	*y*
7	*H*	*t*	*l*	*w*
8	*I*	*t*	*l*	*w*
9	*J*	*t*	*w*	
10	*M*	*t*	*w*	
11	*N*	*t*	*w*	
12	*O*	*t*	*w*	
13	*P*	*t*	*w*	
14	*R*	*t*	*w*	
15	*S*	*t*	*w*	
16	*U*	*t*	*w*	
17	*V*	*t*	*w*	
18	*W*	*t*	*w*	
19	*C*	*t*	*w*	
20	*D*	*t*	*w*	
21	*E*	*t*	*w*	
22	*G*2	*t*	*w*	
23	*L*2	*t*	*w*	
24	*T*2	*t*	*w*	
25	*X*	*t*	*w*	
26	*Y*	*t*	*w*	
27	*Q*	*t*	*w*	
28	*C*	*O*	*v*	*x*
29	*D*	*O*	*v*	*x*
30	*E*	*O*	*v*	
31	—	*O*	*w*(*GC*) *w*(*LD*) *w*(*TE*)	

TABLE A5

ACE PILOT MODEL MACHINE INSTRUCTIONS

Eqn. No.	Code	Eqn. No.	Code	Eqn. No.	Code	Eqn. No.	Code	Eqn. No.	Code	Eqn. No.	Code	Eqn. No.	Code	Eqn. No.	Code
	2 6	6	8 29		7 21		0 21		0 6		2 6		2 6		7 25
	7 0		2 6		0 6		2 6		1 31		1 11		1 17		0 6
	0 6		1 6		1 27		1 7		7 5		5 31		5 31		1 1
1	8 0		1 0		7 30		5 31	23	8 23	28	8 11	33	8 17	38	8 25
	2 15	7	8 6	12	8 21	17	8 7		2 6		3 21		2 26·5		0 11
	7 1		2 55		0 24		0 8		1 5		7 6		1 16		2 6
	0 6		7 21		2 6		2 6		7 20	29	8 31		2 6		1 26
	1 0		0 6		1 20		1 8		9 5		2 55		1 18		5 1
2	8 1		1 6		5 27		5 31		3 23		2 44·5		5 31	39	8 26
	2 6		2 24		7 29	18	8 8	24	8 5		5 16	34	8 18		2 15
	1 3		5 29	13	8 20		0 18		2 28		7 24		2 64		7 4
	1 0	8	8 30		0 32·5		2 6		1 13		0 6		2 15·5		0 6
3	8 3		0 24		2 6		1 9		7 12		1 31		5 15		1 19
	2 29		7 29		1 19		5 31		0 6		7 6		7 14	40	8 4
	7 19	9	8 29		5 27	19	8 9		1 31	30	8 24		0 6		
	0 6		0 32·5		7 28		0 18		2 18		2 6		1 21		
	1 3		7 28	14	8 19		7 8		5 5		1 6	35	8 14		
	2 72	10	8 28		7 3	20	8 8	25	8 12		7 21		2 68·5		
	5 1		7 29	15	8 31		0 8		0 28		9 6		7 15		
4	8 28		7 30		2 29		7 7		2 6		3 24		0 6		
	2 6		7 1		2 21	21	8 7		1 13	31	8 6		1 21		
	1 5		0 6		5 7		3 20		5 31		2 26·5		2 15·5		
	1 0		1 27		7 22		7 5	26	8 13		1 18		5 14		
5	8 5		2 56		0 6	22	8 31		2 43		2 42·5	36	8 15		
	2 70		5 29		1 31		2 70		1 11		5 17		2 5		
	7 20		2 92		7 3		2 46		7 10		7 16		7 2		
	0 6		5 28	16	8 22		5 13		0 6		0 6		0 6		
	1 5		2 52·5		2 6		2 18		1 31		1 31		1 1		
	2 82		5 1		1 3		5 12		2 41		2 44·5	37	8 2		
	5 28	11	8 27		7 19		2 41		5 5		5 6		2 14		
	2 90·5		2 18		9 3		5 10	27	8 10	32	8 16		2 11		
	5 1		1 20		3 22		7 23		0 43		0 42·5		5 26		

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TABLE A6

NEAREST TELEPHONE AND TRAVELLING TIMES OF EACH GROUP OF MEN

No. of Men in Group	Nearest Telephone		"*a*" Time from nearest Telephone (Units of Time)	"*b*" Time to travel to Pit Bottom (Units of Time)
	Letter	Code		
15	*B*	002	3	8
2	*F*	003	1·5	1·5
1	*K*	005	1·5	77·5
1	*K*	005	22·5	98·5
6	*H*	011	11·5	122·5
10	*I*	012	8·5	127·5
21	*J*	013	8·5	145·5
11	*M*	014	7·5	194
3	*M*	014	26	216·5
30	*N*	015	6·5	236
17	*N*	015	19	252·5
12	*O*	016	5	172·5
34	*P*	017	6	200·5
23	*P*	017	16	210·5
3	*R*	018	4·5	157
14	*S*	019	10	175·5
17	*S*	019	21	187
12	*U*	020	4·5	193
6	*U*	020	18·5	192
31	*V*	021	9	243·5
20	*V*	021	22	239
32	*W*	022	9	232·5
28	*W*	022	23	237
21	*W*	022	39	231·5
3	*C*	101	6·5	67·5
13	*D*	102	8·5	88
19	*E*	103	7	104·5
9	*G*	104	5·5	95·5
6	*L*	105	3	151
11	*T*	106	1·5	154
3	*T*	106	28·5	181
9	*X*	107	5	16
1	*Y*	108	4·5	34·5
2	*Y*	108	25·5	55·5
5	*Y*	108	49·5	35
2	*Y*	108	67·5	97·5
5	*Q*	109	1·5	43
2	*Q*	109	22·5	64
1	*Q*	109	37·5	85·5
1	*Q*	109	40·5	76
4	*Q*	109	58·5	94

Abstracts and Reviews

Un Metodo Analogico per i Problemi di Programmazione Lineare (in Italian)
A. GAZZANO and G. POZZI.
Tecnica ed Organizzazione, 1957, **8** (35), 41–48.
Description of an electrical analogue device for the rapid approximate solution of linear programmes. A 12×5 numerical example is solved by the method and compared with the Simplex result.

This number also contains a four-page report, in Italian, of the International O.R. Conference at Oxford.

An Analysis of the Claim Records of a Motor Insurance Company.
N. L. JOHNSON (University College, London) and F. GARWOOD (Road Research Laboratory).
J. Inst. Actuaries, 1957, **83** (365), 277–294.
An analysis of 725 policies to assess accident proneness, the variation of claim rates with age and duration of policy, and secular trends in claim rates. A theoretical model for accident proneness is presented and tested against the data.

Necessité de la Prévision (in French).
A. LE CLERCQ.
L'Industrie française, Achats et Entretien, 1957 (64).
Two-page article advocating use of time series analysis for forecasting.

Une Étude Statistique Portant sur des Données Hétérogènes (in French).
J. MAZEL.
Travail et Méthodes, May, 1957.
Statistical study, by a method of successive elimination of variables, of response to working conditions of an oil refinery catalytic reforming unit.

Learning Curves in Production Planning.
J. A. C. WILLIAMS.
Time and Motion Study, 1957, **6** (5).
Fitting of theoretical curves to observations of process-times, where these times decrease as production experience is gained.

La Dactylographie et les Problèmes Humains. L'Analyse Temporelle du Travail Dactylographique. Étude des Fautes de Frappe en Dactylographie (in French).
H. KOCH, T. COURY, H. RUMEAU and M. GENEST.
L'Étude du Travail, 1957 (78), 10–35.
Symposium on studies into typewriting performance. The first paper is straight ergonomics. The second describes a technique of detailed analysis of operator movements revealed by typing on to a moving paper band. The third is a statistical study of the frequency of errors in the typing of characters in different parts of the keyboard. Not O.R. except in so far as sophisticated techniques are employed.

Utilisation des Ordinateurs Electroniques (in French).
D. N. CHORAFAS (I.B.M. Corporation and Catholic University of America).
L'Étude du Travail, 1957 (78), 36–44.
Thorough description of the use of electronic computers in industry. Distinction made between uses for data handling, technical research, routine production and stock control, and operational research. Computers, automation, data handling, and operational research are carefully distinguished from each other and their inter-relations illustrated. A three-year time-table for computer installation in a typical firm is detailed, from inception of the idea to commencement of routine operation.

La Recherche Opérationelle dans une Entreprise Suisse (in French).
F. VENIBERG.
Hommes et Techniques, 1957 (150).
Case history of a study into delivery delays.

The Cost Accountant and Automation in the Factory.
A. KENYON (Guest, Keen & Nettlefolds).
Inst. of Cost & Works Accts. Eighth Summer School, Cambridge. *Sept.* 1957, 1–18.
Costing aspects of Automation. Notable for recognition that accountants should avail themselves of statistical techniques so that more useful figures can be submitted to management.

The Economics of Plant Replacement.
J. A. SCOTT (Hayeshaw Ltd.).
Inst. of Cost & Works Accts. Eighth Summer School, Cambridge. *Sept.* 1957, 19–32.

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Comprehensive discussion, free from orthodox accounting conventions, of alternative measures of effectiveness to be maximized when making decisions on plant replacement, taking account of different possible limiting factors.

The Cost of Variety.
C. E. POWER (Brit. United Shoe Machinery Ltd.).
Inst. of Cost & Works Accts. Eighth Summer School, Cambridge. *Sept.* 1957, 33–43.
Lists the savings resulting from variety reduction, and emphasizes shortcomings of conventional procedures in assessing these savings.

Input Preparation for Automatic Data Processing Equipment.
J. E. DUNKLEY.
O. & M. Bulletin, 1957, **12** (5), 228–232.
Illustrated by examples of accounting and stores work.

A Modified Form of Procedure Chart.
WAR OFFICE (O. & M. Dept.).
O. & M. Bulletin, 1957, **12** (5), 232–233.
Useful technique for plotting data-flow in an organization.

Test for Detecting Real Changes in Frequency. Stafford Beer (Samuel Fox & Co. Ltd.).
The Engineer, 7 *Dec.*, 1956.
Useful new chart for carrying out the χ^2 test, at the 5% level, in a 2×2 contingency table. Its use is explained in simple lay terms.

Economical Acceptance Sampling Schemes.
G. HORSNELL (Ministry of Supply).
J. Royal Stat. Soc. (*A*), 1957, **120** (2), 148–191. *Discussion* 192–201.
Close theoretical study, with numerical tables, of choice of single and double schemes when the criterion is the effective cost of accepted items of normal quality. The discussion brings out the difficulties in economic design of such schemes.

Cost Accounting & Statistics.
J. A. REECE.
The Incorporated Statistician, 1957, **7** (4).
A short history of the characteristics of, and co-operation between

accounting and statistics, such as it is, is followed by examples of O.R. problems where greater co-operation is needed.

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A Note on a Linear Programming Problem.
J. ABRHAM.
Czechoslovakian Mathematical J., 1957, **7** (1), 124–129.
Some theorems on the behaviour of the solutions of a transportation problem in L.P. Purely theoretical.

The above abstracts are provided by ORbit (Operational Research) Ltd., Empire House, St. Martin's-le-Grand, from whom the relevant journals may be borrowed on payment of postage.

Natural Disaster and Political Crisis in a Polynesian Society (An Exploration of Operational Research).
J. SPILLIUS.
Human Relations, 1957, **10**, *pp.* 3–27.
While in Tikopia (British Solomon Islands) on a conventional anthropological study, Spillius found himself drawn into the role of interpreting the Government to the Polynesian population, and of the latter to the former during the crisis caused by the destruction of crops by a hurricane. Though some of his activities were such that any intelligent outsider could have undertaken, many of them depended on his being in some sense a member of both sides and, more important, of having the specialist knowledge and insight that allowed him to find the significance of some of the apparently unreasonable activities on both sides. Moreover, he found that this combination of participation and research directed at immediate advice led him to understanding that a more detached study could not have produced, and also forced him into a very immediate comparison of such predictions as he made and actual happenings. Following Dobbs, he calls this mixture of research, participation and interpretation "operational research" (though to the reviewer it seems that 'action research' as described by John Collier in "Operational Research and Action Research" might be a better name). Spillius points out that for such an activity to be profitable, the whole system must be studied, there must be some agreement on basic aims, and that the research worker must assume professional responsibility for his actions (which includes awareness of how his own prejudices, which cannot be eliminated, affect his sociological analysis).

Order Size and Cost Calculation in the Wholesale Trade (in German).
MARIO KNUSLI.
Industrielle Organisation, **25** (4), *p.* 99.
The article is based on conditions in the Swiss wholesale food trade and demonstrates the effect of the size of order, length of transport and the paying habits of the customer on the various factors which make up the costs to the seller. Such correlations in turn indicate the conditions necessary to make a particular order profitable.

Organization du travail de récolte et d'usinage du sisal (Organization of the Work connected with Harvesting and Finishing Sisal).
PIEL-DESRUISSEAUX, J., and BRETON, J. F.
Institut d'Organization Scientifique du travail en Agriculture, February 1957.
A survey on sisal harvesting and processing has been carried out in the French colonies. This report describes the operations which have the greatest effect on working costs, and suggests means of reducing these costs. Intensive cultivation produces good results and is very important. A 50 per cent reduction in growing and harvesting costs can be made when the leaves are from 31½ in. to 48 in. in average length. It has been found that a smaller number of workmen does a greater quantity of work, especially when they are equipped with knives of the correct shape, and are organized to work as a team. In the same way, the reduction of the number of operations and workers concerned with processing, and less handling in transport, increase output. It is recommended that records be kept of all operations carried out on each plot of land, so that any defects or inefficiencies in the system may easily be seen.

Maintenance Wage System based on Performance (in German).
OTHMAR HAGI.
Industrielle Organisation, **25** (9), *p.* 335.
The author discusses some principles of American maintenance work practice which have resulted in maintenance efficiencies considerably higher than those normal in Europe.

How Management Uses Queueing Theory.
F. GORDON FOSTER.
Technology, 1958, **1**, 394.
An elementary account of stochastic processes and the use of analogues and of Monte Carlo methods with random numbers, and of the kind of

problems to which the methods are applicable. A bibliography compiled by the London School of Economics listing 500 articles is referred to.

Prediction of Performance Failures.
GERALD D. COHEN.
Machine Design, 3 *Oct.*, 1957, **29**, 106–111.
A general method is given for estimating the results of a design decision in the preliminary stages. Possible environmental variations are analysed as well as the variety inherent in the product itself. A method for arriving at a numerical estimate of reliability is shown. The general case is formulated, and the special cases of constant environment and of repeated environment are considered and multicomponent reliability is briefly treated and an example is worked of the strength of a bolt in shear. Design reliability is referred to four parameters, mean product strength and variability, mean environment and its dispersion. The cases considered are illustrated graphically.

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Operational Research: Theory of Games.
MARKO A. PAKER.
Revue de la Société Royale Belge des Ingénieurs et des Industriels, 1957, (12), 478–483.
An attempt is made to give a simple account of games theory. The matrix type of game is described to which Neumann showed that all games could be reduced by the use of the concept of "strategy" among others. This concept being here defined as representing a complete plan of action not to be disturbed by chance of the acts of the opponent, takes account of all possible actions, even catastrophic acts. Another concept was that of Combined Strategy, and the essence of Neumann's theory is that it is always possible to assign a probability to any strategy and to find a number called the "game value" with the property that if one player chooses a strategy according to the probabilities disclosed the other cannot get a greater number; the distribution of probabilities are called "good strategies", and the fundamental theory is that every game has both good strategies and a "game value". The theory gives a rational method of playing where the opponent is intelligent and has the same intentions. An example is worked out where Victor takes Hélène to one of an array of restaurants, one to secure that a minimum price is not exceeded and the other that no less is spent. A wartime case of partly armed aircraft protected by fully armed aircraft is then explained. Brown's Monte Carlo

method is similarly set out. Economic problems to which the methods apply are briefly listed. A bibliography of 10 titles is added.

Planning Product Strategy for Long-range Growth and Profit.
PHILIP R. MARVIN.
Machine Design, 13 *June*, 1957, **29**, 110–117.
The use of business analysis charts, as a preliminary, the determination of the present position of a company, the critical evaluation of resources and the estimation of the value of new products are reviewed. Drawing up programme timetables, estimation of future potential and similar matters are considered, all in the most general terms.

Developing Ideas for New Product Programmes. (11 *July*, 114–119.) Methods for collecting useful ideas in quantity are indicated, four methods of approach being distinguished. Ideas are classified as referring to new functions, improved functions, or to lowered costs. Major forces in the general economy leading to prospects of growth are noted and it is stated that searching should be divorced from screening. A new products panel should be set up and its functions are indicated.

Profitable Fields for New Products Development. (8 *Aug.*, 76–81.) Seven steps in planning are listed and six questions are given to evaluate developments. To this point the articles are practically compilations of the obvious, but so minutely classified that the completed picture may contain suggestions that are not obvious at all. A general review of the kinds of directions leading to novelty is then given, miniaturization, new materials, energy sources (the solar battery for example), agriculture, textiles, technical tools and transportation being cited as examples of recent surprising advances and considered in so far as they may point the way to new products or new uses of old ones.

Screening and Appraising New-Product Ideas. (5 *Sept.*, 78–83.) Six steps are distinguished and the data necessary for a realistic decision are defined with suggestions for their collection, correlation and use.

Research For Results. (17 *Oct.*, 110–116.) The function and objectives of research are considered and organization and duties of the administrators are outlined. The great value of operational research, i.e., research into the research process itself, is pointed out.

Putting Chemical Products to Work.
CLIFFORD F. RASSWEILER.
Chem. and Ind., 1957, **45**, 1463–1469.
The Chemical Industry Medal Address of the American Section of the S.C.I. Various examples are quoted of the value of new products depending entirely on the development of uses for them, and the special nature of scientific development work is discussed. It is argued that this work is of a special nature as standard laboratory techniques and the codified knowledge of the accepted scientific disciplines may not give the answer, apparently because the problems are too complicated. The difference between the scientist and the inventor is considered and the importance, not only of scientific principles but also of knowledge of practical field situations and even the personal idiosyncrasies of possible purchasers. The need for special training is then developed and an approach to the principles of Industrial Experimentation is indicated by reference to books and other publications already in print. The importance of the ability to communicate results is also stressed.

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Ergonomics—(Official Publication of the Ergonomics Research Society). *Volume* 1, *Number* 1, *November* 1957. *Price* £1 5s. *per part; Subscription per volume* £4 15s. *post free.*
The term "ergonomics" means literally "the customs, habits or laws of work", but the disciplines and studies which have come to be grouped together in acceptance of this title for themselves have not had quite so wide implications as the literal meaning of the term would suggest. Both from their wartime genesis and by their present preoccupation, they tend to be concerned largely with physical work or with that part of human work in which there is a significant interaction with the physical environment. Within this category, there is a variety of occupations ranging from machine operators, truck drivers etc. to aircraft pilots. Excluded is the majority of that kind of work called "executive" and many technical and scientific occupations. The recognition of this boundary is important in appreciating both the contributions that can be made by workers in this field and the limitations of the field itself. From the point of view of the present reviewer it is perhaps a pity that the boundary is not more explicit but has to be derived from the material presented before an effective appraisal of the present journal can be made. Moreover an interesting relationship to operational research is suggested, for this latter tends to be concerned more with "executive" work and to impinge on a work

system at those points where decisions have to be made about the interaction of a wide variety of processes and factors. Broadly, operational research is thus concerned more with managerial work, while ergonomics is concerned with operative work.

The present journal is intended to provide regular publications of work within this field. The articles in the first number are of moderate scientific standard. The addition of multi-lingual summaries to each article is a good idea but one wonders whether more disciplined writing conventions should not be imposed by the editors. In all fields it is becoming increasingly difficult to keep up with scientific advance and the insistence on the stating of aim and scope, results and conclusions in a clear and direct manner might make both for higher standards of scientific work and for its more effective communication.

The journal as a whole is attractively laid out and printed and has good reproductions of both diagrams and photographs.

J. M. M. H.

A Course in Multivariate Analysis.

M. G. KENDALL.

Charles Griffin & Co. Ltd., London, 1956. 185 *pp.* 24s.

The field of multivariate analysis has been extensively explored by mathematical statisticians since 1930. Considerable progress has been made with the theory, but practical applications remain relatively few and unambitious, largely due to the prohibitive amount of computing necessitated by many of the techniques. With the coming of high-speed automatic computers, this barrier to progress is being removed and we may expect further developments in the theory to follow the practical testing of those theoretical tools which are now available. It is therefore very welcome to have the present status of the subject surveyed by so lucid an expositor as Professor M. G. Kendall.

Following a brief introduction, the book opens with a chapter on component analysis. This contains some of the necessary theorems in matrix algebra, and shows how a set of correlated variates can be replaced by an equivalent set of principal components, themselves uncorrelated, which (in a sense not too clearly explained) "account for as much of the variation as possible in descending order". This chapter also describes the centroid method widely used by psychologists, and a very simple ranking approximation to the first principal component devised by Professor Kendall in 1939. The next chapter deals with factor analysis, and

it is welcome to see a strong distinction drawn between this and the superficially similar concepts of the previous chapter.

Chapter 4 deals with the estimation of a functional relationship, an important problem dealt with meagrely if at all by most text-books. The theory of this subject is not in a satisfactory state, and the various paradoxes that arise are clearly pointed out. Chapter 5 comes back to more familiar ground with a description of canonical analysis—Hotelling's "most predictable criterion". As a preliminary, Professor Kendall discusses ordinary multiple regression, and suggests transforming the independent variates to principal components before calculating the regression—a practice which the present reviewer regards as misconceived.

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So far the techniques have been purely descriptive. Chapter 6 takes up some of the sampling problems. A fair amount is known in this field, notably Wishart's distribution of the sample variances and covariances, Hotelling's generalized T-statistic, and the latent root distribution due to Fisher, Hsu and Roy, but many practical problems still await a solution.

Chapter 7 is devoted to a brief history of multivariate analysis and contains a large number of useful references. Chapter 8 discusses tests of homogeneity, some of which can be regarded as multivariate analogues of the analysis of variance. Included in this chapter is an ordinary-looking analysis of covariance, the interpretation of which strikes me as highly misleading. The final chapter deals with discriminatory analysis in some detail. There is an extensive bibliography and a short set of exercises.

Altogether, this book may be recommended as an excellent introduction to the theory of multivariate analysis, at least for readers who possess the "considerable body of prerequisite knowledge" referred to in the preface —this includes "matrix algebra, 3-dimensional coordinate geometry, beta functions, and statistical theory up to the theory of correlation and regression, the bivariate normal surface and tests of significance based on normal theory". However, Professor Kendall also claims that it has been "prepared with practical considerations very much in the foreground". On this aspect, my recommendation cannot be so whole-hearted. I have commented above on two points where the author's advice seems to me more likely to mislead than to enlighten. In a book of this size and scope, many details have necessarily to be omitted; but the discussion on the identification and interpretation of factors and components is essentially incomplete without some mention of "rotation"—the factors or components carve out a subspace in the variate space, but the axes within

this subspace remain at choice. This is perhaps of little statistical interest, but its practical importance is considerable. A further topic worth inclusion (on which much work remains to be done) is that of regression and functional relationship when the errors are not known but can be estimated by replication. Tukey's important paper (*Biometrics*, 1951, **7**, 65–110) is not included in the bibliography. The description of the practical evaluation of latent roots and vectors does not give Aitken's δ^2 process for accelerating convergence (in fact the only reference to Aitken's basic paper is extraordinarily misleading) though here the author may claim that he is covered by his advice to consult an expert.

The book has a soft cover and is printed by reproduction from typescript, measures which have kept the price down to a level which must to-day be considered reasonable considering the large expanses of mathematical formulae contained in it. The layout of the pages makes them easy to read, but a number of misprints occur, including a troublesome "now" for "not" on page 31, and "independent" for "dependent" on page 69.

M. J. R. H.

Digital Computer Programming.
D. D. McCracken.
John Wiley, New York; Chapman and Hall, London. 253 *pp.* 62*s.*

People are not as scared of computers as they were, but this new book, dedicated to people "with no previous knowledge of computing", may well help to restore the balance.

The author combines complete clarity in expounding the minutiae of programming with what one can only term academic zeal that the pupil shall travel with him the whole weary length of the winding trail that computer designers and programmers have pursued during the last decade. He adopts the admirable device of postulating a "typical" computer, in whose code all examples are worked, and then drags the student through chapter upon chapter of methodology that is triumphantly revealed to have been substantially outmoded and unnecessary when the last secrets of TYDAC are unveiled during the second half.

It is undeniably difficult to find a satisfactory sequence for introducing the concepts of programming. One must not bemuse the reader by offering statements that are sure to remain unintelligible until a later chapter, nor must one lose his respect in the early stages with too unsophisticated a treatment. It is, in particular, hardly the time to instruct at length on how not to programme until the student has derived a little confidence from a concise course on how he should do it.

In fairness it must be noted that we are warned that "at least half the chapters may be omitted or included at will". This might be taken to authorize reading the book from back to front, a procedure I feel inclined to recommend.

These are, however, matters of judgment. The author has taken his stand on the historical approach, and some readers may indeed endure it, though I would give marks to any aspiring programmer who confessed to having turned to the end to see whodunit. (Typically, he would find there an 11-page octal-decimal conversion table.)

The suitability of this book for readers in this country is in any case open to doubt, for TYDAC is not typical of most computers here. It is a decimal machine, and in this connexion it is disconcerting to find so much of the book devoted to decimal/octal/binary conversions. This would be excessive even in a book on pure binary computers.

TYDAC in fact typifies the consequence of standardizing for mass-production an *ad hoc* design, and comparison with the more recently and rationally designed computers is revealing. A TYDAC routine achieves linear interpolation in 43 orders, inevitably reduced in a later chapter to 28 when order-modification is at last presented as if it should still be regarded as a clever afterthought. With a rational order code, still single-address, the routine needs only about a dozen orders, none of them any more difficult to handle than TYDAC's.

Terminology is notoriously variable in the computer world; the newcomer should be warned of this confusion. The author's use of such terms as "initialization" and the total absence of any reference to "loop-stops" are indications of what to expect. His arguments for a standard set of symbols for flow diagrams are eminently sound, although the notation he proposes is by no means universally convenient.

But much can be forgiven for the sake of Mr. McCracken's insight into the difficulties of programming, and I treasure his list, in connexion with writing a loop for n cycles, of "mistakes which will result in doing it: (1) not at all; (2) $n-1$ times; (3) $n+1$ times; (4) $2n$ times; (5) until the power fails or the machine breaks down". D. G. O.

News and Notes

Our New Look

THE *Operational Research Quarterly* has been redesigned by Mr. Jack Bowles, M.S.I.A.

International Conference Proceedings

THE *Proceedings* of the International Conference on Operational Research held at Oxford in September 1957 are now published. They are obtainable, price 50s., through booksellers or from the publishers, English Universities Press, 102 Newgate Street, London, E.C.1. In the United States and Canada they will be obtainable from the Operations Research Society of America, price $7.50.

The volume contains all the papers presented, with résumés of the discussions both at the full sessions and at the panel meetings. A full list of delegates is also given.

Two-Day Conference

THE Operational Research Society is to hold a two-day open Conference at the Old Swan Hotel, Harrogate, on 21–22 May. The Conference will give representatives of commerce and industry an opportunity to discuss current applications of Operational Research, and will have decision making as its theme. The fee will be seven guineas, to cover accommodation on the night of 21/22 May, a reception and dinner on the evening of 21 May, meals on 22 May, and conference papers.

Further details and application forms may be obtained from the Honorary Secretary of the Operational Research Society.

Film on Work Sampling

THE University Extension of the University of California has released a 16 mm. film on *Making a Work Sampling Study*. This shows how a firm used work study to ascertain the utilization factor for each production line in its works. It follows the definition of the problem, the preparatory

steps, the design of the study and the observations made, the analysis and presentation of the data and results. The film is designed as a sequel to the University Extension's previous film, *Introduction to Work Sampling*, but is complete in itself and can be shown separately.

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Productivity Study Courses

FUTURE arrangements for the series of short, residential executive courses at the Institute for Engineering Production, University of Birmingham, are: *Operational Research* 17 to 28 March, 16 to 27 June, 15 to 26 September, 8 to 19 December. *Work Study* 28 April to 9 May, 21 July to 1 August, 13 to 24 October. *Production Planning and Control* 12 to 23 May, 1 to 12 September, 24 November to 5 December. *Organization and Methods* 2 to 13 June, 10 to 21 November. *Queueing Theory and Practice* 14 to 18 July. *Statistical Quality Control* 29 September to 10 October.

Applications for reservations and for further information should be sent to the Director of the Institute.

'Making the Pound go Further'

A ONE-DAY conference under this title, to be sponsored jointly by the O.R. Society and the British Institute of Management on 27 February, was in an advanced stage of planning when the *Quarterly* went to press. The intention was to follow an introduction by Sir Owen Wansbrough-Jones with a panel of O.R. workers who would describe their scientific background and their approach to O.R. problems. Invitations to sit on the panel had been accepted by Dr. T. E. Easterfield (D.S.I.R.), John Harling (ORbit Ltd.), L. T. Hems (Field Investigation Group, National Coal Board) and Mrs. A. H. Land (London School of Economics).

The intention was to follow this with talks by senior executives from some firms using O.R., explaining how O.R. groups fit in, what they do, and the effect it has. At the time of going to press, invitations had been accepted by Courtaulds Ltd. and the Steel Company of Wales Ltd.

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Operational Research Open Conference

MUCH of this issue is devoted to the proceedings of an open conference organized by the Operational Research Society at Harrogate on 21 and 22 May. The scope and nature of the Conference are indicated in the Chairman's opening remarks. The Conference programme was:

Session 1. Chairman: Dr. K. Pennycuick.
Data for Decision Making. A. P. M. Purdon.
Case Study 1, *Passenger Flow in Subways*. B. D. Hankin and R. A. Wright.

Session 2. Chairman: R. T. Eddison.
Case Study 2, *A Study of Congestion in the Melting Shop of a Sheetworks*. J. Banbury and R. J. Taylor.
Symposium, *Problems in Decision Taking*. E. H. C. Crack; J. P. Wilson; Z. M. Tarkowski.

Session 3. Chairman: R. S. Gander.
Case Study 3, *Operations Research Study of Manufacture at a Group of Factories*. J. H. Hepburn.
O.R. as a Staff Service for Management. B. M. Brough.
Summing up by Conference Chairman, Dr. K. Pennycuick.

Chairman's Opening Remarks

When an announcement was made in 1956 of the intention to hold an International Operational Research Conference at Oxford, the interest aroused was rather dampened by the announcement that the Conference was to be devoted to Methodology and that it was intended for experienced workers. Also the total number of Conference delegates was to be limited to 250. As this number was smaller than the British Operational Research Society membership, proposals were made for the supplementing

of the International Conference by a National Conference. The Society set up a Sub-Committee under the Chairmanship of R. S. Gander, the other members being Dr. T. E. Easterfield, Dr. K. Pennycuick, and J. Stringer. This Sub-Committee advised that there would appear to be a demand for an Open National Conference, and was instructed to prepare detailed plans for a one- or two-day Conference. At this stage Mr. Gander passed the Chairmanship of the Sub-Committee on to its present Chairman, Dr. Pennycuick. The present Conference is the result of this sequence of events.

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The theme is *Aids to Decision Making*. In preparing the programme an attempt has been made to provide an introductory paper and talk on some of the considerations that run through the starting of an operational research project. This, of course, brings up the perpetual difficulty of what is operational research. Discussing this point at long length will cut across what the invited speakers have to say, but at this stage perhaps we can consider operational research as a way of looking at problems macroscopically. There are many well-known definitions of operational research, and most of them can be paraphrased into "The application of scientific methods to decision making". You will see that this definition agrees with the published theme of our Conference. The introductory paper enlarges on this theme and considers in much more detail the nature of operational research and the sort of information required to conduct it. This paper also discusses the major stages in an investigation.

The first case example is an illustration of the collection of data and its analysis in what appears to be a simple problem, namely that of passenger flow in underground subways. It exposes many of the difficulties being incurred in carrying out such work. The study is so new that this public statement is presented before the official report on this work. This is a very great honour for us. It is extremely generous of London Transport to allow a greatly simplified version of this complex problem to be so treated. The simplicity of the presentation does not obscure the very real difficulties behind this type of problem. The information presented is vital information whenever underground subways are to be constructed for passenger traffic. This work is associated in the main with the proposed route C, but it is, of course, extremely valuable when any underground station is being reconstructed.

The second case example which we will come to tomorrow morning is a much more complex problem, but again associated with readily measurable processes. It illustrates an operational research approach to a system

where there can be considerable time variations and where straightforward assumptions of uniform behaviour are quite unjustified. The link between the first and second case examples should be quite clear.

The Symposium is something quite new in operational research circles; none of the three speakers is a member of the Society. They have all of them kindly agreed to come and talk about decision taking from their own particular point of view. We have therefore, Mr. Crack, who is a Managing Director of a firm of tanners, Mr. Wilson, who is a very well known Cost Accountant, and Mr. Tarkowski, who is a consultant in the field of human problems. The expression "decision taking" is here preferred to the expression "decision making". In operational research work generally there is (in industry, at any rate) a possibility of distinguishing between the work leading up to making a decision possible, agreement on all heads, i.e. the decision is made, and what one might describe as the taking of the decision, i.e. the act of going and doing something about it.

Perhaps the difference between making and taking can be regarded as the difference between the scientist who arrives at the decision and the executive who takes it. The time interval between these two stages will, to a large extent, be inversely proportional to the respect which the executive has for his scientific management adviser. It is interesting to note in the military sphere, that this confidence is well established, and that senior military commanders are coming more and more to ask what decisions should they take rather than to be faced with a number of different options without proper guidance as to the final selection to be made.

The contributors to this Symposium by their presence here have added a great point to the fact that this is an open Conference. The Operational Research Society is delighted to welcome here those delegates who are not among its members. These delegates will do much to help the operational research workers present to an understanding of mutual problems.

The third case example is unusual. It is an instance of an operational research study of a top-level management problem. This is not to suggest that operational research studies of top-level management problems are very rare. What is rare is permission to publish them with the kind of detail given here. This study is made particularly interesting by the fact that the solution to the problem posed appears contrary to common sense, i.e. it is rather unexpected.

The last paper of the Conference is devoted to the problem of organization of operational research and how it aids business effectiveness.

Data for Decision Making

Presented at the Operational Research Society's Open Conference at Harrogate, 21 May 1958

A. P. M. PURDON
Imperial Chemical Industries Ltd.

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THE purpose of this paper is to serve as an introduction to the case studies and symposium, by outlining in general terms the objects of operational research and the service that it can provide. The paper is mainly addressed to those members of the Conference who are not themselves O.R. workers.

The title *Data for Decision Making* may seem a little strange to industrialists attending this conference, as so much of their time is spent in taking decisions for which the data are inadequate or non-existent. Were it not so, then business would be much less difficult than it is. Often there are very good reasons why full information is not available when a decision has to be taken, an obvious one being lack of time—the situation arises before it is possible to marshal all the facts or to check those that are to hand. Another reason is that the relevant facts may not have been recorded at all. In addition, the manager has to allow for qualitative factors that cannot be measured precisely and presented as data. In any event he has to rely on his personal knowledge of what is going on at the moment and what he judges is likely to happen in the future. Of course, he has usually some information to aid him; the technical and commercial advice of his staff, records and financial reports produced within the concern, and so on. But these are generally prepared for day-to-day purposes and may be of little use for policy decisions out of the normal routine; even in routine matters they are not always sufficiently up to date.

There are many occasions, however, when it is possible to foresee the sort of situation that may arise, or when there is enough time to give proper consideration to a problem that has already come to light. In those circumstances operational research can help the manager by determining what is the right information to use and how to use it, and presenting it in such a way as to cover the different eventualities. The decision involved may be one of major policy which, once taken, it would be difficult to reverse; for example, whether to build a new factory or warehouse, and if so where. Or, at a much lower level, doubts might have arisen in the course of ordinary departmental work as to whether current practice was really giving the best answers, or there were "puzzles" that it was not easy

to resolve by conventional methods, and the department may seek outside help in deciding the best action to take.

On the other hand O.R. is unlikely to be concerned when early executive action is needed to meet an unexpected situation, though in many such cases the results of a previous O.R. investigation may help to guide the manager in taking a decision *ad hoc*. Nor would it normally contribute to ordinary departmental problems (e.g., staff and labour, commercial, engineering, etc.) unless specifically consulted as suggested above.

The Nature of O.R.

What is operational research, and how does it set about its task? It has been described as the application of scientific methods to policy problems. Although this is a fair description of what it sets out to do (provided one is quite clear as to the precise meaning of "scientific method"!) it may not sound very convincing to the industrial manager whose prime concern is the exercise of judgement in the many problems of human and business relations. In the words of Wilfred Trotter, the great surgeon and essayist,

> "The scientific worker among other qualities must have an especially severe standard of evidence and proof; he must draw no conclusion that is not strictly justified by the evidence, and he must be content to leave in suspense any decision for which the materials are not quite complete. Now the last thing a doctor is free to do is to exercise the scientific suspense of judgement.... The advice to think scientifically would seem, therefore, to risk paralysing his judgement rather than activating it."

If for "doctor" one reads "manager", this seems to be closely applicable to decision making in industry.[1] This analogy has already been remarked.[2]

Commenting on Trotter's remark, Professor Woodruff asks,[2]

> "Does this mean that it is impossible for anyone to maintain a position in both camps . . . ? The answer, of course, is 'no', and . . . I believe that the number of people qualified to fill this role of middle man, and to translate into clinical terms discoveries in the basic sciences, is steadily increasing, and this is indeed fortunate, for such people act as catalysts and can bring about in a short time changes which would otherwise occur very slowly, if at all."

The O.R. worker in industry is one of those whose function it is to act as catalyst, or middle man, in helping to bring about the right changes.

Research has been defined by Sir Alexander Fleck[3] as "The process of producing new knowledge by the systematic study of phenomena or by

the critical analysis of existing data". As applied to the operations of a system such research can be used to determine how the system will behave in response to any conditions imposed upon it, and so help to provide management with information on which to base its policy for the future of the system or of its operations.

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This leads to the following description of Operations Research:—

"The study and measurement of the behaviour of a system, under different conditions of operation, to provide data for decision making."

The system, in this context, may be of any size or complexity: it may be international—as exemplified by N.A.T.O., which uses military O.R.—national, or purely local, as in a small concern of two or three employees that has taken thought, if not to increase its stature, at least to improve its efficiency. The conditions of operation are any that may be imposed on the system (by politics, economics, or business considerations), including those already experienced and documented. It usually happens that the decision relates to the employment of capital resources.

Problems such as the following may arise.

(1) What is the best strategy to follow to anticipate competition, allowing for different policies that competitors might be expected to adopt?

(2) Revision of stock-holding (inventory) policy—most economic allocation of working capital to raw materials, process (buffer) stocks, and finished goods.

(3) A company finds that its plant is out of balance (perhaps for perfectly good reasons, such as a change in the pattern of demand for its end products) and it has to decide whether to remove the bottlenecks and balance production at the higher level, or to reduce fixed and working capital and balance production at the lower level: on what criteria should such a decision be taken and what measurements are required?

(4) If total plant capacity in a factory is sufficient to manufacture all the goods ordered, but not to produce each by its most economic process route, how should the work be allocated?

Each of these problems requires that some manager shall take a decision; but on what is that decision to be based? Partly on experience, partly on intuition, and partly perhaps on the sheer need to make some decision even if it be the wrong one! The field of operational research is the investigation of problems of this nature to provide the sort of information that the manager requires and, all too frequently, has not got.

There are two important things that must be said at this point. In the first place O.R. is not some new branch of science: it is a way of thinking, a method of working, that can be followed by a good manager and his staff. Secondly, O.R. does not always claim to be able to produce information that the manager could not obtain for himself, if only he weren't so busy managing—and taking decisions. Nevertheless, there are times when the experienced worker can devise new and more informative methods of measuring phenomena, and others when the application of special techniques to available data may lead to definite conclusions that otherwise the manager could only have reached intuitively, if at all. Besides, if he is quite honest with himself the manager will admit that a decision based solely on experience is not always the best one; yet without a full analysis of the situation how could a better answer be found or the effects of different decisions compared? Analyses of this sort, supported by proper data, are precisely what O.R. tries to supply. Just what can and what cannot be done is a matter of experience, and here the case studies presented at this Conference should serve as a guide. The O.R. worker is well aware that no industrial manager will rely on him in a matter of great importance until he has proved his practical knowledge of business affairs on a smaller and less important scale. This has certainly been done in the military sphere, but not yet to the same extent in industry.

Study of the System

The first function of O.R., as defined above, is to study the behaviour of the system. Take, as example, a medium sized company with some 5,000 employees. Its board of management is responsible for watching the markets, both for its raw materials and for its end-products, and for deciding in principle how the company should modify its business to conform to changes in those markets. But it is not collectively responsible for determining how the details of production should be altered to follow board policy; that is a matter for works and plant managers. In any case it is unlikely that more than one or two members of the board would have a sufficiently detailed knowledge of each stage of the process to be capable of advising on the best changes to make. On the other hand, it is equally unlikely that the most economic development of the whole process—whether improved quality, increased yield, or reduced operating costs—could be obtained by leaving each section manager to do what he considered best for his own section without being able to consider its probable effect on all the other sections. None of these criteria of yield, cost,

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profitability, etc., can be applied properly without considering the company (system) as a whole, to determine how every section operates and how each depends on any or all of the others, on the supply of raw materials, and the demand for end-products—in fact, to find out how it works and what makes it tick.

The next step is to reduce the results of this investigation to a form convenient for further study, to devise some means of representing or writing down the working of the system in a manner sufficiently concise for top management to be able to view it as a whole, yet sufficiently detailed to show the effects of different policy decisions at each main process stage. This representation is usually referred to as the "model", and when that word is used in the context of O.R. it is understood to mean a simplified representation of the system under investigation. In some cases it may be, literally, a physical model or analogue; in others it may be a production flowsheet, with tables to show the inter-relations between different sections; and in other cases it may be a series of statistical or mathematical relations, or a list of costs relating the expected behaviour to different modes of operation. But in all cases it must represent what really goes on in such a way that changes in the system are reflected in the model.

Measurement of Behaviour

Hand in hand with the study of the system goes its measurement. If the model is to produce useful data it must be a working or behaviouristic model in statistical agreement with the real system. To talk of "using the model to provide data" is, admittedly, jargon; but the convention is useful if properly understood. When, as sometimes happens, the model is a graph, then the working model may be nothing more than a set of graphs, and "using it" would mean taking off a series of readings for different operating conditions. Of course it will often be something more complex, but the significance of its construction and use is just the same as in the case of a simple graph.

The working model, simple or complex, can only be constructed when it is possible to assess the facts and label them with some sort of measure. This involves much work in the collection of figures on plant performance and capacity, in determining optimum use of equipment and the "break-points" between different practices, in measuring the relative dependence of each process stage on the others, and finally in relating the whole to the throughput of material. It usually requires close co-operation with other departments: with Work Study for measured performance (as opposed to

design specification), with Technical, Engineering and Research departments, and with the Commercial departments for the wider setting of the whole picture.

It may go much further than this, however, since it is not just the system itself that is being measured but its behaviour; and behaviour cannot always be represented by the simple units of time, length, weight, temperature, etc. In the first place there may be qualitative factors not susceptible of measurement; the impact of trade practice, or questions of human relations, may come into this category. Then there are facts that obviously could be measured if only one knew just how to do it, or what units to employ. A brilliant example of this occurred in agriculture when Thornthwaite[4] showed how to plan very large scale commercial planting of vegetables so that they should mature at the proper rate and time for mechanical harvesting and processing. He was able to prove that vegetables have a fundamental growth-unit such that each variety matures when it has received its specific number of those units; by relating these to a calendar of climatic conditions the appropriate planting dates can be determined. Another example is that of a process in which the product may take different chemical forms at different stages, at which both yield and quality may be important; here the "break-back" principle is often used, and each product measured in terms of the equivalent quantity of the main raw material of standard purity. Finally there are cases when it is quite easy to measure the facts, but not so easy to see that the right measurements are being made. As an example, consider the rolling of a strip of metal; the number of passes through the rolls that will be required depends on several factors, two of which are the width of the strip and the amount of reduction to be made. Now it may be just possible to make a given reduction, in a strip of a certain width, in three passes; but for a second strip 1 in. wider than the first the same reduction may not be possible in three passes, so that four are required. What measure is to be used? Is it total time, which would not distinguish other reductions that could also be made in four passes? or is it fuel and power consumed, which might measure the mechanical but not the manual work? or is it total cost—in which case, how should one treat allocated overheads?

In each investigation it is necessary to construct—sometimes to invent—a scheme of measurement that can really represent behaviour. This has to be agreed with management as an acceptable basis for decision making, and it has to be related to the ordinary criteria that the manager may wish to apply in taking the decision. The scheme of measurement may be quite

simple, and not involve any special units at all; but it is necessary in each case at least to see that the right things are being measured, and in a right and meaningful way. Metrology, in this sense, is one of the two main practices of O.R., the other being representation by model. Of course they are often met in other fields, but the application of O.R. to military, industrial, or commercial problems is characterized by both.

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The Right Data and the True Objective

The third function of O.R., in our definition, is to use the model to provide data according to the purpose of the investigation. If that were an analysis of requirements—for example, a preparatory study for a new enterprise—then the model would be used to show the different ways in which the intended result could be obtained, to estimate the cost and effectiveness of each, and to specify the true requirements. Whereas if it were the analysis of a system in current operation, or process development, the model would be used to determine the effect of changes within the system itself (e.g., re-balancing plant capacities) or in the mode of operation, and the likely response to changes in the environment such as in supply and demand or the effect of competition. In none of these cases can the O.R. man "leave in suspense" any conclusion for which "the materials are not quite complete". When the data are required he must be ready to make such approximations or extrapolations as may be necessary (he, too, must exercise judgement) and, knowing the limits within which his model is representative, to estimate the accuracy of the data and agree them with the manager.

This part of the work also demands close co-operation with the different departments concerned, but at this stage O.R. begins to provide a service. Thus, the discussion of alternative solutions, or of internal changes, provides information on the most fruitful fields for engineering design, research and development, market research, or the allocation of priorities to different projects; it may save expense by showing that the results of certain lines of enquiry are never going to be wanted, whatever they may be. Again, measurements taken to compare different methods of operation may be used later in production planning. And the overall relationships of processes to one another and to changes in the environment form a basis for process costing and the preparation of a price structure.

To illustrate the last point, consider the costing of two products A and B that have to be made on the same plant. The investigation may show that, in the time taken to make three tons of A followed by one ton of B (this

being the pattern of the order book), it would have been possible to make five tons of A in a continuous run. So, besides incurring a cost of change-over and set-up, the total production is one ton less than it might otherwise have been. Also, when two products have to be made then minimal stocks are higher, more working capital is tied up, and the cost of stockholding rises (an item rarely shown in accounts since it is seldom known). Does the normal price structure allow for all of this? Is there a case for standardization? Should the firm attempt to modify demand by price changes or advertisement? Similar problems may arise when there is a choice of process routes for different products; it may not be enough just to know the cost of using a particular route for a particular product, for the cost of its not being available to other products may also be important. Unless such information is determined and given to the cost office it will be no fault of theirs if it is not taken into account.

For purposes of decision making it is important not merely to obtain data, but also to relate them to the criteria that management may wish to apply. Suppose, for example, that the problem is that of planning maintenance. The objective might be to balance the cost of plant down-time against the cost of additional labour, and then to minimize the total; or if there were not enough skilled men available the object might be to deploy them over a section of the works so as to maximize the mean throughput; or it might simply be to obtain maximum plant availability subject to rigid safety regulations regarding maintenance. Each of these criteria would probably lead to a different answer, but they can all be expressed in terms of the fundamental measures of the system, once those are known, and the model used to determine the best action to take in a particular case. In any investigation it is essential to be quite clear about the objectives and to determine the limits within which they are compatible with one another. What is it that has to be optimized? What are the key factors? When there are several of these they may operate against one another; it may not be possible to reach all the objectives and a choice has to be made. This the manager will make, of course, but how much easier that decision would become if he already had an estimate of the likely outcome of applying such different criteria as output, yield, profits, or what have you?

Presentation of Results

The last stage in operations research is the presentation of results. The questions of "Who does O.R.?" and "What is their place in the organization?" are to be discussed in the final paper of the Conference. Whoever

they may be, if they are to be of real use to the concern they must have the art of presentation. Although the investigations are technical, they are all carried out for some manager at some level; if O.R. is indeed to provide a service then the conclusions and data have to be presented in a form acceptable to the manager both as a basis for decision making, and for implementation by whomsoever he may designate. This is a problem of understanding and good relations, and it is as fundamental a part of O.R. as mathematics, accounting, technical know-how, or the ability to write clear concise English. No suggestion for change need be presented as a surprise, or as a criticism of past action, but the team leader will keep the manager in close touch with the work and allow him to reach the conclusions for himself. The greatest success a team can achieve is to have some of its results accepted and used whilst the investigation is still in progress: that proves both confidence and co-operation.

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Conclusion

The decision to start a team investigation depends on the scale and urgency of the problem. There is in each case some threshold value, above which it is more economic in cost and time to appoint a team to work full time in liaison with the manager, than to leave the work to the manager and his staff, or to a part-time committee. Investigations are usually started to analyse the requirements for some new project, or because there is some symptom in current operations that has disturbed the management. In the latter case the management will, of course, take remedial action—first aid; the purpose of O.R. is to discover the true cause of that symptom—the real disease—and to examine the effects of different ways of treating it. What is the final outcome? However well the job may be done there remain the imponderables, the immeasurables. The manager must still exercise his judgement, but the field of uncertainty is very greatly reduced. In different situations he knows by measurement not merely which are the factors that really matter—experience may already have taught him that—but also just how sensitive the system is to changes in each, how it will behave according to the decision he takes.

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Case Studies

The following three case studies were presented for discussion by the Operational Research Society's Open Conference at Harrogate on 21–22 *May* 1958.

Passenger Flow in Subways

B. D. HANKIN and R. A. WRIGHT
London Transport Executive

OPERATIONAL research journals, particularly those from America, may give their readers the impression that operational research is frequently a complicated, highly mathematical and rather theoretical business. In our experience there are countless examples in industry where the operational research approach can also be applied to relatively simple problems with equally useful results. This example is a case in point and has been chosen so that no specialized knowledge is required to understand it.

Initiation of Work and Terms of Reference

The Operational Research Section received a request for the work in a letter from the Chief Civil Engineer which included the following sentence:

> "Our present rough and ready yardsticks for measuring the capacity of subways and deciding widths are reasonably satisfactory but it might be desirable to have some additional work done so as to arrive at a more logically founded theory."

After discussion with representatives from the Chief Civil Engineer and the Operating Department, operational research personnel agreed the following terms of reference for the work:

> "To study the flow of passengers in subways including the effect of constrictions such as stairs and corners in order to assist in the design of new facilities."

The new facilities referred to are connected with the Victoria Line which is a major project now being planned by London Transport to link Victoria, Green Park, Oxford Circus and various stations to Walthamstow.

Study of References

A preliminary study of references was made. Some of the figures for pedestrian flow did not agree with the yardsticks in use in London Transport and varied considerably with the conditions and the authority

quoted. No satisfactory theory explaining the mechanism of pedestrian flow was found and all the indications seemed to suggest that the subject was less straightforward than might at first be supposed. It was evident that London Transport would have to investigate more fully the conditions and flow rates which applied to its own conditions.

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Scale of Effort

It was clear that this was one of those tasks where research would probably be rewarding but also rather time-consuming. A text book on passenger flow could have been produced but was not required. The work therefore had to be limited by the effort and time available in competition with much other work of equal or higher priority. It was hoped that it might be completed in three months but in the event (because of other priorities taking precedence) it has taken about eight months to produce a report. If overheads are ignored, the man-hours and costs can be estimated approximately as follows:

Graduates	.. ..	20 man weeks
Others	.. ..	7 „ „

Hence Total Labour Costs = £500 (excluding overheads).

Early Work

In the early stages of the work observers visited a large number of subways and stairways and timed passengers passing a point in order to calculate the flow (passengers per foot width per minute). The principal difficulties were:

(a) Considerable variations occurred in results for a given place due to the difficulty of deciding when it was functioning at *maximum* flow;

(b) Flow in a subway is affected by what is happening on either side of the section under observation, e.g., "blocking back" may slow down the flow and give a false measurement.

These difficulties were overcome as far as was possible by taking a large number of measurements on occasions when a subway appeared fully loaded and by using only the highest three readings to derive an average "maximum flow" for a given site.

It became clear at this time that the mechanism of passenger flow was less straightforward than had at first been supposed. There appeared to be a gradual slowing down in speed as the passageways became more crowded.

As a denser crowd appeared to travel more slowly, the flow past a point was not necessarily increased by having a crowded subway. It was obviously necessary to investigate the relationship between speed and concentration (passengers per square foot) and the resultant effects on passenger flow. This had to be done in a way in which we could be certain that extraneous effects such as "blocking back" were not affecting the measurements. How could we gain the co-operation of a large number of passengers in a carefully controlled experiment on this aspect? It was decided that the expense of hiring film extras or using large numbers of our own staff was prohibitive and that our passengers could not easily be involved in such an experiment. Eventually, the co-operation of the Headmaster and boys of a school in the south of England was obtained and a controlled experiment successfully carried out with their help.

Experiments at the Boys' School

Two concentric rings of chestnut paling were held up by boys standing on an asphalt surface to form a circular passageway initially 4 ft 3 in. wide and 30 ft internal diameter. Over 200 boys were gradually fed into the ring, starting with a few boys walking round and ending with a dense crowd. Speed measurements were made at various concentrations for several passage widths and the shape of the speed–concentration curve established as that shown in Figure 1. The flow–concentration curve follows from it and is of the form shown in Figure 2. It was fully realized from the start that boys are not quite the same as passengers. We were not seeking the absolute values but we wanted the shape of the curve relating the variables so that we could better understand what was happening in London, especially at the higher concentrations.

Further Measurements in London

Further measurements were then made in London to check the relationship between speed and concentration. It took some time to arrive at a simple and satisfactory method of measuring this, but in the end a remarkably simple and flexible method was evolved. Two observers with stop-watches stationed themselves in a subway at a measured distance apart. At a given signal both observers started their stop-watches. The up-stream observer immediately joined the stream of passengers moving in the subway and walked with them at the passenger speed until he reached the other observer, at which point he stopped his stop-watch. In

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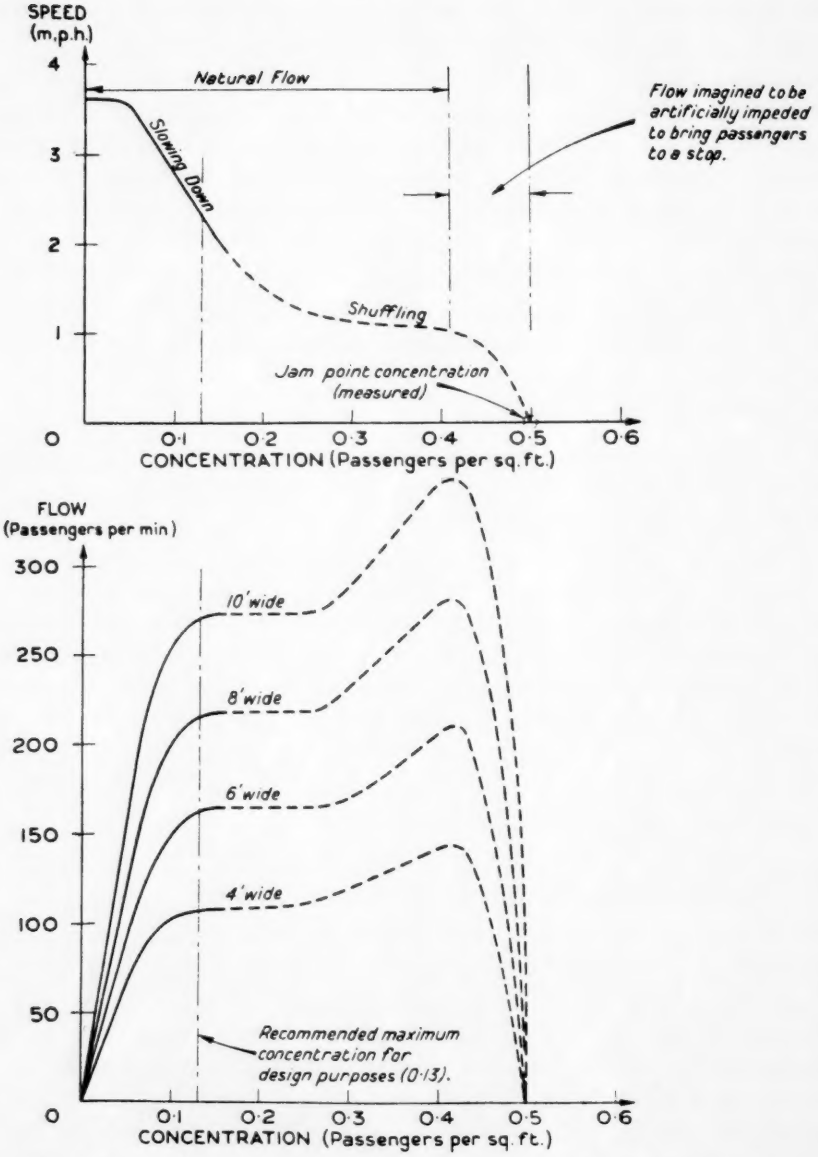
LEVEL PASSAGES



FIGURES 1 (*above*) *and* 2 (*below*). *Speed and flow–concentration curves obtained with schoolboys.*

the meantime, the other observer counted all the passengers that passed him between the time of starting his stop-watch and the second observer reaching him. In this way the total number of passengers originally in a measured area between the two observers was obtained (hence the concentration), and also the time of the movement (hence the speed). By doing this in a number of subways at different concentrations the general shape of the curves obtained with the boys was confirmed and absolute values suitable to passengers in London were obtained.

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Results from a Technical Point of View

From all the data then available it was possible to produce the curves for unidirectional flow on London Transport subways and stairways shown in Figures 3, 4, 5 and 6. Some of the more important conclusions are as follows:

(a) Above a certain minimum of about four feet the maximum flow in subways and on stairs is directly proportional to width. Below four feet, multiples of shoulder widths become important. Under certain circumstances a centre handrail dividing a passageway into two narrower ones can reduce the capacity of the passageway as a whole.

(b) Movement on stairs is markedly slower than in subways and stairs are therefore likely to form a bottleneck in "passenger movement systems" operating under full load. (The figures now available will enable designers to allow for this as necessary.)

(c) When a subway becomes more crowded there is an unconscious slowing down due to the natural desire to avoid treading on the heels of the person in front. This results in the flow reaching an effective maximum beyond which increased crowding (and consequent discomfort) is compensated by reduced speed. The net effect is that for practical purposes no gain in flow is obtained. Somewhat higher flows are possible but they are very uncomfortable and should be avoided in practice.

(d) A summary of the principal results in numerical form appears in Table 1.

Conclusions from the Point of View of Management

At a cost of about £500 in scientific and observer manhours the following results have been obtained:

(a) Existing yardsticks have been confirmed as approximately correct but have been refined.

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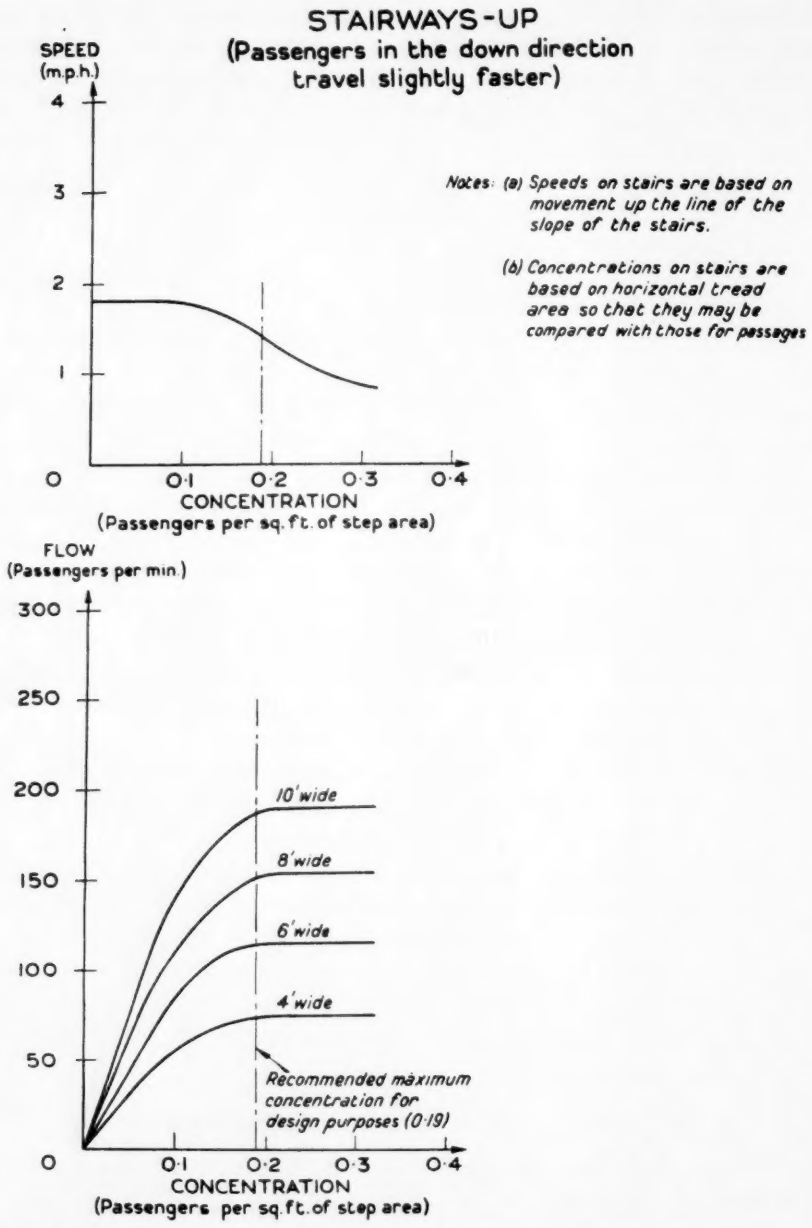
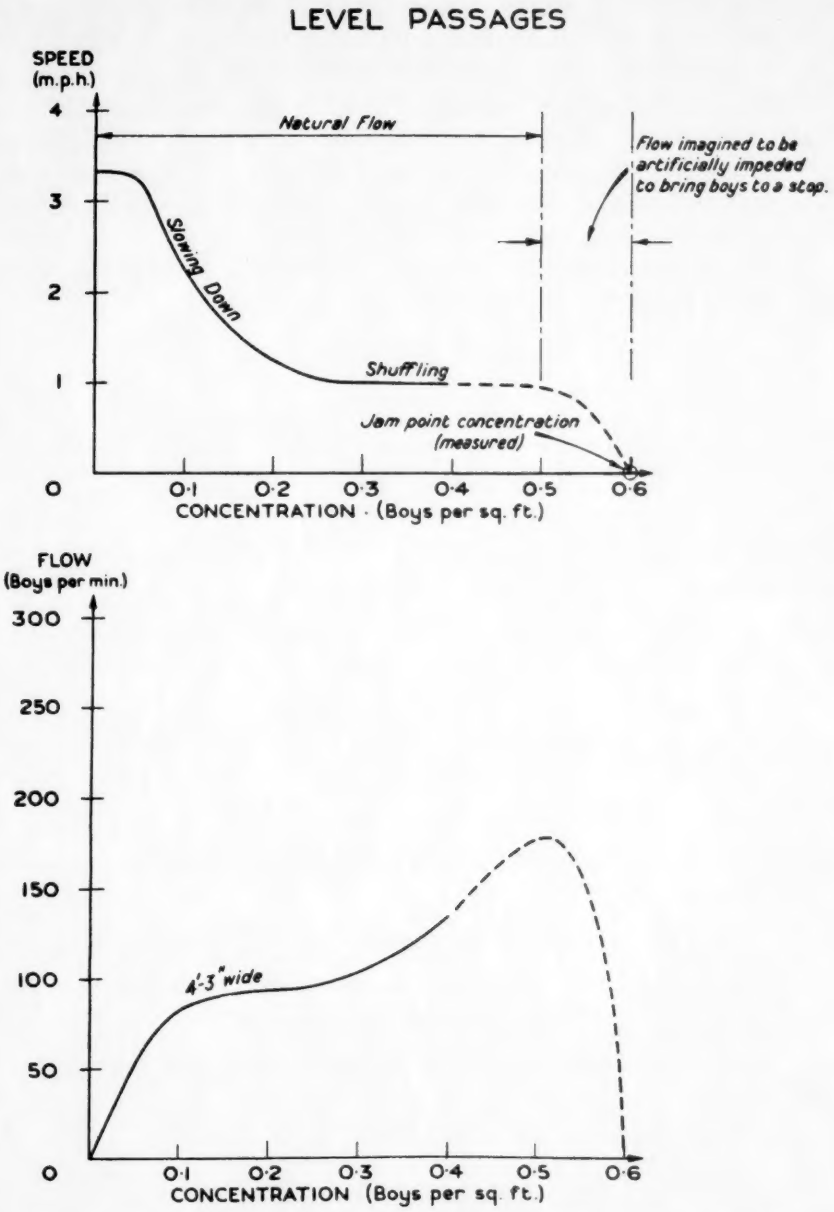


FIGURES 5 (*above*) *and* 6 (*below*). *Speed and flow–concentration curves obtained on London Transport stairways.*



FIGURES 3 (*above*) *and* 4 (*below*). *Speed and flow–concentration curves obtained in London Transport subways.*

(b) A great deal more is now known by London Transport about the mechanism and quantitative aspects of passenger flow. This could have useful repercussions in a number of related situations where the flow of passengers is an important part of the transport problem.

(c) Designers of subways should be able to take advantage of the information and figures now available to design "passenger flow systems" of maximum efficiency for a given standard of passenger comfort.

(d) Useful foundations have been laid for further study if it should be thought useful to go further in this subject.

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TABLE 1

Type of Footway	Free passenger flow speed in empty passage or stair. m.p.h.	Suggested maximum flow for design purposes. Persons per ft width per min.	Concentration of passengers per square foot for suggested design flow	"Shuffle" speed in m.p.h.
Level Subways	3·6	27	·13	1·1
Upward Stairs	1·8	19	·16	Not measured
Downward Stairs	2·2	21	·15	Not measured

Notes (i) Traditional London Transport figures in use prior to this study were:—

Passages .. .. 30 passengers per foot width per minute
Stairways .. .. 17 „ „ „ „ „ „

(ii) It is of interest to note that a detachment of guardsmen *marching* in threes down a subway of appropriate dimensions to accommodate them, would produce a flow of about 37 soldiers per foot width per minute.

ACKNOWLEDGEMENT

Permission by London Transport Executive to publish the material contained in this paper is gratefully acknowledged.

REFERENCES

A number of references were studied but the following were found particularly useful:

Post War Building Studies No. 29. *Fire grading of Buildings, Parts II, III and IV* (Section 212–268). H.M.S.O., 1952.

Traffic Engineering Handbook. Chapter IV, "The Pedestrian". Institute of Traffic Engineers, New Haven, Connecticut, U.S.A., 1950.

A Study of Congestion in the Melting Shop of a Steelworks

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1. *Introduction*

1.1. *The Aim of the Work*

1.1.1. This paper describes an investigation of a congestion problem in the melting shop of a steelworks. The problem mainly centred around the delays that arose in charging the three furnaces in the shop. The company concerned felt that the congestion was having a marked effect upon their present rate of production, and that this effect would become serious when their plans to attain greater production were put into operation.

1.1.2. The practice in the shop was to run one furnace continuously (Furnace 1) and the other two intermittently (Furnaces 2 and 3) for a six-day week; the average output when no furnace was down for repair was 1,600 tons a week. The intention was to increase the output to 2,000 tons a week by running all three furnaces continuously for an 18-shift week. It was hoped to produce an annual output of between 90,000 and 100,000 tons of steel.

1.1.3. The aim of this investigation was therefore to estimate the effect of congestion with the present practice, and to see if the effect on the planned future production would be serious; if it looked as though it was likely to be serious it would be necessary to find ways of reducing it. At this stage the costs involved would have to be considered.

1.2. *Description of Existing Practice*

1.2.1. The essentials of the steelmaking process can be described, in this instance, by considering the flow of materials through the shop. The raw material was mainly steel scrap and pig iron which were delivered by road and rail, and put to ground in the scrap bay adjacent to and parallel to the main line of the melting shop (see Figure 1). The scrap and pig were handled by two overhead cranes (A and B) fitted with magnets. These cranes also loaded the scrap into long boxes (scrap pans) when it was required by the furnaces. The magnets on the cranes were then replaced by slings, and the scrap pans were picked up two at a time, weighed, and set down at one of the points of access to the melting shop. They were then picked up by a crane on the melting shop stage which set them down on a gantry until the furnaces were ready. In order to charge the furnaces, the

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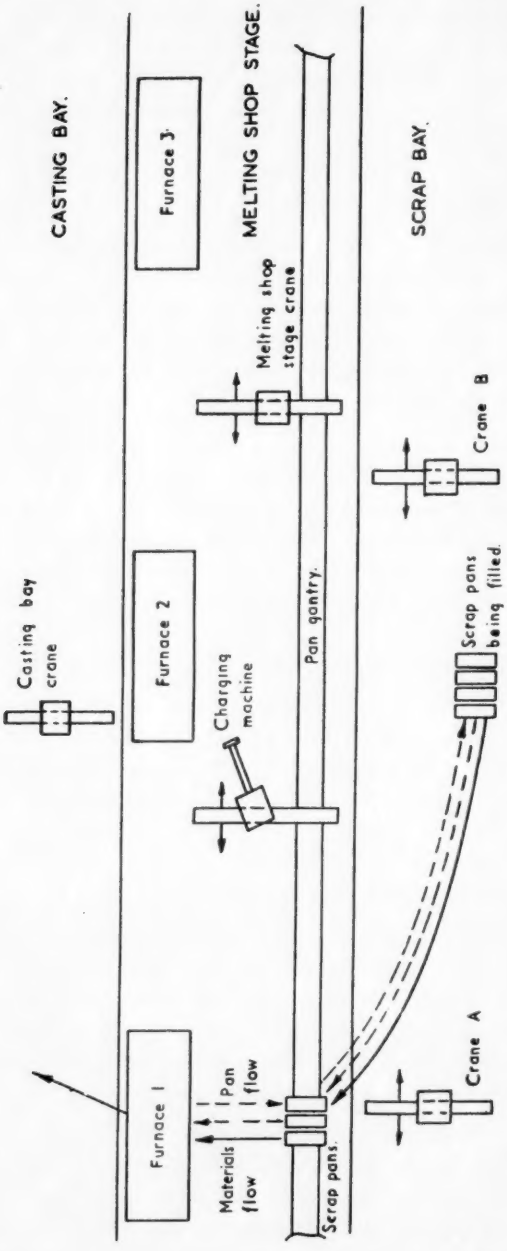

FIGURE 1. *Schematic diagram of the melting shop.*

pans were picked up one at a time by the charging machine, and the contents emptied into the bath of the furnace. Other materials were also charged, but the quantities involved were so small that their effect on congestion was negligible. The empty pans were then put back on to the gantry, and eventually returned to the scrap bay by the stage crane and one of the scrap-bay cranes. The bulk of the scrap was such that generally the furnaces could not be charged to capacity in one operation; a period of time had therefore to be allowed for the first charge to be melted (and thus reduced in volume) before more scrap could be charged. The steel was then melted and refined, and was eventually emptied (tapped) into a ladle on the side of the furnace remote from the scrap bay; the ladle was held by the single overhead crane in the casting bay.

1.2.2. After a furnace had been tapped, minor repairs were carried out to the hearth (fettling), and the furnace prepared for its next charge. Periodically the refractories making up the furnace lining had to be replaced; this represented the main cause of furnace down-time.

1.2.3. In this particular shop, the single charging machine and the stage crane ran on the same track and therefore both could not be operated at the same time.

1.3. *The Types of Congestion Studied*

1.3.1. During a furnace charging period two main types of delay might arise which could be characterized as follows:

(a) "Pan delays", when more than one furnace required charging at a given time.

(b) "Bunching delays", when more than one furnace required charging at a given time.

1.3.2. Other delays might arise outside a charging period, for example, when more than one furnace was ready for tapping at a given time, or when a piece of equipment broke down.

2. *Preliminary Investigation*

2.1. *The Characteristics of Existing Operating Practice*

2.1.1. A study was first made of the existing practice in the melting shop, using time studies, and works records for the previous year.

2.1.2. The furnace production data were studied in detail and the results are summarized in Figures 2 and 3. Figure 2 shows the frequency distribution of tap-to-tap times (including all the delays recorded in the works'

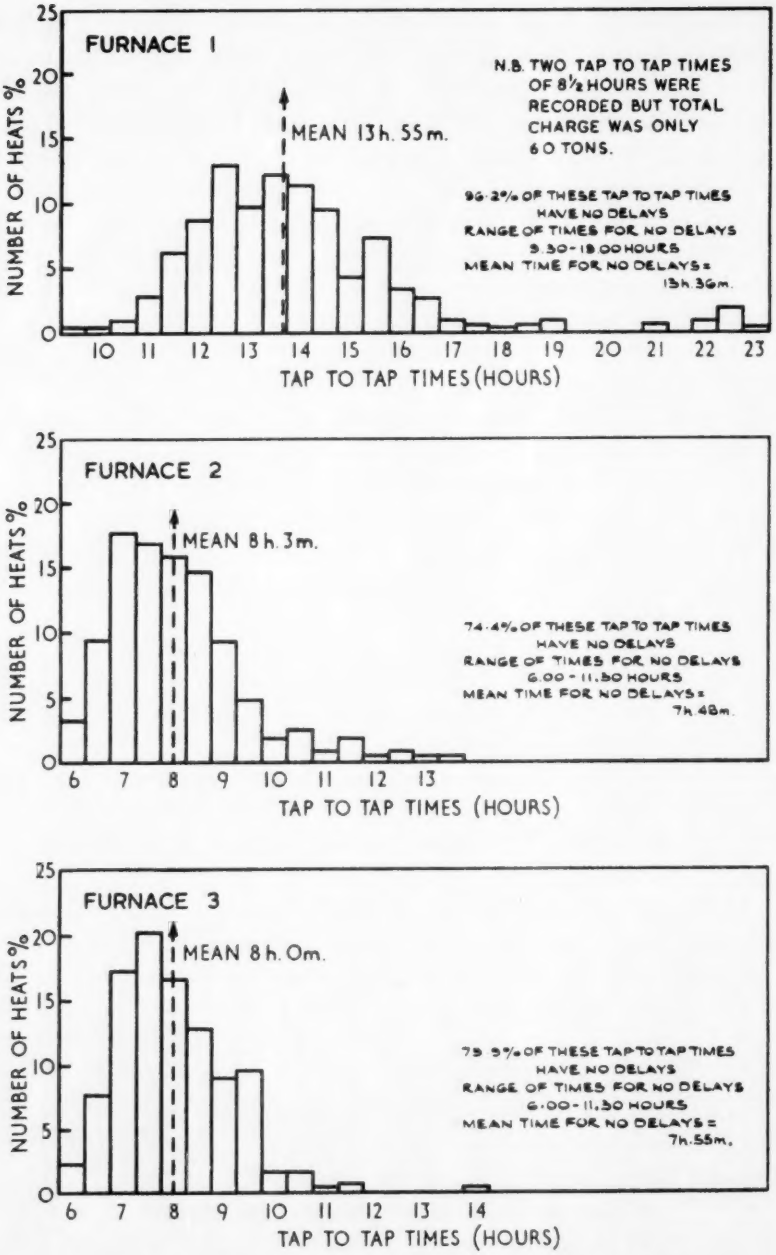
FURNACE 1
N.B. TWO TAP TO TAP TIMES OF 8½ HOURS WERE RECORDED BUT TOTAL CHARGE WAS ONLY 60 TONS.
MEAN 13h. 55m.
96·2% OF THESE TAP TO TAP TIMES HAVE NO DELAYS
RANGE OF TIMES FOR NO DELAYS 9.30-19.00 HOURS
MEAN TIME FOR NO DELAYS = 13h.36m.
NUMBER OF HEATS %
TAP TO TAP TIMES (HOURS)
FURNACE 2
MEAN 8h.3m.
74·4% OF THESE TAP TO TAP TIMES HAVE NO DELAYS
RANGE OF TIMES FOR NO DELAYS 6.00-11.30 HOURS
MEAN TIME FOR NO DELAYS = 7h.48m.
NUMBER OF HEATS %
TAP TO TAP TIMES (HOURS)
FURNACE 3
MEAN 8h.0m.
79·9% OF THESE TAP TO TAP TIMES HAVE NO DELAYS
RANGE OF TIMES FOR NO DELAYS 6.00-11.30 HOURS
MEAN TIME FOR NO DELAYS = 7h.55m.
NUMBER OF HEATS %
TAP TO TAP TIMES (HOURS)

FIGURE 2. *Distributions of tap-to-tap times (including recorded delays).*

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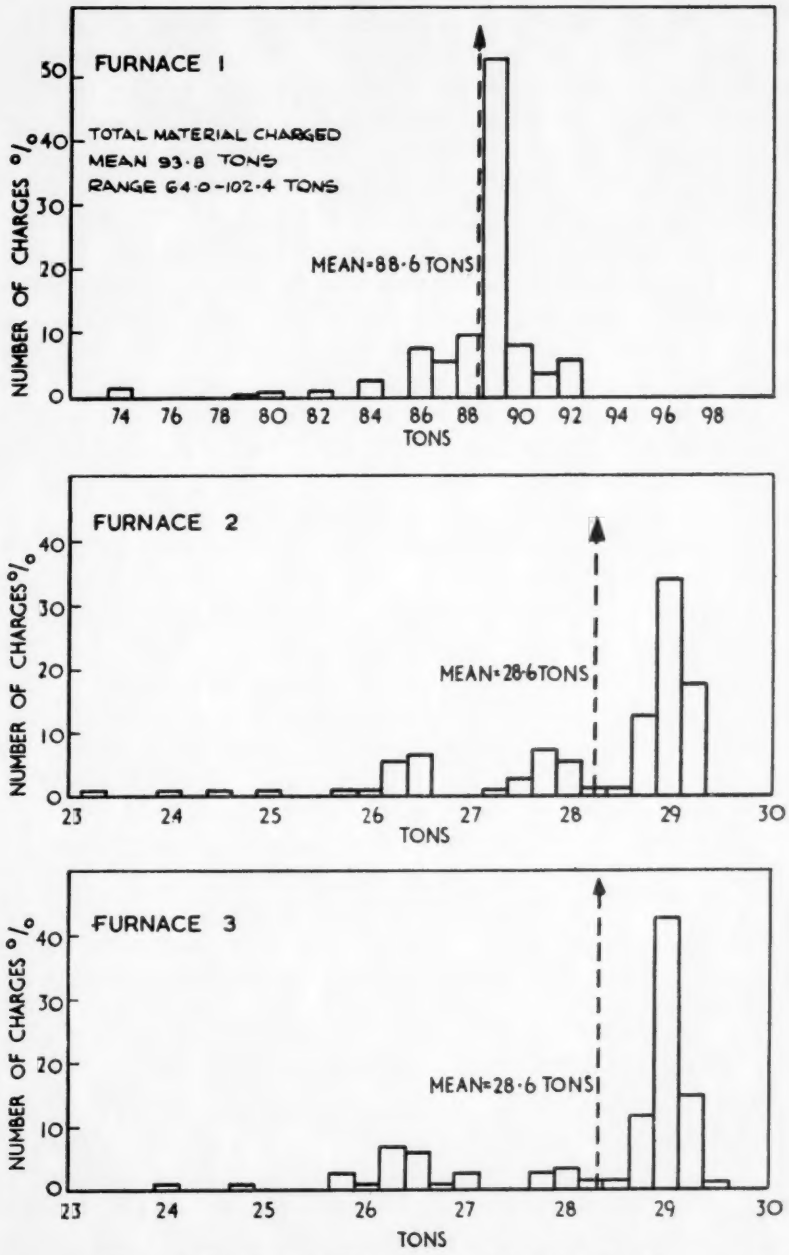

FIGURE 3. *Distributions of furnace charges (pig and scrap only).*

books) for each furnace,* the average times being 14 hours for Furnace 1, and 8 hours for Furnaces 2 and 3. Figure 3 illustrates, for each furnace, the variation in the weight of pig and scrap charged. The averages were 88·6 tons for Furnace 1 and 28·6 tons for Furnaces 2 and 3.

2.1.3. Figure 4 shows the distributions of the weekly production of the melting shop both including and excluding weeks when furnaces were out of action. The output for the year considered was about 70,000 tons.

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2.1.4. It should be noted that, at this stage of the investigation, only the delays actually recorded could be taken into account. It can be argued that no one would bother to record the short delays that occurred and hence that the assessment made on this basis would underestimate their importance. This may be so, though since the bonus scheme was such that an allowance was made for time lost due to delays, there was an incentive to record all the delays. In any event, for the later stages of the investigation, the unrecorded delays were also considered; for the preliminary analysis the approximation was thought to be justified.

2.1.5. Before attempting to estimate the effect of the existing delays during charging, it was important to realize that there was a restriction to any saving that could be made in the furnace tap-to-tap times by eliminating these delays. Some melting of the scrap (and pig) must occur during the time charging was actually taking place, and a reduction in the charging time thus implies that part of this melting time would be lost. The actual associations between charging time and melting-plus-refining time are shown in Figures 5 and 6 for the three furnaces. It can be seen that as charging time decreases, subsequent melting time increases; for example, a saving in charging time of one hour is associated, on average, with an increase of 36 minutes in melting-plus-refining time, which is equivalent to a saving of only 24 minutes in the tap-to-tap time. This relationship is obviously very important when estimating the effect of charging delays both with existing practice and with the proposed increase in production.

2.2. *Preliminary Assessment of the Effect of Charging Delays*

2.2.1. The running of the melting shop was reconstructed on paper, from actual works' records, for a period of 26 weeks. Each week was analysed separately to determine the nature and effect of all recorded delays, and by this means it was possible to assess the effect of eliminating

* The tap-to-tap time is the sum of the times occupied by fettling, charging, melting, refining and tapping, i.e., the furnace cycle for a complete heat of steel.

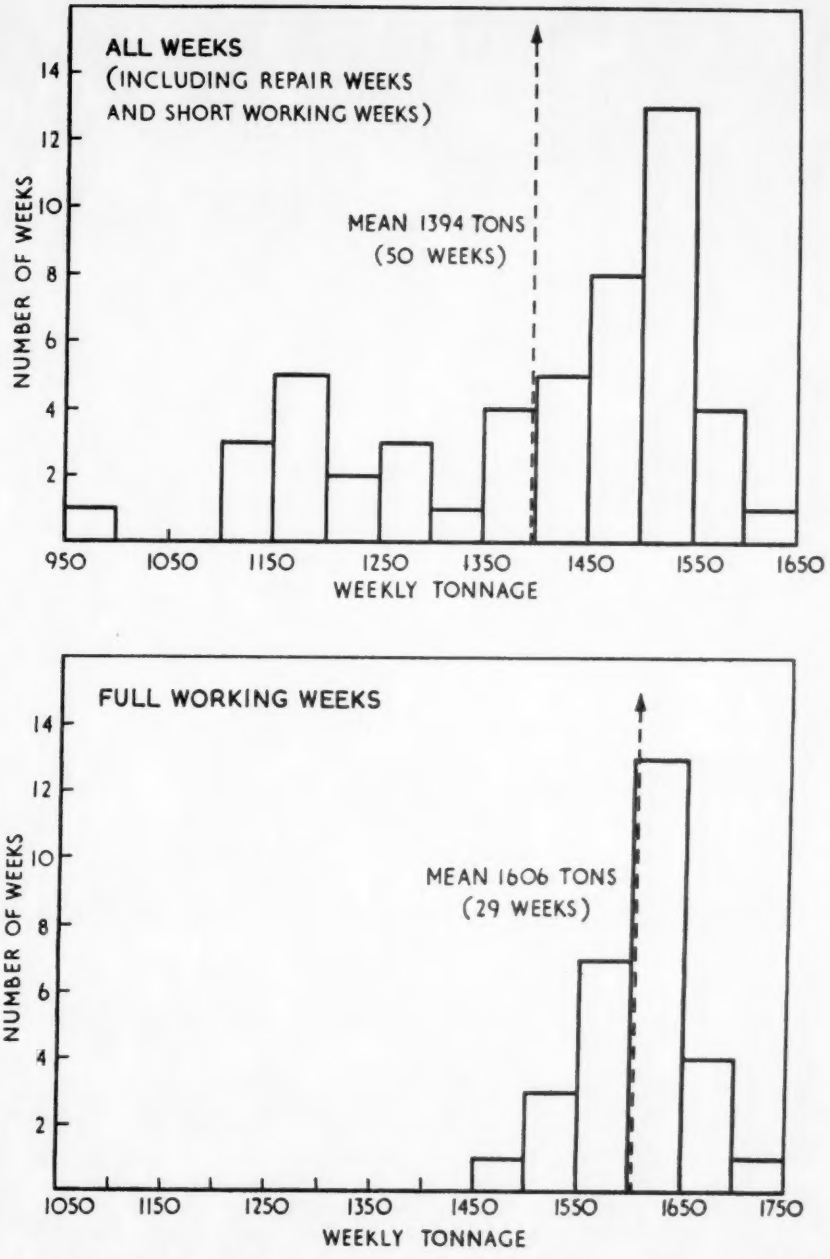

FIGURE 4. *Distributions of the weekly production of the melting shop (one year's records).*

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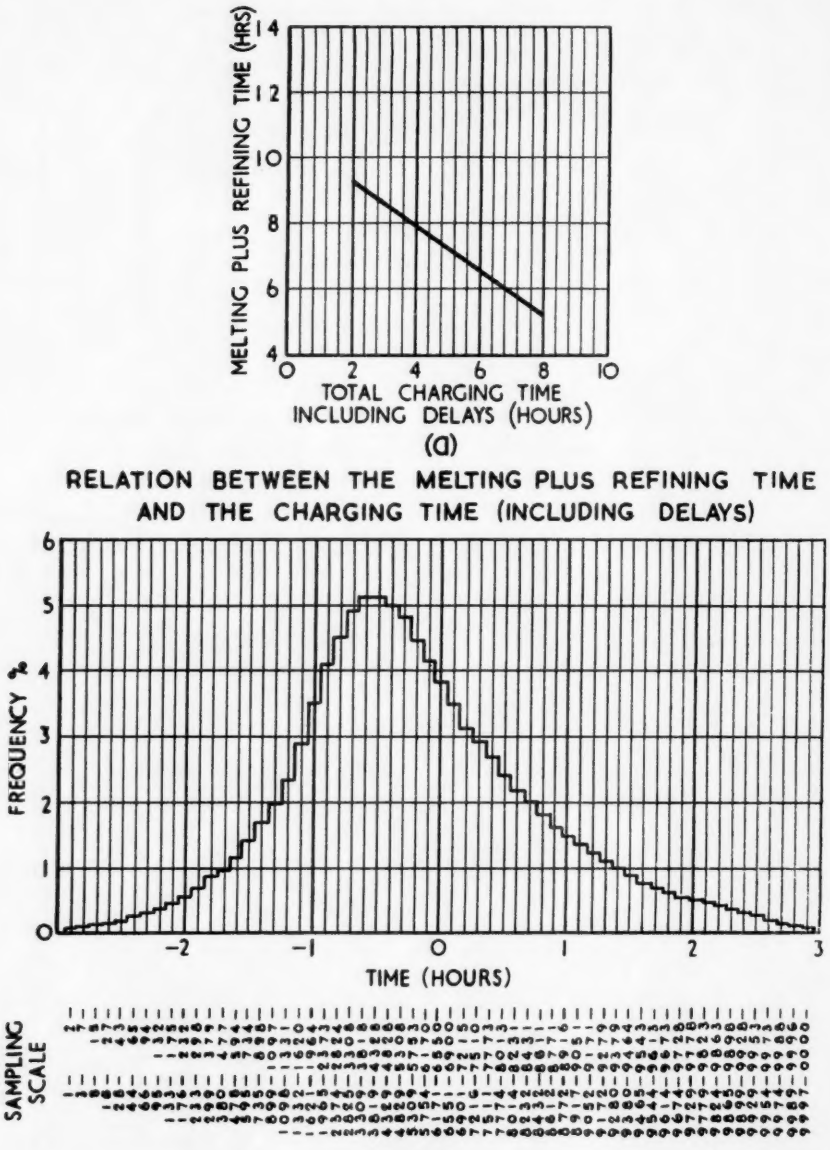

FIGURE 5. *Production times for Furnace* 1.

FREQUENCY - %
TIME – HOURS
SAMPLING SCALE
(b)
DISTRIBUTION OF CORRECTIONS TO TAP-TO-TAP TIMES FOR A GIVEN NUMBER OF PANS
TAP TO TAP TIME – HOURS
TOTAL NUMBER OF PANS CHARGED
(a)
RELATION BETWEEN TAP-TO-TAP TIMES (WITHOUT DELAYS) AND NUMBER OF PANS CHARGED

FIGURE 6. *Production times for Furnaces 2 and 3.*

pan delays. This showed for each week separately the number of heats (individual makes of steel) produced, the charging delays, the total of all other delays and the time available for further production if these delays had not occurred. The minimum tap-to-tap time for Furnace 1 was 9½ hours. In no week could an additional heat have been obtained merely by eliminating pan delays (bearing in mind the relationship discussed in paragraph 2.1.5 above). For Furnaces 2 and 3, the minimum tap-to-tap time observed was 6 hours. If the minimum time could have been achieved and if pan delays had been eliminated, an extra heat might have been obtained in three out of the 23 furnace weeks. However, the average tap-to-tap time was 8 hours for these furnaces, and to obtain these extra heats it would generally have been necessary to work extra time.

2.2.2. If all delays had been eliminated, this analysis showed that an additional heat could have been obtained during 3 out of 14 weeks for Furnace 1 and 13 out of 23 weeks for one or other of Furnaces 2 and 3 (though sometimes extra time would have been required).

2.2.3. These results are considered in the next section.

2.3. *Estimated Maximum Output of the Melting Shop*

2.3.1. The maximum output of the melting shop was then calculated assuming that all congestion delays were eliminated; the results are given in Table I.

2.3.2. The average tap-to-tap times (which include inherent metallurgical delays such as heavy fettling), the average breakdown times (due to electrical and mechanical failures), and the average furnace yield of steel, given in Table I, were obtained from works' records. The expected frequencies of number of heats per week were calculated from the observed variation in tap-to-tap times.

2.3.3. It can be seen that, given a combination of lucky circumstances, Furnace 1 might produce 11 heats and Furnaces 2 and 3, 37 or 38 heats in the same week, resulting in an output of just over 2,000 tons. But this would happen in only a small proportion of weeks (about 9 in every 100, on average). The maximum average output would be about 1,920 tons a week (81,700 tons a year after making allowance for furnace down-time).

2.3.4. These figures were calculated on the assumption that all congestion delays were zero and, with existing practice, these delays did, in fact, appear to be negligible. However, if by working all the furnaces full time, production was increased, these delays might be expected to increase as well—though there would be no simple and direct relationship between

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increase in production and corresponding increase in delays. It was possible to estimate this relationship by a method of graphical simulation: this is described in Section 3 of this paper.

TABLE I

MAXIMUM MELTING SHOP OUTPUT

(Retaining present furnace driving rates and burdens)

	Furnace 1		Furnaces 2 and 3	
Average tap-to-tap time	13·6 hours		7·9 hours	
Average breakdown time	0·1 hours/heat		0·2 hours/heat	
Pan delay	assumed zero		assumed zero	
Bunching delay	"		"	
Other congestion delay	"		"	
	No. of Heats/week	No. of Weeks (%)	No. of Heats/week	No. of Weeks (%)
The frequency with which the given number of heats would be achieved in any week	9	1	17	21
	10	59	18	63
	11	40	19	16
Average number of heats/week	10·4		18·0	
Average furnace yield	84·6 tons/heat		19·0 tons/heat	
Average output	1920 tons/week or 81,700 tons/year			

3. *Graphical Simulation*

3.1. *The Need for Simulation*

3.1.1. The steelmaking process is inherently very variable; for example, the range of tap-to-tap times for Furnace 1 was 9½ hours to 23 hours (see Figure 2), and hence any attempt to forecast the future production would be unrealistic if it were based simply upon average furnace times. The effects of congestion, which are an indirect consequence of this variability, would be ignored. A method of analysis must therefore be used which takes account of these moment-by-moment variations.

3.1.2. For this reason, the different production systems considered were simulated graphically to provide more accurate estimates of the output that could be expected. (The technique was described to the Conference in detail in an appendix to the paper.)

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3.2. *An Explanation of Simulation and its Use*

3.2.1. If sufficient data were available, a week's operation of the melting shop can obviously be represented graphically, with the state of each furnace, the operation of each piece of ancillary equipment (including its breakdown time) and the whereabouts of full and empty scrap pans recorded against a continuous time scale. It is also possible to prepare a similar chart for what might be called a representative week. Thus, instead of using a particular observed time for a particular day, a choice is made from the complete range of times obtained from the works' records. Thus when, say, Furnace 1 is to be fettled, the fettling time is selected from the range of observed values which, in this instance, extends from half an hour to over six hours. The resulting "simulation" of a week's production probably would not be exactly the same as any already observed, but it would be typical, in the sense that it certainly would represent a week which might arise sometime.

3.2.2. In practice, the simulation of such an operation is by no means straightforward. First, the logic of the operation must be appreciated; for example, the sequence in which operations occur must be known, in addition to the reasons why certain actions must take place at certain times. Secondly, possible associations between different production factors must be evaluated (e.g., the relationship between charging time and tap-to-tap time, previously discussed, and the relationship between the number of pans charged and the charging time). These associations may not be immediately obvious, and the job of checking their existence or otherwise can become tedious; the advice of the technologists on the spot can certainly be very helpful. Finally, the selection of the data, to simulate a particular operation, must be made without bias from the mass available; there are well-established ways of doing this.

3.2.3. Generally, the present practice is first simulated to check that the operation of the system has been adequately understood, and the model is sufficiently representative.* Production methods which have as yet not been tried in practice can then be simulated on paper. Many elements of the process are independent of the production system in use (e.g., the time taken to fill a scrap pan), and others may easily be corrected (e.g., the number of pans required to fill the furnace can easily be calculated from a postulated furnace capacity). An important point to note is that the delays due to congestion are not determined beforehand, but arise during the

* Mr. Purdon has indicated the obvious need for this in his paper "Data for Decision Making".

course, step by step, of the simulation, in the same way as they would in practice.

3.2.4. One other important point is that the simulation of the shop can be used to show whether the present "rules of operation" are the best possible. For example, if the charging of one furnace is at present given priority over the charging of another, the effect on output of reversing the priority can be checked; again, if the crane drivers are told to unload lorries in preference to filling scrap pans, the effect of giving priority to pan filling can be measured in terms of the resulting decrease in delays and hence the increase in output. This property of the simulation can be very useful in establishing the order of priorities necessary to make best use of the facilities provided. These priorities can, of course, be drawn up against a background of cost.

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3.3. *The Results of the Simulations*

3.3.1. It was not possible to determine the absolute precison of the simulation in this instance as the furnaces had not at that time been run continuously for an 18-shift week. In addition, it was impossible to simulate the present method of working owing to the difficulty of completely defining the actual rules of operation. However, the details were discussed with the staff of the company concerned and the eventual formulation agreed.

3.3.2. The results of the simulation were given in detail to the Conference in an Appendix and are here summarized.

3.3.3. The operation of the shop was first simulated to determine the rate of production and the effect of congestion delays when present practice was merely extended to running all three furnaces continuously for an 18-shift week. The present furnace driving rates (that is, the rates at which heat was put into the furnaces) and capacities were retained. Production decisions were based upon those working rules, developed during the course of the simulation, which maximized production (see paragraph 3.2.4), whereas operational procedures were copied exactly from present procedures.

3.3.4. The maximum average output under these conditions if congestion delays were negligible has already been shown to be 1,920 tons a week (see Section 2.3). The results of this simulation demonstrated that, at most, congestion delays would reduce this figure to about 1,850 tons a week. Thus their effect would be unlikely to reduce output by more than 70 tons a week, of which about 30 per cent would be due to pan delays,

40 per cent to bunching delays and the remaining 30 per cent to other congestion delays.

3.3.5. It was apparent that some alteration would need to be made to the operational procedures before 2,000 tons a week could be achieved, as, even if it were possible to eliminate all the congestion delays, output would still be about 80 tons a week below this figure.

3.3.6. The annual production would be about 79,600 tons (with a maximum of 81,700 tons if all congestion delays were eliminated), when allowance is made for furnace down-time. If this down-time could be reduced by, say, 50 per cent—by speeding up the repairs—an increase to 85,200 tons a year might be expected. The extra cost of reducing furnace down-time could be compared with the value of the extra steel produced.

3.3.7. In view of the results discussed in paragraph 3.3.5 and the Company's proposal to increase production to 2,000 tons a week, it was next necessary to determine the effect of congestion were this output to be achieved. The changes in operational procedure which might make it possible to obtain this output had therefore to be simulated. Two changes were considered:

(a) Increasing the furnace driving rates but retaining their present capacities;

(b) Increasing the capacities of the furnaces but retaining their present tap-to-tap times, i.e., effectively increasing their driving rates as well but not to the same extent as in (a).

In view of the limited time available for a solution no attempt could be made to determine which of these two alternatives was more suitable, and hence both had to be simulated in turn. In the first simulation, the furnace driving rate was increased sufficiently to reduce the melting-plus-refining time by about 14 per cent, and in the second simulation the furnace capacities were increased by about 11 per cent. In each instance the increase was sufficient only to obtain the desired production rate of 2,000 tons a week; their feasibility was checked against the past performance of the furnaces.

3.3.8. Congestion delays could be expected to increase as production was increased by these means, because of the greater rate of flow of materials. For this reason, the following equipment modifications were incorporated:

(a) The stock of pans in circulation was increased from 100 to 125 pans.

(b) The pan storage capacity on the melting shop stage was increased from 80 to 100 pans.

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(c) The 4-leg slings used by the cranes (see paragraph 1.2.1) were replaced by 6-leg slings.

The system of working priorities was operated as before.

3.3.9. Generally the effect of congestion was to reduce production by about 70–80 tons a week, though the required rate of production was achieved. The loss of production due to pan delays was about 10 tons a week.

3.3.10. The capital outlay on the extra handling equipment which would be required to reduce congestion delays still further was such that the expense would not be justified in view of the small increase in production which would result.

3.3.11. The weekly production of 2,000 tons and over could not be achieved without increasing the furnace driving rate, but this generally also increases the frequency with which furnaces require to be repaired. However, even if this effect was small, annual production under the conditions simulated would not be greater than 86,500–88,000 tons, which was below the desired amount. The required *annual* output could only be obtained if either furnace down-time were reduced by about 50 per cent and/or the furnace driving rates were increased still further. The results obtained in this analysis are summarized in Table II.

TABLE II

MELTING SHOP OUTPUT FOR DIFFERENT PRODUCTION SYSTEMS

(All three furnaces run continuously for an 18-shift week)

Production System	Average Weekly Output (tons)	Average Annual Output (tons)	
		With Present Repair Times	Repair Times Cut by 50%
Present Furnace Operation—			
(a) Delays as they occur	1,850	79,600	85,200
(b) Pan delays eliminated	1,870	80,200	86,000
(c) Pan and bunching delays eliminated	1,900	81,100	87,200
Furnace Operation Extended* (2,000 tons/week)			
(a) Delays as they occur	2,030†	87,400	93,400
(b) Pan delays eliminated	2,040	87,700	93,800
(c) Pan and bunching delays eliminated	2,090	89,200	95,800

* Estimates have been obtained by taking the average of the results from simulating two different methods of achieving 2,000 tons/week.

† Production only possible if the modifications to equipment and its utilization are adopted (see paragraph 3.3.8).

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4. *Conclusions*

4.1. *Conclusions Drawn from the Results Obtained*

4.1.1. The following conclusions were drawn from the analysis described in this paper:

(a) The effect of congestion delays on present production was small and could be made negligible by operating the shop with a system of working priorities.

(b) If production were increased merely by increasing the number of shifts a week for which the furnaces were operated, and the system of working priorities were used, congestion would still be negligible. However, it would not be possible to achieve the desired output by this means.

(c) If the desired output were attained by altering the operating procedures in the shop, and if the system of working priorities were used and certain minor changes made to the equipment, congestion would still remain negligible.

(d) The loss of production due to congestion was not sufficiently great to justify purchasing the extra handling equipment which would be necessary to reduce it effectively to zero.

4.1.2. These conclusions were put before the Company concerned and accepted by them. The changes in working priorities and in the equipment are now being put into effect.

4.2. *General Conclusions which can be Drawn from the Use of Simulation Techniques*

4.2.1. Simulation techniques are a powerful way of tackling problems of this sort which, at present, cannot be tackled mathematically without making unacceptable simplifying assumptions.

4.2.2. It is a technique which is generally more readily appreciated by management than pure, mathematical analysis. The results are therefore more likely to be accepted.

4.2.3. It is unfortunately all too easy to misuse simulations. For example, there may be only an inadequate check that the logic of the system being studied has been understood, and hence that the model adequately represents the real situation. Again, the simulation may not be continued for a sufficiently long time to allow the full range of adverse conditions, which are likely to occur in practice, to arise during the simulation.

4.2.4. Simulations can be very tedious by paper and pencil methods. However, the technique is one which readily lends itself to analysis by an electronic computer.

Operational Research Study of Manufacture at a Group of Factories

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IN a recent paper[1] Wanty enumerated what he considered were the main stages in an operational research study and pointed out that the greater part of the published literature has concentrated rather exclusively on two particular stages which could be described as the setting up of a representation or model of the system under examination, and solution of this model to determine the optimum conditions. I am in agreement with Wanty particularly in so far as he says that the emphasis of the published literature is on the solution of models, but to my mind the stages in a study are so closely linked that I would telescope his first five stages into three:

(1) Definition of the problem.

(2) Setting up of a *valid* representation of the system.

(3) Solution of the model.

The difficulty is in finding a valid model of the system and, of course, this must entail enumeration of the parameters and verification of the pertinence of the basic data, although not necessarily as distinct stages. However, once a valid model has been found, and it need not be a mathematical or statistical model, Wanty's sixth stage—experimentation with the solution—is superfluous, and even impossible if the study affects major policy decisions.

In many studies the model must be a financial one, but here there may be just as much difficulty in obtaining a valid representation of the system as there is when it is necessary to represent a sales pattern by some statistical distribution. The difficulty is usually in finding valid costs. In the study I am going to describe, although our costs were in some respects valid, they could not be used to validly represent each stage of the system and allow comparisons to be made. Furthermore, experimentation with the optimum solution found might have had disastrous effects.

To any who have always considered that the essence of operational research lies in the setting up and solution of a mathematical model and have difficulty in reconciling their approach with mine, let me make it quite clear that a cost is as much a model of a business undertaking as a differential equation is of machines awaiting service in a repair shop. Further, in my experience, the more likely that the results of a study will

be used as the basis for a major policy decision, the more reason there is for a simple model which is readily understandable by all who have to take the decision.

The Problem

Three factories, which we shall call A, B and C, were engaged in the manufacture of the same group of products. The manufacture was an intricate assembly operation which, at the time, had to be done by hand, although machines were being developed which would enable the assembly to be mechanized within about five years' time. It was estimated that fairly considerable reductions in cost could then be made, but meanwhile growing competition in the export market meant that unless costs could be reduced now there was a distinct prospect that it would become unprofitable to sell in some markets. Competitors in this country were also assembling the product by hand but we had no difficulty in selling at competitive prices in the home market; nor did we expect any since we were well ahead in the development of mechanization.

Comparison of Efficiencies

The three factories were situated in different parts of the country and produced mainly for customers in their particular areas. Factory A, the largest, produced about 46 per cent of the total output, factory B 30 per cent, and factory C 24 per cent. Half of factory A's production was for export but factories B and C produced only for the home market.

There were many different varieties of the same product manufactured, but as far as the home market was concerned about 90 per cent of demand was for two different types and so the study was concentrated on these. A comparison of costs indicated that plant costs at A were about 30 per cent above those at C, while those at B were about 5 per cent above those at C. At first sight, then, the investigation appeared to be one simply of finding means of bringing A's costs into line with C's.

It was necessary first to make exact comparisons of the manufacturing methods and the assembly of two particular items was traced from start to finish at each factory. Although the factories all produced the same finished article, for historical reasons they exhibited many differences in layout, method and sequence of operations, which made exact stage-by-stage comparisons difficult. Further, although a breakdown of costs

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showed more or less marked differences between the factories at each stage of manufacture, these costs were not strictly comparable as they did not always refer to the same operations. The fact remained, however, that in total the wages costs were different at each factory. Work unit values for each element in the assembly operation were therefore compared and in some instances fairly large differences in values were found for the same operation at the various factories. This merely confirmed the experience of previous workers who have carried out countrywide Work Measurement surveys.

Obviously, changes in work unit values could not be made unless there had been an accompanying change in method. In the comparison of the methods at each stage it was found that no factory was best or worst at all stages and by taking the best of all worlds there would require to be changes in method in most processes at each factory. Because of the changes in method there would require to be a re-study of all operations and this would provide the opportunity of bringing work unit values into line.

A wages cost was built up on the basis of work unit values and this was checked by comparing it with actual total costs for the whole assembly. Discrepancies were found which could not be explained until the whole question of labour was gone into, including sickness absence and turnover, whence it was found that the age distributions of workers were quite different at each factory. Operatives were mainly female and reached their maximum wage rate at the age of 21. At one factory, where there was little alternative employment in the immediate surrounding district, it was agreed (at the request of the local trades union) that all girls should be dismissed on marriage. The girls tended to marry young and the average hourly wage was therefore much less than at the other two factories. Thus the difference between derived and actual wages was explained.

We had compared the wages part of cost and showed how the differences between the factories could be evened out, but that was only part of the picture. Oncost charges, i.e., charges indirectly associated with production, were different at the factories, the percentage oncost/wages being 60 per cent at A and 40 per cent at B and C. The allocation of oncost to products was different at each factory and so a correction factor, based on total process wages, was applied and each item of oncost then compared. Factory A was higher than the other two in most items of oncost but it was soon apparent that little could be done about it. For example, one of the main items of oncost was labour charges, comprising payments for

sickness absence, annual holiday pay, service bonus, pension fund, etc. In the neighbourhood of factory A there was little alternative employment for women, labour turnover was small and average service and age were much higher than at the other two factories; hence all items of labour charges were higher than at the other factories.

Items such as steam and electricity usage, rent, rates and insurance were higher at A since it covered a much larger area than the other two. Although its production was, in units, double that at C, all export production, which was of a much more specialized nature, was carried out at A and the total work content was three times that at C. Moreover the factory had originally been built for a different purpose and its area was about six times that of C, which had been built specifically for assembly.

Salaries and wages accounted for more than 60 per cent of other items of oncost and overheads. In many cases salaries were apportioned over a number of items and so further comparison of items could not readily be made. The *numbers* of staff, tradesmen and ancillary personnel at each factory were therefore compared. I do not intend to go into any details on this aspect of the study; suffice to say that in the past years overheads had tended to increase at factory A owing to factors most of which were outside the control of the factory management. It was possible to indicate how further reductions could be made, amounting to roughly about the same as those which could be made as a result of improved methods and work study values. At the same time it was shown that a small amount would have to be spent at factory C to bring certain amenities into line with general Company standards.

The overall effect if all suggested changes were effected was to reduce the plant cost of the product by 2 per cent. On paper, the plant cost at factory A was still 25 per cent above the cheapest factory, now B, but the study had shown that this was mainly due to the method of allocation of overheads which were necessarily much higher at A owing to the additional activity entailed in the production of export orders.

Re-allocation of Production

A 2 per cent reduction in costs, whilst a significant contribution, might not have been sufficient if in some areas the effect of competition was to force a reduction in prices and it was necessary to pursue other means of reducing costs. The next stage in the study was to consider whether there

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was a real need for three factories. It was obvious that if the same output could be obtained from a fewer number of factories considerable savings could be made.

The attainable capacity was assessed at each stage of the assembly operations in the three factories. This was not the straightforward exercise it may seem. Output was dependent on the particular type of product being produced and a weighted average work unit value for all types had first to be obtained for each process. Secondly, in a hand assembly operation such as the one under consideration, output is dependent on the level of operative achievement and it had to be assumed that the prevailing level would continue. Finally, output is dependent on the number of people employed and so a reduction of 10 per cent in the calculated maximum capacity was made to allow for trainees, sickness absence, etc.

With the existing factory capacities the only way of operating with fewer factories was to operate A on two shifts with either B or C on single shift. It was apparent, however, that capacity was restricted by a few bottlenecks and it was found that with small capital expenditure the existing nominal capacity could be increased by 10 per cent.

With this increase in capacity it was seen that the required production could be obtained by concentrating at only two or possibly only one of the existing factories as follows:—

(1) Operate only A full out on 2 shifts
(2) Operate A partly on 2 shifts and B on single shift
(3) Operate A partly on 2 shifts and C on single shift
(4) Operate A on single shift and B on 2 shifts
(5) Operate A on single shift and C on 2 shifts

Under the first alternative there was virtually no spare capacity and in the event of an emergency whereby production was reduced for a few weeks there would be great difficulty in catching up with outstanding orders. Under the second and third alternatives production could be increased by operating A fully on two shifts and the additional cost of production would be merely the additional wages to operatives. Under the fourth and fifth alternatives there was spare capacity in the form of operating A on another shift. If this were done, however, the increase in costs would not be marginal. It was desirable therefore to consider other alternatives whereby additional capacity could be readily available. Further, since there would be a considerable increase in the wages bill to be set against the obvious attractions of shift working, each factory was considered in turn to determine how its capacity could be increased at

somewhat greater capital expenditure so that, if possible, production could be reallocated between two factories working on single shifts.

It was found that by converting existing buildings an 80 per cent increase in capacity could be effected at A and B, but that factory C had almost reached its limit and only a small increase could be made there. Four more alternatives were then drawn up:

(6) Increase capacity at B. Operate B and C on 2 shifts
(7) Increase capacity at A. Operate A and B on single shift
(8) Increase capacity at A. Operate A and C on single shift
(9) Increase capacity at B. Operate A and B on single shift

It now remained to find which of the nine alternatives was the optimum.

In drawing up the cost model, the wages part of cost was determined from work study data as already indicated. The other parts of cost could not be determined by any marginal costing technique such as dividing costs into fixed and variable overheads since at certain levels of production there would be sudden jumps in the overheads usually regarded as fixed. The additional staff, tradesmen and ancillary workers considered necessary were therefore enumerated for each factory for each of the anticipated levels of production on single and double shift, and the additional salaries, wages and associated charges estimated.

Any reallocation of production would affect distribution costs and the additional charges arising were determined for each alternative. Account was also taken of the fact that an income tax rebate would be allowed on any capital written off and that if a factory were closed the site could be sold. It was assumed that any benefits of reallocation would be obtained for a period of four years, until the advent of mechanization, and so the necessary capital expenditure under each alternative was subtracted from the total savings likely to be made in four years.

The model indicated that three alternatives, (6), (8) and (9) showed much higher savings than the others and that the adoption of any of them would effect a reduction of a further 3 per cent in the cost of the product. The difference in savings under these three schemes was such, however, that factors other than strict economies would certainly play an important part in reaching a decision as to which one to adopt.

At this stage some of you may think that an operational research study should end. A quantitative basis is now available on which top management can make a decision, but to my mind we could not justifiably present such a case without first taking into account the possible effects of any decision.

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In this study there was a danger of sub-optimization. Thus, while some alternative might have appeared an attractive proposition because of the savings to be achieved until the advent of mechanization, quite a different picture might have emerged when viewed in the light of the best policy after mechanization. This picture was examined broadly and it was determined that factory C would most certainly not be required after the assembly was mechanized. There were some doubts as to whether both A and B would be required and a further study to determine the best policy for mechanized assembly was indicated.

Alternative (9) was the only one of the three showing largest savings which involved future operation without C. In addition, the workers in the areas of factories A and B depended on these factories for employment and closure of either would cause severe hardship to some employees, whereas in the area of C there was considerable choice of alternative employment and although there might be some disturbance there would be little or no hardship.

Two other factors helped to indicate that alternative (9) was the optimum. Firstly, under alternative (6), showing similar savings, there would have been very little spare capacity and the adoption of this scheme would not have allowed the desired flexibility to meet peak fluctuations in export demand. (A buffer stock could not be kept to meet these fluctuations since export sales are generally of types not sold in the home market. Further, orders for particular markets vary from time to time and it is usually found impossible to be able to forecast anything other than numbers required with any degree of accuracy.)

Secondly, a local distribution depot, which was one of the largest in the country, was administered from factory B. If the factory were closed alternative arrangements would have to be made and it was almost certain that administration costs of the depot would increase since it would be no longer possible to share duties between production and distribution as was done at present. In addition, the very close liaison between production and distribution at this main depot would be lost.

These factors could only be expressed in qualitative terms but they were such that they would help to sway a decision one way or the other, particularly if there was little to choose financially between alternative courses of action. In our report to management all these points were brought out and we indicated that unless there were other factors to be considered and which we had ignored, alternative (9) was the optimum.

Conclusions

The main conclusion from this study is that the optimum solution was the one which at first sight appeared to be the ridiculous one of killing the goose that layed the golden eggs.

Viewed in retrospect the problem appears very simple but it is of interest as an industrial diagnosis[2] on a fairly large scale. There appears to be considerable scope for operational research of this kind at top management level.

The simpler the model the better; it will be the more easily understandable and the conclusions will be more likely to receive acceptance. Furthermore, the tools of accountancy can be just as powerful as other scientific aids in operational research.

Finally, if there is little to choose financially between various courses of action, qualitative factors which might affect a decision should be presented to management.

ACKNOWLEDGEMENTS

The Directors of Imperial Chemical Industries Ltd. are thanked for permission to publish this paper.
The investigation was a team effort and could not have been completed without the very active co-operation of many members of Nobel Division.

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Symposium: Problems in Decision Taking (1)

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THE leather industry is a small industry, particularly if you consider the amount of labour employed. Any one of several companies represented here could employ the total of those in the leather industry. All I feel capable of doing, therefore, is to present the type of decisions I am faced with, or rather the way I deal with them, and perhaps that will open the door for the operational research workers here to solve all our problems.

As I see decisions, the greatest general point that always strikes me is time. There are two types of decisions. In one you have got to make up your mind pretty well instantly, an emergency or something like that, and that type of decision calls for, in an individual, the greatest all-round knowledge of his job. The other is the normal type of decision where there is time available and a man can resort to adequate records and also to advice from experts. I think in industry a lot of people allow themselves to be rushed into making a decision when they should take all the available time permissible. Sometimes it is only after a second or even a third thought that some possible repercussions that may come from this decision become obvious. If you let the Salesman, or whoever it is, rush you into a decision you often live to regret it when you have had a second thought about it.

Now I would like to break down the type of decisions I make into four types. Decisions on buying; decisions on processing; decisions on general policy; decisions on selling.

Decisions on Buying

In all the decisions I have to make the most important thing I find is the keeping of really accurate and adequate records; this can save a great many mistakes. It does not matter what type of decisions you have got to make if you have really top class accurate records behind you, you can at least put away a lot of the doubt regarding the outcome. Take buying first of all, because I think I can give you an illustration in buying as applied to our industry which would emphasise this question of keeping adequate records.

As you are probably aware leather is a by-product industry. People don't raise animals for their hide; they raise animals either for their wool or for their meat and consequently there is no control over the source of supply and the availability of supply or anything else. If there is a glut in the demand for meat, then there are more hides, and so on.

After the last world war the chaos that was left in the human population was very considerable. The chaos that was left in the animal population was even greater, and all our existing knowledge of hides and skins from various parts of the world was obsolete. I was then a technical manager of a factory and we had to start from scratch. I said to the Managing Director "Well, there is only one thing to do. That is, to have parcels of hides and skins as applied to our leather trade from all sources throughout the world and to reassess in really accurate detail for their suitability." We started to do this and we did it for a period of five years. I can assure you that that record is now invaluable to me. To give you one or two of the sort of points that we have to watch for: in an animal, the quality of the hide taken off the animal depends on, first of all, where the animal is living. If it is living in a tropical country you have got to think about tick and various insect diseases and you have got to think of the native taking it off, and you have got to think about the heat and various conditions. Then you have got to think of the age and the health of the animal itself, the standard of living of the people where the animal has been slaughtered. All these have a direct bearing on the health of the animal. Then you get all sorts of peculiarities. In Kenya for instance, where the man is judged on his wealth by the number of animals he keeps, the number of cows he's got, they tend to keep far more cows than they have pasture for, and you get underfed beasts with poor thin hides and so on. All these data have to be related and recorded against every type of hide all over the world. And I don't care how able a man is, or how much experience he has, or what enormous length of memory he has, if he is suddenly offered a parcel of hides and he has not got the adequate recorded details behind him, he is going to find it extremely difficult to buy intelligently in local and competitive markets. And that is the thing I would stress strongly, this keeping of records.

Decisions on General Processing and Manufacture

The next general stage that I have divided my decisions into is General Processing and Manufacture. As you have probably realized I am largely

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a technician, so consequently I find less trouble in technical decisions than in some of the later ones I refer to in selling, where my knowledge of costing and mathematics in general is very weak. The golden rules that I find are roughly these:

One: it is very important to differentiate in a plant between controlled decisions and research decisions, and to see that all research and development is run separately from the manufacturing in bulk. It is surprising how much time and effort is lost by small bulk or semi-bulk experiments interfering with the flow of production. I have always found it advisable in any new plant that I have taken an interest in to set up a Research and Development Section separately and allow the technicians and statisticians and all those concerned in it to work and develop it without interfering with the plant production and then, when that development is ready, to transfer over to the works.

Two: when you put in a process, in general you have got technical management and chemists and so on, and here again they tend to rush and orientate their decisions departmentally. Decisions have to be given time to show that they are not made out of context and applied only to one unit, completely overlooking the interaction of what has come before, and what comes after.

An old saying in the leather trade is, when something is wrong in the warehouse, the foreman of each department passes it back and it finishes up blaming the animal for leading a bad life. That is roughly the way the buck is passed in the factories. This brings me back to the importance of records; if mistakes and what may be called experiences (for that after all is what "experience" is, an accumulation of a lot of mistakes and a few successes) are adequately recorded you don't make the same one twice. I would like to quote an example related to this subject of records. After the war when I was in Germany, and I was talking to a man in a very important position in the I.G. I told him my name and he said "Oh yes, I remember you—you are in our dossier". So I said "What have I done to get on your dossier?" He said "We have a dossier on every technical man employed in any industry with which we have any dealing", and he was able to produce this dossier about me and about others whom I knew very well. And quite frankly you would be amazed at the assistance they give their salesmen, in telling them the weaknesses and characteristics of various buyers. Even to the extent of whether a man preferred a blonde or a brunette.

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Decisions on General Policy

In a business general policy is, if anything, the biggest executive decision, because according to the size of the business, many of the other decisions are made by departmental people. In my particular job, the general policy is largely governed by fashion. We have to watch fashion, we have to read all the fashion magazines, to see what women are going to do next, and there is no logical reasoning in what they do. You take, for instance, high heel shoes. Every woman's legs look better in high heeled shoes than they do in anything else; so they are prepared to walk on great stilts and push their toes down into a very small space and it really is very unhealthy for the foot. And yet, you will get sudden changes of vogue, and we have to watch these kinds of trends, for naturally the whole of our manufacturing programme must be changed to suit them. If the shoe changes from a soft low heel casual shoe to a smart thin leather high heel pump then obviously we have got to change the whole approach. That is the biggest defect on general policy, for we must keep abreast of fashion, and in fact some tanneries have fashion advisers and other experts. The other point that executives have to keep abreast of is the innovations that come about which will call for a revolution in existing methods of manufacture with a lot of new plant and reorganization of existing buildings. In making decisions of that type one has adequate time; so get all the facts and get everyone possible to put a constructive argument together, get them all into a meeting, let them all throw bricks at one another. If everybody co-operates, you get the collective ability of everyone in the business. And then it is up to you to assess all the very able advice which you have got and to make the decision accordingly.

Decisions on Selling

The next section I am going to deal with is selling and here I must confess I am on a very weak wicket. Selling of course fundamentally depends on whether it is a buyers' or sellers' market. A well known colleague of mine, who is a very able man, sums it up very well by saying, "Calf skins have far more veins in a buyers' market than they have in a sellers' market". That is fundamentally the cause of the whole trouble. We take business whether we are facing a buyers' or a sellers' market. In the leather industry, when you are being faced with a sellers' market, your only problems are making enough products. When the market swings as it has—we are now facing a very acute, very difficult time—the sort of

decision that I have to make (and I get virtually no help in it at all) is in an export order; can we make a very large quantity of a specific line at a specific price? Now, if you have a business working ten or twelve lines, and different types, I cannot find a way of allocating overheads in a sensible way.

We try to recommend a way to work out overheads, engage all types of accountants, and none of them has ever given us assistance so that we can take an order and really know we are making a profit on it. That may sound an exaggeration, but I can assure you it is not. It is all very nice to make a product and find out Tom Jones is selling it for £2; then if our costs are less than Tom Jones', we can sell it for 39/–. But if you are not sure about your costs, and an American comes along and asks you to quote, what are you going to say? How are you going to charge your overheads? Where are you going to allocate them? It is all very well if you have got a a plant making one line, but when you have got a mixed production, then I really am stuck.

I am hoping that some genius in the audience can solve it for me because then I shall consider my journey has been worth while. And that is the general approach, gentlemen, to the way I see problems and the way I have tried solving them.

Symposium: Problems in Decision Taking (2)

J. P. WILSON
Tube Investments Limited

IT is always very comforting when one is asked to address a learned Society not because of one's knowledge of the subject but because of one's lack of it. That has happened to me this morning and I am quite appreciative of the low, easy standard that is expected of me. I am merely a Cost Accountant; I feel I ought to tell you straight away that just as a vehicle goes on the road saying "Left hand drive, no signals", I say "Cost Accountant, no Operational Research".

So now we know where we are in one respect. I came here yesterday in the belief that I was to give quite a gentlemanly short paper on Cost Accounting for Decision Making. I was proposing to deal in this paper

with one particular aspect of costing where it is particularly valuable as aiding efficiency on this one narrow subject. After discussing this matter with the organizers last night I found that I was wrong and what I was really required to do was to shout my wares as a cost accountant as raucously as possible. So my aim is to make some claim for cost accountants in their capacity as providing information to management for decision taking. The first thing that I would like to say is that cost accountants are the general practitioners in this business. You members of the Operational Research Society may, of course, be Harley Street; but there are many of us. Incidentally judging from the list of attendances here, you have a very fashionable practice. Let us say the cost accountants are in general practice as the people that have to help management to make its decisions. If I have said that you are the glamour boys in this business, you would probably describe cost accountants as the blunt instrument boys; and I agree that we as cost accountants still have a great deal to learn. Mr. Crack in his paper has complained that the accountants did not give him the answer he was seeking. I will go no further at this stage than to say that cost accountants are still learning and are developing techniques which I hope will help to give better answers to management for decision taking.

Now, what are costing and cost accounting? They are really only an extension of accounting—the function of providing information to management. We claim that function just as you do and we say that we are providing management with information really for two reasons: firstly for taking decisions and secondly for exercising control.

In respect of taking decisions there is one technique of costing which you have probably heard about, and that is marginal costing. Marginal costing is a conception of costing, not a method of costing. It is a conception of costing in which you take account of the fixed expenses and variable expenses of the particular firm. Mr. Crack has been saying that he does not know how to apportion his overheads; perhaps the answer may be that for some of these decisions it is not a problem of apportioning overheads, because fixed overheads at least are not relevant in this particular situation.

What are the sorts of decision in which marginal costing can help management? The first example is selling-price decisions. In this respect please remember my claim that cost accountants are general practitioners and that decisions on selling prices are being made day in and day out in many concerns up and down the country. So the cost accountant can help with regard to selling prices, particularly for example when deciding what

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price we can quote for this Chinese order, or for that order in some other market where we can dispose of some of our surplus production. Another example of a selling-price decision is the determination of the optimum selling price and volume. That of course applies where the demand for any product is elastic. Another decision, more common where there is a sellers' market rather than a buyers' market, is which orders to select or in which particular direction to make the sales effort. In finding what sort of business it is wished to attract, or in deciding whether to make the product or buy it outside, the technique of marginal costing can be of great value.

Then there are decisions regarding methods of manufacture—how to make a particular product. Is it better to have one man working one machine, or two machines, or can he work three machines? In problems of this description costing can help management find the answer. Now if I can come back to this technique of marginal costing for a few moments, just in order to put some solid content into my short talk, the criterion of profitability is what is known as "contribution". Contribution is, of course, a cost accountants' technical term. It is the difference between the sales price of the product, or the revenue to be derived from the operation you are investigating, and the marginal cost of making that product. Marginal costing is really your out-of-pocket cost. So forget your fixed overheads; contribution is what is required for profitability, because your fixed costs go on in any case.

There is also a refinement of this conception, namely that you should seek to obtain the highest contribution per unit of whatever is your limiting factor of production. For example, let us say, as does happen in some industries, that material is in short supply. Now obviously in selecting orders, you would wish to obtain orders for those products which give you the greatest contribution per unit of that limiting factor. So, if material is short you would want the work which gives you that greatest contribution per pound of material used.

Now you might say this is not the traditional costing where you put on all your overheads. I say that it is only one conception of costing, but it is the conception of costing that is particularly useful in certain circumstances of decision taking and it is frequently the only way of giving information to management that will not mislead them. But there again it is something that might be a little dangerous, if it is misused. We all know the danger of cutting selling prices; although you may be making some contribution and so getting something back over and above your marginal cost, if by doing so you are forcing down the level of selling prices in the industry generally,

it may be a mistake. So, this particular technique of costing must be used with great care.

Well now, that is a claim on my part that a cost accountant has a definite technique here which is of value in this problem of decision taking. If I may become more general for my remaining few moments, I would like to ask you to consider the economics of Operational Research, as I assure you that cost accountants are all the time considering the cost of costing—considering to what extent it is economic. The cost accountant is always searching for the philosopher's stone and his particular philosopher's stone is this: the costing system that will give as much information as possible at all times and as cheaply as possible. Now there are limits to what you can do. You can't have all the information on tap all the time at an economic cost. And, therefore, the cost accountant is seeking that sort of costing system which will enable him to have the salient information on tap all the time, or in a form so that he can obtain what information he requires. The cost accountant finds, of course, that there are certain things he cannot do. I will ask you to consider whether Operational Research is always economic, or whether in some cases it is more economic to be ignorant.

A further point that I would like to make in this statement of claims for the cost accountant as distinct from the operational research man, is that the cost accountant does have an advantage in that he sees the business as a whole. He keeps the cost accounts for the whole of the business, and therefore he can see the various problems that management have to face in the context of the business as a whole. I have also said to you that the cost accountant is concerned with producing information which is not only a basis for decisions but is also a basis for management exercise of control. The cost accountant is not making specific investigations only, but he has a responsibility for recording the whole story of the business. And, therefore, the cost accountant might be in a better position to give information to management than the operational research man, if the operational research man is applying himself to one problem at a time and is not seeing continuously the whole picture of the business.

I would readily admit that the cost accountant is only one part of a whole, for at the present moment the cost accountant uses very extensively the results of the Work Study Department in carrying out his job. I rather submit to you that operational research might bear the same relation to costing as work study does to costing: that to a large extent they are complementary. And I rather see the cost accounting process as the last

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process, the final process, using the results of work that has been done by operational research.

Finally I would say this, that on arrival yesterday evening, I met an old friend who asked me: "What are you doing here? and in what capacity have you come—as a disciple or as an unbeliever?" I hastened to tell him, as I assure you gentlemen, that I come as a disciple; perhaps a little reluctantly, for I am not going to apply for my party card, but I would claim to be a fellow traveller.

Symposium: Problems in Decision Taking (3)

Z. M. TARKOWSKI

To take a decision in the absence of adequate information is always difficult. The difficulty is further increased when various experts offer contradictory advice. More especially, when the advice is not obvious or easily intelligible on common-sense grounds and, even worse, goes against the spontaneous inclinations of a manager based on his intuition and experience, it may produce a strong emotional reaction and be a cause of considerable stress.

Examples of this are provided by the well known difficulties of accepting recommendations of an operational research team referring to the level of stocks or the range of products. Objections, predominantly of an emotional nature, are supported by invoking tradition and by arguments based on apparent commonsense. These objections are counteracted by mathematical or statistical reasoning and it is hardly realized that in a state of emotional stress the ability to follow new and unfamiliar ways of thinking is very seriously diminished.

An analogy has been drawn already during the present conference between operational research and medicine, in so far as there are some similarities between medical and industrial diagnosis. It may be useful, therefore, to point out that the understanding of the personality and the moods of the patient are almost as important to the successful practice of medicine as is the understanding of the physical processes taking place in the patient's body. To a biochemist, whose task is to analyse a sample of a body liquid, the personality of the patient may be of no import and he may be able to confine himself to the strict limits of his scientific discipline.

But a practitioner or a specialist, who has to deal with a human being as a whole, must take these various factors into account and his bedside manner is as much a part of the treatment as are the physical measures.

The same remarks apply to operational research work. An industrial organization is composed not only of buildings and machines, but also of human beings. The materials and the processes they undergo are obviously of great importance and so are the methods of financial and production control. No less important, however, are the personalities, the moods and attitudes, of people who form a part of the whole system. As with the biochemist, a backroom statistician or scientist may be able to neglect the human aspect of the situation, but members of an operational research team who are in direct contact with the management or labour must possess an adequate understanding of these factors and must display, accordingly, an appropriate "bedside" manner. Also, the form and content of operational research reports and recommendations must be partly dictated by these considerations.

A great deal is heard nowadays, in the daily press and in the appropriate technical publications, of executive stress. This is a serious matter and a study of human behaviour under stress would amply repay those whose work consists of industrial diagnosis and cure. In stressful situations human beings generally become unreasonable and suspicious, they may become over polite, unduly submissive, or, more frequently, over-aggressive, and such clarity of speech and thought as they may ordinarily possess is greatly diminished. A person under stress does not have a balanced and integrated view of his situation. Some ideas and notions, some objects and some possible remedies for his problems loom large in his mind to the exclusion of others. This has been named the tunnel effect by some psychological writers.

Furthermore, the worry and anxiety may be such that steps must be taken to relieve them even if nothing can be done or is being done to remove the original causes of distress. This often leads to self-frustrating action. Both these phenomena of stress, the tunnel effect and the self-frustrating action,[1] are best explained and illustrated by reference to the story of a sales manager who was from time to time worried and depressed about the unsatisfactory volume of sales. There was a unique remedy which would overshadow, in his mind, almost everything else. He would summon a sales conference for the purpose of hectoring his salesmen and urging upon them the necessity for greater efforts. That would dispel, temporarily, his worry and depression and make him feel better. At the end of the month,

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however, the volume of business would be shown to be smaller than before, due to the fact that the sales force had been taken off the road for a period. This would increase his worry again and thus frustrate his original intentions.

Consequently, in a state of worry people cling tenaciously to what is simple and apparently obvious and are more than ordinarily frightened of the unfamiliar. One of the ways to relieve the worry is to shelve the issues and to stop thinking about them. This is another self-frustrating action, as the trouble itself is not thereby removed.

Taking an important decision should not be rushed. All the information available and the opinions of experts need to be digested, the implications considered and the risks assessed. This is often referred to as the incubation period by psychologists. Only then will a conviction emerge, and, associated with it, the willingness to take the risks. In a state of stress, however, the need to think out decisions imperceptibly shades off into indefinite postponement.

On such occasions a report suggesting an action, apparently contradicting the established practices and the natural inclinations based on intuition and experience, does not help matters. Moreover, the unfamiliar reasoning and language supporting the recommendations will only contribute to a strong tendency to reject the advice off hand or to shelve the matter indefinitely.

There is thus a great need for patience, for persuasiveness, and for clarity of exposition. These are not easy to achieve. Perhaps we may remind ourselves that the purpose of this conference is to give representatives of commerce and industry an opportunity to discuss current applications of operational research. Although the proceedings are primarily for the benefit of the non-specialists and many managers unacquainted with operational research are present here, we have heard the following words used: *triangular* and *exponential distribution*, *sub-optimization*, *Monte Carlo method*, *sampling error; hydraulic*, *gaseous* and *probability models*, *chi square distribution* and *matrix algebra*. It might be salutary to remember occasions when several doctors, in our hearing, were discussing our ailments in technical terms. To quote[2]: " . . . thou hast men about thee that usually talk of a noun and a verb, and such abominable words as no Christian ear can endure to hear".

REFERENCES

[1] Some of these points have been discussed more fully by the author in *Engineering* (Jan. 3rd, Feb. 7th, Mar. 7th, Apr. 4th, May 2nd, 1958).

[2] SHAKESPEARE, W., *King Henry VI, Part II*, Act IV, Scene VII.

Discussion

THE fact that this was an open meeting comprising both operational research and managerial workers could explain why the discussion after each paper rapidly turned to familiar general topics such as "what can operational research do?" "is it different?" and "where does it fit in in industry?"

The case examples were variously described as being appropriate for method study, cost accountancy and hydraulic engineering, such contentions gaining strength from the tactful tendency of some operational research people to describe their specialism as "no more than management would do for themselves if they had the time". The opposite viewpoint, that operational research is "complicated stuff", was also well expressed along with the difficulties of defining problems in the first place and a warning of the dangers of too hasty self-diagnosis by management of their difficulties and problems. Although the final result may appear in one guise or another it is rarely possible at the outset of an operational research investigation to say "this will be costing" or "hydraulics" or what you will. General acceptance of this point would have left more time for discussion of fruitful issues.

Both managerial and operational research representatives saw that there was little point in inter-disciplinary disputes and anticipated B. M. Brough's concept of an internal consultancy service bringing together operational research and other management advisory services as a desirable development. No one suggested a name for it. Several managers, however, doubted the wisdom of burdening the Chief Executive with yet another function reporting to him. Suggested alternative locations included the Research Department, but there was said to be no extant example of a really thriving operational research section so located. It is quite possible for operational research to become just another form of departmental special pleading instead of acting as an organization-wide method of reaching decisions.

Accepted ideas on a proper "span of control" for the Chief Executive or General Manager should be modified to take account of operational research. The top level task of resolving inter-departmental conflicts arises partly from conflicts of objectives, a situation which operational research should be able to put right. Clarification of the process of reaching decisions should help here too in reducing the burden of policy formation.

A decision must involve a choice between acceptable alternatives. If there is no choice, or if one course of action stands out as unquestionably

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the best, there is no decision involved. To select a course which is better than the alternatives requires a scale of values and operational research proceeds by defining such a scale and finding the proper place on it for the alternatives, however complicated they may be. Decision making by operational research, therefore, requires enumeration of alternatives, a scale of values, information, and time. If the large decisions are correctly taken, the need for smaller ones may disappear. The use of operational research to establish optimum working rules was cited as a case in point.

Mention by Mr. Hankin and Mr. Hepburn of the cost of undertaking the projects they described, raised questions of the value obtained. In the latter's case the saving was many times the cost, the conclusion not immediately obvious, and the solution had been implemented. The study of tube station passages largely confirmed previous thought on the subject but did so in numerical terms: the value of the data lies in its contribution to the theory of passenger movement and in particular, to finding the economic balance between those various parts of a tube railway which influence its carrying capacity.

Difficulties in counting the cost are not confined to operational research; several speakers pointed out the shortcomings of costs as a basis for decisions. The need to consider opportunity costs and the problem of allocating expenditure to "fixed" and "variable" categories were mentioned. Mr. Wilson's claim for cost accounting to be the integrative element in a firm was naturally challenged by operational research people. Cost accounting is an essential part of the nervous system of a firm and, because of its continuous influence on decision making, deserves more attention from operational research. As a constructive suggestion, Mr. Crack's pricing problem was cited as one which costing could not solve but which costs plus games theory and linear programming might.

Mr. Tarkowski's strictures on unclear reporting and jargon fell on receptive ears (it had already been necessary to explain to those not in the secret, that JORSA is not the Goddess of operational research—her demise was also remarked upon). Most managers and many operational research people had little sympathy with discussions of "what is a model?" and of the methodology of simulations. On the other hand, the manager who wanted to receive a plain answer to his problems, with no explanation of how it was arrived at, was considered atypical. Reluctant scribes (and readers?) were glad to hear of the greater success achieved with oral presentations of operational research results as an alternative to reports.

Operational Research as a Staff Service for Management

Presented at the Operational Research Society's Open Conference at Harrogate, 21–22 May, 1958

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TO-DAY operational research is gaining acceptance as a means of providing data to aid decision making and its techniques are becoming more widely recognized. But there appears to be far less general understanding of the provisions necessary to ensure the effective employment of operational research in the organization it is intended to serve. The indications are that where the function already exists its work is often unnecessarily restricted because it has been placed in the wrong organizational context; elsewhere, its introduction is being hampered by failure to deal adequately with existing organizational issues and provide a satisfactory environment in which the activity can succeed.

When planning the introduction of any new specialist function into an established organization three principal questions arise. Firstly, where should the function be placed in the structure and with what other activities, if any, should it be associated? Secondly, what responsibility should it carry, and what relationship should it bear to line management and to other specialist functions? Thirdly, apart from intellect and technical skill, what particular qualities are needed in the personnel who are to run it in relation to their organizational role? In seeking an answer to these questions the exact nature of the function and the special contribution it is designed to make towards the objectives of the organization as a whole need careful analysis. Until the problem is approached rationally in this manner no basis exists for determining the conditions necessary to secure the smooth and effective integration of the function with the pattern of activities already in being.

In the case of industrial operational research there is a broad measure of agreement that the function constitutes a management tool to be applied in seeking solutions for specific problems. The activity involved is commonly defined as the application of scientific method to provide a basis for action by executives directed towards increasing the efficiency or

effectiveness of the operations under their control. This, and other definitions like it, determine the research nature of the function and that it is concerned with increasing industrial efficiency, but they do not adequately distinguish it from other activities with a similar objective. In consequence the special attributes of operational research that justify its existence as a distinct specialization are often misunderstood and there is confusion as to its connections with other better known activities also concerned with studying the manner in which work and operations are conducted. To a certain extent the specific nature of operational research becomes more apparent after seeing examples of its application, although the background against which it has evolved warrants examination to obtain a true perspective of the function's place in the typical organization.

Management Problems and the Scientific Approach

The application of scientific methods of enquiry to the problems of business is by no means a new insight as these ideas have been generating and gathering momentum for almost two centuries. Perronet was making time studies of pin manufacture in 1760. Babbage published his work, *Economy of Machinery and Manufacturers*, in 1832. However, it was not until the new industrial economy became established during the latter part of the last century that the thinking in this field began to acquire any coherence and emerge as a recognizable concept. The first notable practitioner of the scientific approach to management problems was F. W. Taylor, whose studies of industrial work were begun in 1881.

Taylor and his associates, of whom Gilbreth is the best known, successfully pioneered the application of scientific methods of investigation to improve and simplify the physical work involved in manufacturing operations. They also approached questions of workshop organization, planning, costing and so on in a similar spirit of scientific enquiry in an endeavour to acquire knowledge systematically which would form a basis for the establishment of some principles of shop management. Their work was almost entirely confined to the type of problem arising at the first level of management: that is, problems connected with the most efficient or economic way of performing a given job operation, or series of job operations. Nevertheless, within this area much creative thought was produced during the early years of this century and generated a philosophy that might have been expected to spread more rapidly into other management fields.

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But the application of this more analytical approach to the wider problems of business management was retarded by lack of people with comprehension of the techniques involved. At this time, before the days of more liberal education, almost the only group of people in industry with any sort of scientific training were the engineers and mechanics who were chiefly employed on the manufacturing side of the metal industries. For this reason the use of systematic techniques of analysis and measurement were for many years restricted to problems occurring within or close to the engineering workshop. Another consequence of this situation was that engineering personnel also became involved with the introduction of other new functions created by this new approach to management for which some mathematical ability was an advantage, such as budgeting, cost control, and wage analysis, even though this work required no basic engineering knowledge. It thus came about that some engineers acquired a special competence in dealing with management problems, and indeed until comparatively recently the majority of consultants in this field were traditionally people trained in engineering disciplines.

Following World War I, there was a general transference of this American philosophy of "scientific management" to Europe but very little progress was made during the twenties and thirties to extend its application outside the engineering field. This period produced very little new thinking and only gave rise to excessive refinement which led to the creation of narrower and narrower specializations. Time and Motion Study, Work Measurement, Work Simplification, Methods Engineering followed each other in quick succession. Although each new wave of doctrine may have stimulated some fresh interest, the repetition of old ideas under new labels contributed little to promoting any wider understanding of the need for the systematic study of management problems in their broadest sense. In Britain, apart from a few notable instances of industrial investigations undertaken by scientific workers from the Universities and research institutes, usually at their own instigation, no efforts were made to organize for the study of work situations or business operations on any more extensive basis until the beginning of World War II.

War-time problems, however, led to two quite separate and important developments. On the one hand, concern at the rapid increase in the size of Government Departments and in the volume of their office work led to attempts to apply to clerical tasks the same techniques of work simplification hitherto confined to manufacturing operations. The study of office methods had, in fact, been begun by a small independent section of the

Treasury created in 1919 and a similar section also existed in the Post Office. But it needed the burden of work arising from the creation of wartime controls to urge the development of this activity. In 1941, the term "Organization and Methods" was coined to designate the specialist units which were then set up in all the large Government Departments to investigate and advise on problems of office management. Although O. and M. activity is generally conducted by people lacking scientific training and does not use any very precise techniques of quantitative analysis, it may still be broadly classed as scientific in the sense that its whole aim is to provide an independent view point from which problems can be studied objectively. In this respect the function may claim to have extended the scientific approach into the sphere of administration even though, from the scientific point of view, the methods of investigation employed often lack precision.

In another direction, war-time problems also produced a significant development which originated in the field of military operations. During 1937 one or two civilian scientists working on the early development of radar had undertaken a study of the way in which information on the movement of enemy aircraft obtained by this new system could be best utilized by the defence organization. Although long associated with the design and improvement of weapons, this was the first instance of trained scientists being employed to study the manner of their use. The results proved so valuable that on the outbreak of war a small group of scientists was stationed at H.Q. Fighter Command to continue this type of work. Soon all three Services began to call on scientists in increasing numbers to assist with the many involved "action" type problems that confronted the military executives. This activity, which became known as operational research, matured rapidly and became one of the principal scientific features of the war. It proved so successful that a very strong sense of scientific advisers at the highest levels of decision making. In particular, it demonstrated that the techniques of statistical analysis could be used to assess complex operational situations in which the phenomena had previously been considered too intangible or random in nature for decisions to be based on anything other than judgement or tradition. Above all it demonstrated conclusively that men with imagination trained in the discipline of the pure sciences could make a valuable contribution to the solution of complex problems, even though these fell entirely outside their particular fields.

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Post-war Developments in Industry

Following the war, the expansion of the economy and a review of our position as a manufacturing nation in relation to world markets, heightened appreciation of the need to improve the operating efficiency of industry, or "productivity" as it is now called. This gave fresh impetus to all activity concerned with searching out just where and how costs could be reduced and output increased.

The old techniques for simplifying manual operation and increasing labour output were revived, collected together, and presented as Work Study. Under this new label methods study has been successfully introduced into many concerns where previously there was no organized activity of this type and generally the function is now widely accepted and has become well established in manufacturing industry. In a number of firms Work Study has been expanded to embrace investigation of the wider aspects of production, such as factory organization, materials handling and planned layout. And in some, the function is making good use of relatively new techniques, such as activity sampling, and is also paying increased attention to human factors in the working environment and in the design of working equipment. However there are clear indications that many firms still hold to the mistaken view that Work Study is nothing more than a synonym for time study and wage incentive schemes. In such cases, the function cannot be said to have achieved anything of its potential as an aid to decision making.

A somewhat similar situation exists with regard to Organization and Methods. This activity has now become widely accepted in central and local government as well as in many large firms, but like Work Study, its application has been largely confined to problems of method. Only in a few concerns has the O. and M. function achieved any status as a management tool capable of being applied to the study of organization structure, and to problems of administrative procedure and information handling in their widest sense. Although the introduction of integrated data processing systems based on the use of electronic computers is now beginning to compel management to employ the function on a far broader basis than hitherto.

With operational research the pattern of post-war application has been less consistent. A few of the larger industrial companies and some of the nationalized industries were quick to appreciate the analogy between military and industrial operations. They recognized operational research

as a new and significant development and set up independent sections specifically to perform this function. Within some of these concerns, operational research has advanced rapidly and is well established; but in the main it has been slow to gain acceptance and its application has progressed only tardily. In the military sphere the use of independent observers to study operations in being was completely novel as were many of the problems involved and the urgency of the situation stimulated attention being given to any fresh approaches. Because the scientists were usually civilian and the executives had military rank, the growth of mutual co-operation between them was assisted rather than hindered in spite of their widely differing backgrounds. Further, the early results were so outstanding that the value of the function was readily appreciated by the military managers. In industry, on the other hand, approaches of this type had frequently been attempted over the years in different guises, by people varying in competence, and with indifferent results. Moreover, ignorance as to exactly what was meant by the old and somewhat misleading term "scientific management" had created some deep seated prejudices and suspicions of any function including the word "scientific" in its definition. In consequence, there has been some tendency among the less well-informed to regard operational research with some scepticism.

Thus in British industry to-day we can identify three quite separate specialist activities concerned with the study of factors having a bearing on productive efficiency. Each is, broadly speaking, attempting to approach problems in a spirit of scientific enquiry using whatever techniques of measurement are appropriate, or it is capable of applying. Each is being employed at a somewhat different level in the organization and is orientated towards a particular aspect of its task. But even though their approach and ultimate objectives are similar, little attempt has been made to integrate their efforts and there is considerable variability in the way they are organized and operated. Moreover, the anomalies of terminology that have arisen throughout the whole field and the degree to which the claims made by the protagonists of the different specializations often conflict, have created a situation which can, and does, lead to overlap of function and confusion of thought. This situation is scarcely conducive to achieving any clearer understanding of these tools of management or to the furthering of their development.

Factors Determining the Effectiveness of Operational Research

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Operational research is distinguished by certain fundamental characteristics from other management tools, such as Work Study and O. and M. First, it is normally conducted by people trained in one of the basic sciences and thus specially disciplined in reasoning through problems in a precise and systematic manner. Second, it makes wide use of mathematical and statistical methods, which make it possible to penetrate complicated action situations involving several variables and analyse them quantitatively. It is these techniques in particular which enable operational research to investigate the interaction between the different functional activities of an industrial organization in relation to the operations of the concern as a whole. This is in contrast to other management tools which in the main have been chiefly employed to improve operations within a given function.

Subdivision of organization into functional elements is frequently adopted in order to gain the benefits of specialization and this creates separate units of organization each with its own particular task to perform; accomplishment of all these individual tasks being important in the attainment of the overall task. But an inevitable consequence of this form of organization is a tendency for each functional unit also to develop its own individual objectives, which frequently conflict with those of other units and may even become inconsistent with the primary objectives of the organization as a whole. This gives rise to a distinct class of management problem for which an overall solution must be sought based on an examination of the alternative courses of action open to the different functional executives and analysis of their likely effect on the complete operation. It requires the optimum overall solution to be obtained before functional objectives can be rationalized and made compatible with the concern's primary objectives. This balancing of functional tasks can be a major factor in securing maximum operating efficiency in all sizeable concerns in which there is any functional division of production activity, and it is the capacity to deal with organization-wide problems of this kind that gives operational research its special potential as a management tool.

However, whatever the problem, the quality and effectiveness of the contribution that operational research can make towards a solution will depend basically on the relationship existing between the operational research function and the executives who constitute the management or decision-making structure of the organization. This relationship is of a

paradoxical nature. On the one hand there are very good reasons for operational research being completely dissociated from line management, yet on the other hand operational research cannot achieve results other than through collaboration with executives.

Unless fully dissociated from executive responsibility operational research loses much of its ability to remain truly objective and hence most of the justification for its existence as a separate specialist activity. Further, the two activities of research and management are in some respects antipathetic. Executives have neither the time nor the outlook to conduct scientific investigations, whereas the research worker on his part cannot concentrate properly on the study of a problem if he is at the same time continuously disturbed by matters of administrative detail. The management outlook which requires that a decision be reached on a problem is in itself opposed to the scientific outlook which defers making any prediction or assessments until the problem has been thoroughly investigated and any theories have been conclusively proved or disproved. This makes it essential for the operational research function to be set up independently of the executive activity it is designed to assist.

To compensate for this organizational separation, strong bonds of mutual confidence need to be created between O.R. workers and line executives. This is the sort of confidence that can only flourish from each having a proper understanding of, and respect for, the other's job. In the industrial world decison making is a process which cannot wait until the ideal or optimum solution has been found for every question. The essence of the executive's job is that he carries the responsibility for making decisions and it will always fall to him to assess the pressure of events and the extent to which decision taking can be deferred, pending the collection of more detailed evidence. Moreover, there are very many executive problems to which operational research can make no contribution, and many in which only certain aspects can be analysed quantitatively leaving the executive to assess the remainder on the basis of his experience. In almost all management questions involving operational research, a satisfactory and timely decision will usually depend on some degree of compounding between the results obtained by research and the judgement of the executive concerned. But the right compounding of effort will not be achieved unless executives have a good insight as to what the operational research tool is capable of achieving and an appreciation of the time and effort required to conduct the necessary studies. Neither will it be achieved unless the O.R. worker has a sympathetic understanding of the practical

distractions which beset the executive and is familiar with his way of thinking and acting. It is essential for him to appreciate that operational research is research with a severely practical goal and that his prime function is to provide a service for the executive as and when his help is requisitioned. His job is to present his result and recommendation in such a manner and in language that will enable the executive to grasp their relevance to the particular problem on hand. But the ultimate responsibility for taking the decision rests solely with the executive.

Communication is also an important factor in the maintenance of these relationships. When engaged on a specific problem it is necessary for the O.R. worker to have free access to all the relevant data. Although, where he is acting as an outside adviser by invitation, the executive concerned will naturally make all existing data readily available and provide for facilities for the collection of any new data that may be needed. Nevertheless, apart from contact on specific assignments, there should be opportunity for frequent exchanges of information between operational research and management at all times. Without it executives cannot be kept in touch with developments in operational research technique and cannot be expected to identify problems that warrant operational research. In addition O.R. staff will be left in ignorance as to the sort of problems that executives are facing. Past experience has shown that some of the big successes of operational research have resulted from the study of problems that executives had not hitherto recognized as significant or amenable to investigation by this approach. For this reason it is desirable that senior O.R. staff should be given an opportunity to sit in as observers on standing and special committees at which current management matters are being discussed, so they can themselves suggest specific problems for investigation.

Responsibility for the establishment and maintenance of the conditions essential to the conduct of operational research falls on both management and operational research workers. The vital element is the creation of a harmonious working relationship between scientist and executive and this can only be born of mutual confidence. Since operational research is only justified if it leads to executive action, the initiation of operational research investigations must be controlled by the executives who are the action takers and the O.R. worker has to acknowledge the limitations of his function. He must remain sympathetic to the executive viewpoint, and respect that he is dealing with people who almost certainly have acquired considerable practical knowledge and experience in their particular

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technical fields. On the other hand it is incumbent on management who are responsible for designing the organization structure to create the right environment in which these relationships can mature.

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The Place of Operational Research in the Organization

The location of operational research function in the structure will depend to some extent on the size of the concern and the basis on which it is organized. The important thing is that operational research should be in a position to conduct overall action studies and investigate impartially all the separate functions involved. If operational research is tucked away in a corner of the organization chart and buried within a particular department or function it ceases to be operational research. For this reason, and to preserve the condition of detachment, the O.R. section should be responsible independently to the executive at the head of the organization it is intended to serve. It cannot study organization-wide problems effectively if its activities are confined to, or restricted by, the work of the individual department or function to which it belongs.

In a large concern such as a transport or public service undertaking in which the primary divisions are functional, one would expect to find the O.R. section coming directly under the chief executive of the undertaking. In a large manufacturing organization or group with several self-accounting divisions or independent plants it may sometimes be advantageous to have a separate O.R. team under each managing director or works manager, with one or two senior operational research people to co-ordinate divisional activities and provide additional central advice and assistance from the group headquarters as and when needed. In general, however, there are a number of advantages to be gained in having one central unit at the head of the organization. Operational research is expensive and unless conducted by suitably qualified people is best not attempted. The efforts of a few capable people are likely to be better utilized if they are concentrated in one section. This then provides a focus for all operational research work in the concern and allows approaches and techniques developed for dealing with a specific type of problem in one department to be readily applied to a similar type of problem in another department. Further, few concerns can provide for all the operational research they are likely to require and it is often preferable to plan for occasional projects to be undertaken by professional consultants or research institutes. But it is not easy to make the best use of this kind of outside help in the absence of a central nucleus

for operational research work within the concern which can be used to identify particular problems and introduce outsiders to the ways and customs of the organization.

In many firms there appears to be a prejudice against the creation of small independent staff units reporting direct to the top management. Consequently attempts are often made to meet the requirements of operational research and still keep what is regarded as a tidy organization by locating the function within some other department that has organization-wide interests, such as the Research Department, and providing for its services to be made available to other operating departments by arrangement. But under these circumstances operational research unavoidably becomes involved to some extent with the interests of the department with which it is associated, and there is inevitably a tendency for this department to monopolize or attempt to direct its activities and for the function to serve its department rather than the organization as a whole. Other departments then find it difficult to accept that their projects will receive the same priority or that any overall studies will be conducted without departmental bias. Once the function loses its impartiality other departments will also become reluctant to seek its assistance and may become suspicious of its motives, particularly if it is associated with any other function, such as Finance, whose primary role is one of control.

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Preserving Internal Relationships

The position of an independent O.R. unit responsible direct to the head of an organization is delicate. It is one that can easily become invidious unless the role of the unit is adequately defined and those concerned are appreciative of the need to establish ethical relationships within the organization and are scrupulous in observing them. Difficulties usually occur in connection with problems that are the sole concern of a particular department. In all except the smallest concerns, decision making on action-type problems takes place in several areas of the organization, and problems of an inter-functional nature can also occur wholly within a department. Full and effective use of operational research requires that it can be brought to bear on these departmental problems as well as the overall problems of top management. But if it is directed exclusively by top management and is imposed on subordinate executives, the relationship between them and O.R. staff will almost certainly be jeopardized.

Should lower management levels come to regard operational research merely as an extension of top management control it will be viewed with

suspicion and there will be reluctance to provide it with access to the data it needs. They will also be discouraged from initiating operational research studies of their own volition due to fear that the results obtained will be used as a check on their performance. Operational research is not a control device but a tool to aid decision making. Its effectiveness depends fundamentally on the existence of a working partnership between O.R. worker and executive. If top management mishandle the tool and attempt to use it in any sense to investigate or assess the efficiency with which subordinate executives are performing their delegated responsibility as defined by the organization, this co-operation will be destroyed and the edge of the tool will be blunted. Executive efficiency is not something which can be subjected to a detailed check or audit. Controls of this nature make little if any positive contribution towards securing improvements and only serve to stifle managerial initiative.

This establishes the need for operational research to exist as an advisory function providing a staff service to executives at all levels. Under this concept the managing director or chief executive does not direct the activities of the O.R. unit, except in relation to problems which must be decided at his level or at Board level. He merely ensures that the function is in a position to provide the type of service needed and creates the atmosphere in which it may be freely used. Subordinate executives are encouraged to consult the unit direct and the chief executive does not interfere except to assign priorities in the event of conflicting demands. Initially it may be necessary to stimulate the application of operational research by developing an appreciation of the function through special executive training. Subsequently if the chief executive feels that operational research participation is necessary in a particular departmental problem he may, of course, advise or order the responsible executive to seek the assistance of the operational research team, but he never gives instructions direct to the operational research unit to investigate activity within a subordinate unit and report back.

When acting in an advisory capacity in this fashion it is incumbent upon the head of the O.R. section to preserve confidences. Subordinate executives will not willingly call in operational research assistance unless they are assured that confidences will be respected. For this reason any reports produced following an investigation should be treated as confidential and given no wider distribution than to the executive requesting the study. Should it then prove necessary for a report to be forwarded to a higher level, it should be sent by the executive concerned up his management line,

not via the O.R. section. Top management need to understand the importance of preserving confidence in this way and should be careful not to compromise the position of the O.R. worker or embarrass him by requesting information which he may have obtained as a result of his privileged position as the confidential adviser to a subordinate manager.

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Controlling Operational Research Activity

The idea of an independent staff unit responsible to top management yet providing advisory service throughout the organization runs contrary to conventional notions of organization and is frequently rejected. The argument is often advanced that close control has to be exercised over operational research activities through direct top management supervision in order to ensure that operational research effort is properly utilized and adequate backing can be given to its recommendations. It should be remembered, however, that the prime object of operational research is to aid decision making, and in any rational organization this is an activity which is already decentralized to an appreciable extent. Therefore, responsibility for initiating operational research assignments should be delegated in the same way, and to the same degree. There is also no reason why top management should need to see every operational research report to maintain control over the use being made of the function. Top management should not be concerned in the detail of investigations made at subordinate, decision-making levels in areas where responsibility has already been delegated. The fact that the operational research section has been consulted by the executive concerned in relation to a particular problem and has made his recommendations should be sufficient. Information of this sort can, without any breach of confidence, be simply listed in a periodic progress report from the head of the O.R. unit to the chief executive. Copies may also be sent to the head of each major department. If it is then considered that the work done by operational research in relation to some particular problem has not led to any significant improvement or change as reflected in other management control data, such as routine accounting figures or production statistics, the chief executive or departmental head concerned can call for more detailed information as to the results of the study and the recommendations made. But if he is wise he does this through the management line and not through the head of the O.R. section.

To think that no worthwhile results will be forthcoming unless top management personally directs the activities of operational research and

takes steps to back up its recommendations is contrary to all the precepts of a management structure involving delegation of authority and responsibility. The production and maintenance of operating efficiency in any concern is unlikely to be forthcoming in the absence of scope for personal initiative and confidence in the individual. This requires that each executive be left reasonably within his assigned area of responsibility to work out for himself the solution to his problems, is provided with the tools necessary to achieve this end, and is instilled with the necessary determination to put them to good use. Thus, in providing the operational research tool, top management should concentrate first on recruiting competent personnel who will apply themselves to giving an impartial staff service for management, and second on creating an environment that will stimulate proper use of the service. Under these conditions, and provided that top management have confidence in the head of the O.R. unit not to undertake any study unless he is satisfied that the solution will contribute sufficiently to the good of the organization to justify the effort needed to obtain it, there should be little doubt that the activity will develop in a healthy manner.

Relationship between O.R. and Other Management Tools

So far consideration has been confined largely to operational research activity. But this is not the only industrial specialization concerned with studying the running of the business, to work in collaboration with executives, or to deal with questions that are organization-wide. There are many multi-functional problems where systematic study by a detached outsider can make a valuable contribution to decision making but where there are severe limitations on the use of the quantitative analysis techniques employed by operational research units. Questions of organization structure, the design of management controls, the development of improved methods of information processing and planning the layout of plant, are examples of this type of problem. In some concerns these and similar overall problems are investigated under the aegis of operational research; however, this would seem to be creating duplication with other established activity such as O. and M., and the methods engineering element in Work Study.

At present, with the anomalies of terminology that exist, the way in which all these different management tools are organized in British industry presents a confusing and overlapping pattern. In some large

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companies we find three quite separate activities of work study, O. and M., and operational research all reporting directly or indirectly to different directors or senior managers. Elsewhere there is evidence of the newer functions being swallowed by older specializations that are more strongly entrenched. Thus work study is frequently associated with production engineering or production planning, and sometimes operational research is also found at this level. O. and M. is sometimes allied to the personnel department and linked with job evaluation, sometimes it comes within the financial orbit and is linked to internal audit. There is one example of O. and M. existing within the orbit of operational research, also at least one case of operational research existing under the aegis of work study. In another undertaking, operational research and some elements of work study are grouped under the heading of O. and M. This situation scarcely helps to promote any clear understanding of the use of these tools or to further their development.

A closer alliance between O. and M., work study and operational research would seem to offer many advantages. Frequently recommendations arising as a result of operational research study require the practical hand of a systems specialist or methods engineer to put them to effect. Conversely, the organization specialist or work study engineer who is familiar with operational research methods gained through close day-to-day association with operation research staff can often in the course of his own investigations diagnose other problems that warrant deeper analysis by more refined techniques of quantitative analysis. His more readily understood approach can then do much to ease the acceptance by executives of the newer and less obvious techniques. However, in any sizeable organization, the benefits of frequent exchanges and close collaboration between the different specializations cannot be readily achieved unless they belong to a common group and are in frequent day-to-day informal contact. Since all the considerations already shown to be important in the satisfactory setting up of operational research are equally valid in relation to these other functions when applied on a broad basis, there appears to be no reason why all three specializations should not be associated in a single unit.

Technological advance and industrial expansion would appear to require at least some teaming up of the various approaches and techniques that can be applied in the study of the more extensive management problems. To-day with the development of electronic devices and the automatic control of operations they make possible, factory and office are beginning

to acquire similar characteristics and become more closely integrated. In the factory one now finds machinery for the automatic processing of raw materials to create the finished product; in the office there is machinery for the automatic processing of raw data to provide the routine control figures required by management. These advances are intensifying the interaction between functional activities and adding new aspects to the process of decision making. It is becoming much less feasible for an executive to seek advice separately on the different facets of a problem. He must increasingly seek an optimum integrated solution for a whole problem which will deal in a practical way not only with the operational aspects but also with the processing of the information involved and with the methods engineering considerations. Wrong decisions taken during the extensive planning required for the automatic factory or the automatic office can have far-reaching consequences and their effect on the economics of the business may take years to correct. Thus the importance of obtaining sound bases for decision making assumes an entirely new significance. The general tendency is for the output of the machine to assume greater importance than the output of its operator, and in time this must lead to a shift in emphasis from studying the productivity of labour to studying the productivity of capital resources.

The Concept of Internal Management Consultancy

The need would seem to be for a comprehensive management staff service capable of providing executives at all levels with competent advice and accurate evidence concerning the problems involved in the conduct and organization of both the operations and administration of the business. This should include within the one unit not only people trained in pure scientific disciplines but also people trained in the more intuitive disciplines such as engineering and accountancy. Thus although it might be concerned in, say, the setting-up of a wage incentive scheme it should not be involved in the administration of it or with undertaking routine time studies or rate-fixing. Neither should it carry any responsibility for the preparation and issue of routine control data for management, although initially it might be called upon to advise on the control indices which are valid and on methods of obtaining them.

In America, the idea of a unified internal consultancy group to serve management is gaining momentum under the label of Industrial Engineering. It is becoming increasingly realized that the application of

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all the modern problem solving tools and techniques of management become more effective when co-ordinated under one senior member of the organization. Some firms still regard the industrial engineer as a functional specialist with direct responsibility for production planning, work standards, materials handling, methods engineering and the like. But the concept is developing of the industrial engineer as the adviser, consultant and assistant to line management concerned with all factors involved in the efficient conduct of production. The use of the term industrial engineering is apt to be misleading since it suggests that those trained in engineering have some special claim to competence in this field over those trained in other disciplines, but in practice people from many different disciplines are frequently found working together under this label. In this context operational research is regarded more as an extension of industrial engineering practice to make better use of scientific methods and analytical techniques than a separate specialist function. Indeed, the graduate course for the typical American industrial engineer will nowadays include training in the use and application of the basic operational research techniques relevant to industry.

The function of these internal consultancy groups does not necessarily replace the services of external management consultants, rather it complements them. Mention has already been made of the need for at least a nucleus of internal effort to aid the diagnosing of problems and facilitate the introduction of outside aid and this is applicable even to small firms that must obtain most of their help from external sources. In the larger concern a well developed internal group has certain advantages over an outside consultant in that it can acquire the confidence of executives through long standing personal contact, can maintain a feel of the organization, and can provide on-the-spot service. Further it is not subject to the same economic pressure to produce results and is thus better placed to handle lengthy fact-finding assignments and detailed planning and installation work. On the other hand, the external consultant, although he cannot hope to acquire an intimate knowledge of the organization, has a broader basis of experience and can often call on a wider range of specialist skills. In particular, being an outsider and completely independent, he is in a better position to deal with the more refractory organizational surveys and management studies affecting individual personalities.

In Britain the concept of an internal consultancy service has been slow to emerge and so far has gained recognition in only one or two concerns.

It is true that as yet we do not seem to have found any convenient and acceptable term to express this idea; in this country the term "industrial engineering" has a different connotation and implies no particular professional status so is not very appropriate. Within our organizations we also still tend to cling to notions of the scientist as someone not to be trusted outside the research department, the engineer as a man who gets things done in a practical way on the shop floor, and the accountant as the only person capable of keeping figures that mean anything. But it is difficult to see how we can hope to make best use of all the various problem-solving tools of management and apply them effectively to aid decision making in this increasingly complex technological age so long as our thinking remains limited in this fashion by semantic difficulties. It needs to be realized that scientists, engineers and accountants, if they have the right inclinations, can in fact work together as a coherent team. More positive thinking is also needed to overcome the thought barriers created by the confused jargon put out by the protagonists of the various specializations, and the extravagant claims made by some as to the comprehensiveness of their own particular technique or approach.

Primarily, however, it needs to be recognized that the scientific approach, using this term in its broadest sense, is not the prerogative of any one particular specialization but one to be encouraged and applied throughout the whole range of management problems. In medicine and in the social sciences the use of precise methods of investigation and measurement exist in harmony with an intuitive approach used in those areas where the phenomena under study are only vague and intangible. Thus, it is not so much the distinction between exactitude and inexactitude that determines whether or not an approach is scientific, but rather the objectivity, independence and reasoning ability of the people conducting it. In this respect the ability to approach management problems scientifically depends not so much on the staff concerned being qualified in one of the pure sciences or being skilled in the use of a particular set of techniques, but more on their imagination, capacity for logical thought, and above all on the attitude they adopt to the subject under investigation which to a large extent will be conditioned by their organization viewpoint. These are all factors for which management carry the responsibility. They are the doers and decision makers who plan the organization and then recruit the people to man it. In the ultimate, therefore, the quality of the decision-making advice they receive can be no better than that for which they organize.

Closing Remarks by the Conference Chairman

K. PENNYCUICK
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RATHER than attempt the impossible in summing up this Conference in the brief time available, I should like to comment on just some of the points that have arisen.

Very important indeed is the continual insistence that wherever possible a problem is taken from a qualitative into a quantitative form—although we have met occasions where that is not possible.

Next, I would remark on the setting up of the problem, and I will once again draw a medical analogy. If a problem is put to an operational research worker in the form that says, "I have the following difficulties, what do you think can be reasonably done about it?" then the problem is put very much in the spirit of the patient going to the doctor who says, "These are my symptoms". It is unusual for a patient to go to the doctor and say, "I am suffering from such-and-such a disease". If you have a problem stated in this way, as I think it will be agreed that Mr. Hepburn had, "Here is a problem of management: we have different costs in three factories, what is to be done about it?", that is reasonable, for the symptom is stated. Problems put in this way are free from "wrong problem-itis".

A further observation arises from consideration of the list of firms represented, and of the people who are operational research workers and the people who are not. It is very noticeable that it is the very large firms which have operational research organizations—and that is not really unexpected. I have heard it said that in a moderate sized business cost accountants have got the situation so well in hand that all the information required is available. I would like to support Mr. Wilson's view in this matter. As business gets more and more complex, it is very unlikely that the individual cost accountant can possibly have his finger on all the affairs in the business and it is in these sorts of circumstances when there are problems that run well beyond the realms of one department that operational research seems to be so very valuable. Now that, I think, leads naturally on to the question of the level at which operational research reports. We have heard of an operational research team consisting of a professional operational research man, an accountant and someone from line management all working together. The team may include

representatives of other functions. If you are cutting across a lot of departments you have to report to somebody who is above the departmental heads. If you don't, you are going to be accused of special pleading. It does not matter, in fact, whether you are indulging in special pleading, but remember that "Justice must not only be done but that it must appear to be done". If you are dealing with a top level problem, then you must report to a top level man. If you are dealing with a problem which is in the province of a factory manager, you report to the factory manager, but if there is a disagreement you will obviously go higher to resolve your problems. I would also like to suggest, if I may, that sometimes we ask too much about "How can you do this rather better?" or "How long does it take you to do it?" I suggest that the real rewards in complex organizations are very often to be found in the answers to the questions: "Why are we doing this?"; "Are we doing the things we ought to be doing?"; "Is there anything we are doing purely for historic reasons which are now nonsense?"

One speaker talked about making the right range of products. This is a most important question and I think operational research has something to contribute. I have heard it said that if you are classifying various branches of management science, various branches of control in a firm, and having listed everything that is normal, such as Work Study, Cost Accounting, Legal Department, Secretaries, Personnel, Management and so on, unless you have a category of "everything else" you are going to leave something out. Operational research, in my experiences, is only too accustomed to taking on the things that no one else is willing to have a go at. That makes me think that operational research workers, on the whole, are rather rash. That may be so, but it is great fun.

There is one problem which I don't really know how to resolve. It arises from a suggestion by Stafford Beer that operational research isn't very easy. If you go on and take the plea that operational research must always be expressed in very simple terms, you end up with the dilemma that operational research has been so simplified that it apparently is easy—but in fact it isn't. It deals with complex matters where you may have to go into extremely deep arguments. It offers a way of thought into many of these problems which I don't think any other discipline can deal with, though I am not suggesting that operational research is the panacea of all ills. If you take these two poles—great simplicity, highly complex reports—may I suggest to my friends in the Operational Research Society that they have an elegant problem in optimization to reach a happy point between the two extremes.

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Can You Profit from Operations Research?

R. H. ESHELMAN.

Iron Age, 16 *Jan.*, 1958, **181**, *pp.* 71–73.

O.R. is defined as the application of mathematics to management problems to be carried out by a group of people with diverse backgrounds. The key to a good programme is given as:

Definitions of objectives, top management support, trained personnel, continuity as benefit may be months in coming, and good basic data. Record systems may have to be revised. An example is briefly reported of the experience of Argus Cameras Inc.

The Operational Value of Iron Ore.

P. M. FOURT.

Annales des Mines, Feb. 1958, 93–111.

The problem treated is whether it is preferable to enrich iron ore by concentration or to import rich ores from overseas territories. There is a limiting price for new ore such that these are equal for a given cost of iron production; although there is no true market for lean ore, this limiting value is estimated. It can then be decided under what conditions it is profitable to decide to be a purchaser of new ores; whether it is preferable to enrich local ores or to invest in new mines, and whether it is preferable to build new blast furnaces or to enrich the charge. The problem is presented as a typical example of O.R. bringing out the analytical power of the method. The properties of the ores and the requirement of furnace operation are set out and relations between the variables are shown. The findings are then applied specifically to the Lorraine Region for which a limiting price is worked out as a function of pig-iron price, of mining costs, and of blast-furnace replacement. Appendixes give the solution of the four linear equations by the method of determinants and also actual cost and price figures, and the case where the demand for pig-iron is limited is also treated.

How to Rate Executives: Take a Realistic Approach to Rating Programmes.
H. B. CLAYTON.
Iron Age, 30 *Jan.*, 1958, **181**, 63–70.
Difficulties of the rating of managers are outlined in general terms. Performance should be related to specific organization goals which can be refined down to departmental objectives, and communication problems between executives and their superiors are noted. Rating should also be linked with selection and development techniques. The Target technique is described and a survey of over 400 companies is discussed in very general terms. A page of tips on the rating of executives in Do and Don't form is added.

An Application of Operational Research.
JACQUES MELÈSE.
Usine Nouvelle, 1958, Edn. Mensuelle, Mar.; 61, 63, 65.
An actual example is given (largely as an introduction to O.R.) of the planning of a plant for the manufacture of pump parts. An output of 800 pumps daily, each of six essential parts and numerous accessories, was produced in two workshops; the second for assembly, testing and packing, the first in five specialized lines, one producing the accessories. The object of the study was to draw up optimum production schedules with particular attention to maximum service to customers as well as to costs. Three kinds of data were included: statistics of requirement for each main part and of general demand to arrive at stocks for finished products; technical elements such as tool changing, machine stoppages and time for sharpening, and cost elements. In all 25 criteria were drawn up for each pump and coded on punched cards. The data established included cost of service to the customer; costs of any programme for time between start and finish for any period between 4 and 8 days shown on cumulative tables; demand for 2 months ahead, and a process for working out the programme for each week in 2 hours. Possible bottlenecks in case of changes in design could also be predicted.

Automation: A Break with the Past. Rethinking for Management Strategy Outlined.
JOHN DIEBOLD.
Times Rev. of Ind., April, 1958; 10–11.
The experience of a supplier of automobile parts during 1952–57 whereby a loss of about $10 millions was incurred is analysed in terms of the

philosophy of production and defects in planning to make the point that automation represents if not something quite new, a great jump ahead, which existing organizations may be ill-prepared for. Five of the changes required in organization are chosen for illustrations.

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Operational Research.
G. BRIGHAM.
New Scientist, 1958, **3**, 24, 41.

A letter drawing a distinction between O.R. and what is known in the U.S. as "industrial engineering" by means of examples and exploratory definitions. The design of double-deck vehicles is regarded as the latter and the arrangement of routeing schedules as the former.

Decision-Making and the Communication of Information in an Industrial Setting.
R. SERGEAN.
Nature, 1958, **181**, 1020.

Communication structures of four light engineering factories are surveyed with regard to the making of decisions or estimates, especially of times of delivery. In two it was carried out on the shop floor and in the others by office staff. Estimated and actual delivery times are compared for the same four weeks and the range of error is tabulated. It appeared that the estimators with the most direct, continuous and immediate information gave the best results, so that the position of the estimator in the communications network should be studied. (A letter.)

Statistical Study of Materials Fatigue.
E. J. GUMBEL.
Revue de Statistique Appliquée, 1958, **5** (4), 51–86, reprint.

The application of a statistical study, with the development of the theory of limiting values, to the number of cycles to failure of test-pieces at constant constraint is worked out to draw the attention of engineers to the possibilities of the method. It is doubted whether the estimations generally employed are valid. This series is based on a study of 49 test pieces under 15 different constraints, and the probability that the values are correct is evaluated; at least 10 constraints can be clearly differentiated and the effects of 6 on the number of test-pieces remaining unbroken can be predicted closely.

A Practical Application of the Warehousing Problem: Location of Depots in a Petroleum Company

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THE activity of the company concerned is the distribution of various petroleum products (benzine, gasoil, fuel oils, lubricating oils, and others) in Belgium, from refineries through depots to consumers, all transport costs being at charge to the company.

The problem consisted in the study, in a definite area about half the size of Belgium, of the optimum depot network, viz.: the number of depots to be maintained; the location of each of these; the products to be stored in each depot; the storage capacities to provide; the distribution area of each depot.

The criterion adopted was to minimize the total costs resulting of the summing up of: the supply costs (refinery to depots); the depot operational costs; the distribution costs (depots to customers).

The comparison of the ideal network with the existing one was to be—and eventually was—a basis of decision in regard to additions or suppressions of depots and to the building up of new storage capacities.

The Model

The total cost function to minimize can be written in the following way, G being the total monthly costs:

$$G = \sum_k \sum_i \sum_j (a_{ik} + c_1 + c_2 D_{jk}) Q_{ijk} + \sum_k F_k, \tag{1}$$

where: subscript k stands for a depot (existing or hypothetical);

subscript i stands for a product;

subscript j stands for a customer sub-area; the whole area under study was subdivided in a number of customer sub-areas, defined empirically from the map, each one of them small enough to be represented by its centre of mass;

a_{ik} is the unit supply cost of depot (location) k in product i;

Q_{ijk} is the monthly average amount of product i to be consumed (in a definite future) in sub-area j; subscript k means that sub-area j lies in the distribution area of depot k;

D_{jk} is the distance, by road, between location k and centre of mass of sub-area j (return trip included);

c_1 is the distribution unit cost (depot to customer) independent of distance;

c_2 is the distribution cost per unit *and* kilometre run (costs proportional to distance);

F_k are the monthly fixed depot operational costs.

The unknowns were, let us recall:

(a) The number of depots to be located.

(b) The exact geographical locations of each.

(c) The optimum distribution area of each.

There was an obvious interaction between those unknowns. (a) and (b) being fixed, the determination of the various (c) is quite easy; the dividing lines are isocost lines. But indeterminateness in (b), and even more so in (a), complicates the problem.

If we look at equation (1), we see that the specific unknowns are:

(1) Distances D_{jk}, being functionally dependent on the location of depot k and on its distribution area, this one being in turn dependent on the number and the location of the other depots.

(2) The number of depots k, that is the very number of terms in the sums $\sum_k$ of equation (1).

Three major difficulties were an obstacle to a formal analytical study:

(a) Variables D_{jk} could not be expressed analytically in terms of the location coordinates; indeed, road distances do not follow any mathematical formulation.

(b) Cost parameters a_{ik} were discontinuous all over the country; the supply cost structure was highly dependent on the communication network and especially on the waterways network.

(c) The number of terms itself in expression (1) was one of the main unknowns.

Steps of the Study

It soon appeared that four major products shared between them more than 85 per cent of the total sales. So it was decided to base the study on

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those four products only, adjustments to take the remaining ones into account being postponed until the end of the study.

The successive steps of the research were:

(a) *Definition of distribution unit standard costs c_1 and c_2.* This does not need any particular comment. The most profitable type of vehicle was considered, account being taken of the geographical and topographical characteristics of the region.

(b) *Analysis of location unit supply costs a_{ik}.* The problem was not to know what were the supply costs of the existing depots but to provide a supply cost grid for the entire area where depots could be located. As the variation of the unit supply costs from one point to another could not be expressed in mathematical terms—because of the geographical discontinuities—it was necessary to establish a grid tight enough to allow for easy interpolations.

It was decided to take as standard unit supply cost of location k in product i the lowest of all unit costs of the various possible transport means (rail, road, waterway).

As the whole study was projected into the future, planned improvements to the road, rail and water network were taken into account at this stage.

(c) *Analysis of the depot operational fixed costs F_k.* Of the depot operational costs taken as a whole—all depots included—some appeared to be proportional to total throughput and others to be tied up to the existence of depots; these were called "fixed operational costs". A standard mean fixed monthly operational cost F_k was defined by a regression analysis; F_k included technical amortization charge and interest charge.

(d) *Consumption forecasts per sub-area j.* Past consumption in each of the main products was analysed by sub-area on the basis of existing statistical data; punched cards were of help for this purpose.

Those data were then projected in a forecast for three periods of the future: 10, 15 and 25 years hence. So three values were obtained, with confidence intervals, for each Q_{ij}.

(e) *Search for the optimum.* The nature of the model was such that it was necessary to use a step-by-step iterative procedure.

A feature of the problem was that there were, *a priori*, few hypotheses to be made about the number of depots—as was confirmed ultimately. It was decided to consider successively the hypothesis of one, two, three, etc. depots in the area under study, and to look for the optimum location(s)

in each of those hypotheses. So, for the whole large problem was substituted a succession of problems each of them much simpler.

In the hypothesis of one and two depots, the optimum locations were searched for by a systematic exploration of the whole area, within the unit supply costs grid. That was in fact a trial and error process, systematized by the drawing up of total costs graphs. It appeared that the most important single factor was the unit supply cost.

In the hypothesis related to a number of depots greater than 2, say n, the iterative procedure followed was to suppose the locations of the previous $(n-1)$ depots fixed, and to look for the most convenient location—economically speaking—for an nth (marginal) depot.

A criterion to appreciate quickly the profitability of a marginal depot was easy to devise. For example, to go over profitably from 2 to 3 depots it was necessary to have the following condition

$$Qc_2\overline{\Delta D_j} < -F_k + (Q_1 - Q'_1)(\overline{a_1} - \overline{a_3}) + (Q_2 - Q'_2)(\overline{a_2} - \overline{a_3}),$$

where: Q was the total average monthly demand for all products in the whole area;

$\overline{\Delta D_j}$ was the average decrease in total distance between depots and customers sub-areas (easy to evaluate);

F_k was the fixed monthly operational costs for the marginal depot;

$\overline{a_1}$ was the average unit supply cost (over all products) of depot in location 1;

$\overline{a_2}$ was the average unit supply cost of depot in location 2;

$\overline{a_3}$ was the average unit supply cost of depot in location 3 (marginal depot) (probably $\overline{a_3} > \overline{a_1}$ and $\overline{a_3} > \overline{a_2}$);

Q_1 and Q_2 were the monthly average deliveries by depots 1 and 2 before setting up of depot 3;

Q'_1 and Q'_2 were the monthly average deliveries by depots 1 and 2 after setting up of depot 3 (of course, $Q'_1 < Q_1$ and $Q'_2 < Q_2$).

Note. In the iterative process, every time a depot location was changed, a new drawing up of the isocost lines and of the distribution areas had to be made. A device was set up to recompute quickly the isocost lines.

Finally, an optimum solution was detected, along with six near-optimum ones.

(f) *Comparisons with the existing depot network.* All those solutions were valued and compared to the existing one in terms of changes to bring about and of profits to expect at different periods in the future.

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(g) *Planning for action.* One solution was then retained as a basis for action and a long-range planning of action was set up.

(h) *Determination of the storage capacities: adjustments.* Storage capacities were then computed for each depot and each product for different periods of the future. Account was taken at this stage of the range of possible variations in seasonal demand and in supply lead time.

Adjustments were made to take into the picture the remaining products (15 per cent of sales).

Conclusions

The computed profits resulting from the proposed rearrangement of the depot network are quite high. As the whole study has been projected into the future, comparison between expectation and reality will only be possible when reality will have materialized, in many years' time.

It was, however, interesting to find out that a great part (about 45 per cent) of the possible profits were of a short-term nature, depending not so much on the complete reshuffling of the depots but rather on improvements to actual supply, operation and distribution rules, within the existing network; this was an important by-product of the study.

It may be of interest to point out that the whole study took up the time of two researchers full-time and of one part-time during five months, plus some clerical help.

Two Years of Linear Programming

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THE technique of linear programming is used to solve problems in which the variables are restricted in the following ways. Firstly, the variables are subject to linear constraints, either equations or inequalities, which do not define a unique solution. For example, the simultaneous equations

$$\begin{aligned} x+2y-z &= 3 \\ 4x-3y+2z &= 5 \end{aligned}$$

merely serve to specify an infinity of solutions which lie on a line. Secondly, it is desired to find non-negative values of the variables which either maximize or minimize a linear function.

As a trivial example consider the following problem, in which the constraints are not equations but inequalities:

$$\begin{aligned} x+2y &\leqslant 6, \\ x-3y &\leqslant 1, \\ y-2x &\leqslant 2. \end{aligned}$$

Maximize $F \equiv x+y$ for non-negative x and y.

The non-negativity condition and the constraints are satisfied only within the polygon *ABCDE* of Figure 1 and on its boundaries. The shading indicates areas which are forbidden. A typical line $x+y=F$ is shown. To increase F this line must move in the direction indicated by the arrow and, therefore, the required solution, which satisfies the conditions and for which F takes its maximum value of 5, is the point D whose coordinates are $x=4, y=1$. By linear programming methods any problem of this type can be solved systematically, provided that a solution exists.

One of the most fruitful spheres of application for linear programming has been the economical mixing of materials: the work of Charnes, Cooper and Mellon[1] on the blending of aviation fuels may be quoted as an excellent example. Another example of this type, the application of linear programming to the use of recovered metal in making up alloys to specification, an account of which has already appeared in *Applied Statistics*,[2] has now been in operation for two years. In this article the

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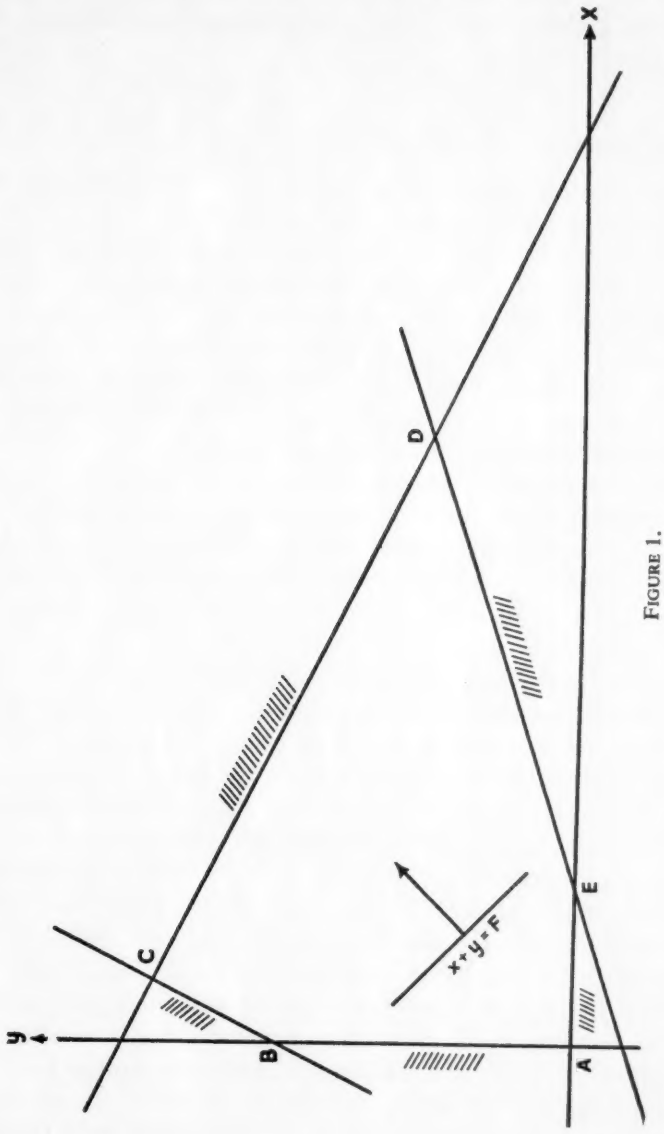

FIGURE 1.

problem and its solution are restated together with a geometrical interpretation. Ways in which the method has been modified and improved are described and the value of the linear programming technique in this case is assessed.

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The Problem and its Solution

Many plain bearings consist of a steel shell on the inside of which is cast a thin layer of whitemetal. The composition of this alloy varies according to the purpose for which the bearing is designed, but in general a large proportion of tin is included with smaller percentages of antimony, copper, lead and possibly cadmium or nickel. Both the surface finish and the thickness of the whitemetal are carefully controlled so that a final machining operation is necessary. The resulting swarf is collected and remelted: other sources of salvaged whitemetal are scrap bearings from which the whitemetal is melted and surplus whitemetal from other ancillary operations. The recovered whitemetal is cast into batches of ingots, each batch comprising those ingots from a common source and having the same nominal composition. Each batch is given a serial number, its composition is determined by chemical analysis and it is stored until it is required. The weight of a batch, now known as a "recovered melt", is of the order of one ton.

A recovered melt may be combined with other recovered melts and with the correct quantities of virgin metals in order to produce whitemetal of a required specification; or it may be sold at a price dependent on its tin content and its contamination, but of the order of 95 per cent of the value of its tin content. It is desirable to use as much recovered metal as possible in making up further whitemetal specifications. The number of specifications required per day is usually about four. For each specification, a prescription form is issued to the foundry and this gives the weights of recovered melts and virgin metals to be used. Before the advent of linear programming the prescriptions were calculated by a "rule of thumb" method which could not claim to reach the best solution in every case, but merely aimed at using equal weights of recovered metal and virgin metal. If this object was attained the prescription was regarded as quite satisfactory.

The problem will now be reduced to a form suitable for the application of linear programming. Consider the case in which the specification required comprises T, A, C and L lb. of tin, antimony, copper and lead

respectively and suppose that only three recovered melts appear to be worthy of consideration. Records of available recovered melts are subdivided in such a way that those suitable for a particular specification are quickly found and in practice only three or four usually seem suitable.

The weights of tin contributed by the recovered melts, together with some virgin tin, add up to the weight of tin required by the specification, i.e.

$$t_1 x_1 + t_2 x_2 + t_3 x_3 + x_4 = T, \tag{1}$$

where

t_i = proportion of tin in the ith recovered melt $(i = 1, 2, 3)$,
x_i = weight in lb. of the ith recovered melt used,
x_4 = weight in lb. of virgin tin added.

Similar equations arise for antimony, copper and lead,

$$a_1 x_1 + a_2 x_2 + a_3 x_3 + x_5 = A, \tag{2}$$

$$c_1 x_1 + c_2 x_2 + c_3 x_3 + x_6 = C, \tag{3}$$

$$l_1 x_1 + l_2 x_2 + l_3 x_3 + x_7 = L, \tag{4}$$

where a_i, c_i and l_i are the proportions of antimony, copper and lead respectively in the ith recovered melt for $i = 1, 2, 3$, and x_5, x_6 and x_7 are the weights used in lb. of virgin antimony, copper and lead respectively.

For each recovered melt the weight used, together with the weight unused, must give the weight available, i.e.

$$x_1 + x_8 = W_1, \tag{5}$$

$$x_2 + x_9 = W_2, \tag{6}$$

$$x_3 + x_{10} = W_3, \tag{7}$$

where

x_{i+7} = weight in lb. unused of the ith recovered melt,

and

W_i = available weight (in lb.) of the ith recovered melt.

If the value of a recovered melt is defined as the sum of the values of its constituents at virgin metal prices, then whatever prescription is given, the cost of making up a given specification remains constant. On this basis, either the value of the recovered metal prescribed should be maximized or that of the virgin metal should be minimized. Of these equivalent alternatives, the former is taken, i.e.

$$\text{maximize } F \equiv m_1 x_1 + m_2 x_2 + m_3 x_3, \tag{8}$$

where

m_i = the value per lb. of the ith recovered melt $(i = 1, 2, 3)$,

subject to the restriction

$$x_j \geqslant 0 \quad (j = 1, 2, \ldots, 10).$$

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The "simplex" method of computation, which has been described in detail elsewhere,[3] is used to reach the final solution systematically. However, before it can be applied, one more item of information is needed. By inspection the solution

$$\begin{aligned} x_1 &= x_2 = x_3 = 0 & x_4 &= T, \\ x_8 &= W_1 & x_5 &= A, \\ x_9 &= W_2 & x_6 &= C, \\ x_{10} &= W_3 & x_7 &= L, \end{aligned}$$

satisfies the constraints and the non-negativity condition and will serve as a starting-point. The physical interpretation of this "basic feasible solution" is the case in which the specification is made up from virgin metals only.

In order to simplify the model the fact that commercial virgin tin contains 0·07 per cent lead has been ignored. This is justifiable in practice as the error introduced is never sufficient to cause the lead content of the alloy produced to lie outside its tolerance limits.

It is interesting to observe that this problem is capable of a geometrical representation. It is a special case in that the difference between the number of variables and the number of constraints is three and for this reason a three-dimensional model could be constructed to illustrate the processes of the analytical solution. Three mutually perpendicular axes are taken to represent the directions of increase of x_1, x_2 and x_3. All admissible points lie in the positive orthant. Each constraint is represented by a plane which divides admissible points from non-admissible points. For example, all points which satisfy constraint (1) lie within the region

$$t_1 x_1 + t_2 x_2 + t_3 x_3 \leqslant T,$$

and the boundary plane in this case is, therefore,

$$t_1 x_1 + t_2 x_2 + t_3 x_3 = T.$$

Further, since all admissible points lie on one side only of each of the seven planes defined by the constraints (1) to (7), the volume containing the admissible points is bounded by a convex polyhedron. The plane representing the linear function (8) can be moved by varying the value of F, and when F has its maximum value consistent with its containing at

least one admissible point, this point gives the final solution. The argument here is basically the same as that in the two-dimensional case, a simple example of which has already been given. It is not suggested that it would be useful to build the model described above: in most linear programming problems the task would be impossible as the number of dimensions involved would exceed three. However, the geometrical approach affords a valuable insight into the analytical processes of linear programming.

The mathematical model specified by conditions (1) to (8) does not apply as it stands to those specifications containing nickel, which create an interesting problem. Virgin nickel, due to its high melting point, cannot be added directly and, as the percentage of nickel required is always small, it is convenient to fill it in by adding the appropriate weight of "nickel hardener", an alloy whose composition is approximately

tin, 90 per cent; copper, 5 per cent; nickel, 5 per cent.

To ensure that virgin nickel does not appear in the final solution, an arbitrarily large negative value of $-M$ per lb. is assigned to it. This effectively excludes it because the programme is designed to maximize the value of the metals used. Nickel hardener is treated as a recovered melt but no value is attached to it, since it is not wished to maximize the weight used. The solution which is taken as a starting-point for the simplex method is still the case in which the specification is made up entirely from virgin metals, even though this initially gives an arbitrarily large negative value to the linear function F which is to be maximized.

Modifications and Improvements

During the last two years changes have been made with the object of simplifying and speeding up the calculations and eliminating the possibility of error.

A very great saving in time arose when it was realized that in a substantial number of cases, those in which lead is tolerated as an impurity but with a maximum permitted amount of the order of 0·2 per cent, the solution given by linear programming could be predicted by a simple rule. In these cases, known as "lead max" specifications, it is merely necessary to calculate the quantity of a recovered melt which has to be taken to contain a certain fixed weight of lead and to assign a value to it. The order in which recovered melts should be taken in making up a specification is found by arranging them in descending order of these values. In practice, those recovered

melts with the lowest lead contents are used first. When the permitted amount of lead has been supplied by the use of recovered melts the other elements have to be made up to their required amounts by the addition of virgin metal.

As stated above, the programme is now designed to maximize the value of the recovered metal used in a given specification, defining the value of a recovered melt to be the total value of its constituents at current virgin metal prices. The original intention was to minimize the total cost of virgin and recovered metal used, but the value to be associated with the recovered melts was difficult to define, since the price for which they could be sold varied. Moreover, this approach, which involved attaching a value to each metal, led to heavy computation. This has been reduced by allowing virgin metals to enter the programme with price zero.

The insertion of all available recovered melts into every programme has now been abandoned as wasteful of time and paper since all except three or four took no part in the calculation in most cases. Instead, three or four recovered melts are selected for the programme. This is not in fact a handicap as linear programming ensures that the stocks of suitable recovered melts are kept low and the choice is, therefore, not wide.

Those readers familiar with the simplex method of linear programming will know that at each stage the vector chosen to enter the basis is that having the most negative $z-c$ value (using the notation of Charnes, Cooper and Henderson). In the present application this corresponds to the choice of the recovered melt which gives the largest increase in the value of recovered metal to be used for an addition of unit weight of that recovered melt. In practice, it is found that the solution is not reached in the least number of tableaux by this method and the recovered melt now chosen to enter the basis is the one which will give the largest increase in the value of the recovered metal to be used for the weight of that recovered melt actually to be added. Figure 2 illustrates a hypothetical example, having four constraints and six variables, in which it is assumed that the initial solution is represented by the origin A. Let $f(x,y)$ be the linear function to be maximized and let its value at the origin A be F_A. For the first step in the calculation, the alternatives are to move either to point B or to point G. The increase in $f(x,y)$ in moving to point B is proportional to the distance between the parallel lines $f(x,y) = F_A$ and $f(x,y) = F_B$. It can be seen from Figure 2 that this distance is less than that between $f(x,y) = F_A$ and $f(x,y) = F_G$, which indicates that the greater increase in $f(x,y)$ is to be obtained by moving to the point G as the first step of the

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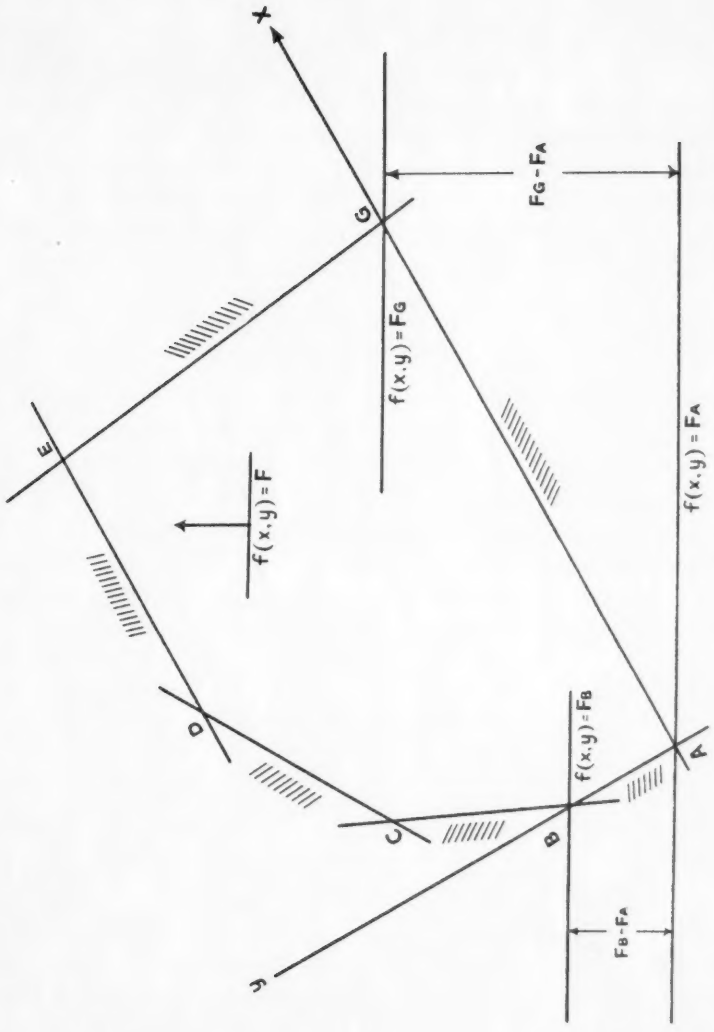

FIGURE 2.

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calculation. Hence, if the modified method is used, the anticlockwise route is followed and the solution is reached in two steps. On the other hand, the choice of the most negative $z-c$ value corresponds to comparing the increases in $f(x, y)$ obtained by movements of unit distance along the x and y axes. Movement along the y axis produces the greater increase per unit distance, indicating that the first step in the calculation should be to move to the point B. The final solution would then be reached in four steps by following the clockwise route. In this particular example, the use of the modified method reduces the calculation by half but it cannot be claimed that its superiority is always as marked as this.

Figure 3 shows the simplex computation necessary to provide a prescription for 750 lb. of the following specification:

tin, 75 per cent; antimony, 12 per cent; copper, 3 per cent;
lead, 10 per cent.

It is realized that the meaning of the calculations shown will not be clear except to those readers already familiar with the simplex method. The heavy lines occurring in the first three tableaux indicate the vector to enter the basis when they are vertical and the vector to leave the basis when they are horizontal. All spaces left empty represent zeros. The P_0 column in the fourth tableau provides the following final solution, the accuracy of which is impressive but unnecessary, since the foundry can weigh metals only to the nearest half-pound:

	lb.
Recovered melt Y15	59·15
Recovered melt Y51	66·31
Recovered melt Y93	568·00
Virgin tin	8·29
Virgin antimony	48·23
	749·98

Figure 3 will serve to show the size of a typical simplex computation and to illustrate some further modifications. Column totals are used to provide a check on the arithmetic, since they can be derived independently from the preceding tableau. For example, the total of the Y51 column of the third tableau, 91·8138, should be approximately equal to

$$92{\cdot}71-\frac{0{\cdot}06}{4{\cdot}65}(70{\cdot}47-1).$$

	Sn	Sb	Cu	Pb	M_1	M_2	M_3	P_0	Y15	Y93	Y51
Sn	1							562·5	72·83	88·86	9·65
Sb		1						90·0	9·85	6·31	0·15
Cu			1					22·5	4·65	3·47	0·06
Pb				1				75·0	12·67	1·36	90·14
M_1					1			2·75	1		
M_2						1		5·68		1	
M_3							1	1·60			1
z–c									−30·53	−35·74	−8·29
CHECK Tot.	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*760·03*	*70·47*	*65·26*	*92·71*
CHECK Act.	1	1	1	1	1	1	1	760·03	70·47	65·26	92·71
Sn	1					−88·86		57·7752	72·83		9·65
Sb		1				− 6·31		54·1592	9·85		0·15
Cu			1			− 3·47		2·7904	*4·65		*0·06
Pb				1		− 1·36		67·2752	12·67		90·14
M_1					1			2·75	1		
Y93						1		5·68		1	
M_3							1	1·60			1
z–c						35·74		203·0032	−30·53		−8·29
CHECK Tot.	*1*	*1*	*1*	*1*	*1*	*−63·26*	*1*	*395·0332*	*70·47*	*1*	*92·71*
CHECK Act.	1	1	1	1	1	−63·26	1	395·0332	*70·47	1	*92·71
Sn	*1*		*−15·6657*			*−34·5143*		14·0699			8·7105
Sb		*1*	*− 2·1187*			*1·0401*		48·2482			0·0229
Y15			0·2151			− 0·7462		0·6001	1		0·0129
Pb			*− 2·7253*	*1*		*8·0944*		59·6719			89·9766
M_1			*− 0·2151*		*1*	*0·7462*		2·1499			−0·0129
Y93						*1*		5·68		1	
M_3							*1*	1·60			1
z–c			6·5670			12·9585		221·3243			−7·8962
CHECK Tot.	*1*	*1*	*−13·9427*	*1*	*1*	*−11·4213*	*1*	*353·3443*	*1*	*1*	*91·8138*
CHECK Act.	*1*	*1*	*−13·9430*	*1*	*1*	*−11·4215*	*1*	353·3443	1	1	91·8138
Sn	*1*		*−15·4018*	*−0·0967*		*−35·2983*		8·2940			
Sb		*1*	*− 2·1180*	*−0·0003*		*1·0380*		48·2330			
Y15			*0·2155*	*−0·0001*		*− 0·7474*		0·5915	*1*		
Y51			*− 0·0303*	*0·0111*		*0·0900*		0·6631			*1*
M_1			*− 0·2155*	*0·0001*	*1*	*0·7474*		2·1585			
Y93						*1*		5·68		*1*	
M_3			*0·0303*	*−0·0111*		*− 0·0900*	*1*	0·9369			
z–c			*6·3277*	*0·0876*		*13·6692*		226·5603			
CHECK Tot.	*1*	*1*	*−11·1921*	*−0·0094*	*1*	*−19·5911*	*1*	*293·1173*	*1*	*1*	*1*
CHECK Act.	*1*	*1*	*−11·1913*	*−0·0080*	*1*	*−19·5947*	*1*	293·1257	*1*	*1*	*1*

FIGURE 3.

The entries in the second tableau from which this expression is derived are marked with an asterisk. In part of the calculation the unit used is 100 lb. instead of 1 lb. This change is made so that the compositions of the recovered melts may be entered as percentages as opposed to proportions. The computation is printed in its entirety, but in practice the figures in italics are omitted.

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Organizational Details

The computation is performed by a girl using an electric calculating machine and the keeping of records of metal stocks is also her responsibility. Since a training period of a week is necessary to ensure competence, a second computor has been trained and is kept informed of any changes in procedure so that she can take over whenever needed. The time spent in computing and keeping the records up to date is about three hours per day, but as prescriptions are sometimes required by the foundry at short notice, the other work done by the computor may suffer from interruptions. The computations are not checked by a second person as the built-in checks have been completely satisfactory.

The smooth running of the system is the responsibility of a research investigator. He clears up difficulties in the computation, which are now very few, makes any modifications to the method which seem advisable, and periodically checks the records.

The Value of Linear Programming in this Case

The linear programming technique was applied in this case with the main object of achieving the *best* utilization of recovered metal, as opposed to using what seemed a reasonable amount of recovered metal in each prescription. Before the method was put into operation it was used to recalculate prescriptions for a ten-week period taken at random from the first six months of 1955. It was found that the use of linear programming would have produced an increase of 5 per cent in the amount of recovered metal used during the period, which is sufficient to produce a worthwhile financial saving. A corollary was a quicker turnover of metal, resulting in lower storage costs. The savings currently being made due to linear programming cannot readily be measured quantitatively since quite arbitrary amounts of recovered metal were utilized under the previous method and this rules out a direct comparison of the two methods under existing

conditions. A very high level of utilization of recovered metal is now being achieved when suitable melts are available. Recently, a prescription was issued in which 99½ per cent of the total weight of 1100 lb. came from recovered metal and percentages comparable to this are not uncommon. The ability of the linear programming method to take advantage of a good supply of recovered metal gives it a flexibility which the "rule of thumb" method did not have. This was demonstrated at the end of 1956 when the output of whitemetal specifications was low but the receipts of recovered metal were undiminished. The accumulation of stocks which would have occurred under the previous method was avoided as the linear programming technique allowed a higher proportion of each specification to be made up from recovered metal.

Some further advantages of the new system over the old may now be mentioned. The "rule of thumb" method, involving as it did arbitrary decisions and the ability to comprehend a mass of data, had to be applied by a research investigator, whose time was relatively valuable. It is more economic to have the job done by a girl computor, and moreover the superiority of the calculating machine over the slide-rule ensures greater accuracy. Prescriptions for any required specification of alloy can now be provided by a routine and systematic procedure and those problems which do occasionally have to be resolved are matters of administration or policy.

It is generally found in applications of linear programming that a thorough insight into the problem is gained and that useful conclusions often emerge as by-products. In our case attention was drawn to the severe restriction on the quantity of recovered metal usable imposed by the numerous "lead max" specifications mentioned previously, and consideration is being given to reducing the lead contamination in recovered metal and to the possibility of relaxing the restrictions on lead content in many specifications.

The time spent on computation using linear programming methods is longer than that spent by the research investigator using a slide-rule to apply the "rule of thumb" method. This is not considered a serious disadvantage as an urgent demand for a prescription can be answered within the hour.

ACKNOWLEDGMENTS

The author wishes to thank the Glacier Metal Company Limited for permission to publish this article and to acknowledge the encouragement and assistance given by his colleagues in its preparation.

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News and Notes

Operational Research Society

A SPECIAL meeting was held on 24 March, 1958, to discuss the future development of the Society. The President was in the chair. In his opening remarks he explained that the meeting was held because, in the opinion of the Executive Committee, the activities of the Society had reached or even exceeded the limit that could be sustained by voluntary effort. The time had therefore come when if the Society was to continue to expand a paid Secretariat and office must be provided. To do this, however, more money would be required, but in order to obtain this in the form of a regular income it was considered that more extended services must first be provided; this, however, could not be done without a paid Secretary. The position was, therefore, the circular one illustrated in Figure 1.

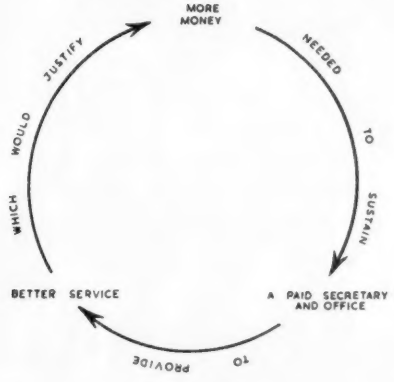

FIGURE 1.

The Chairman went on to explain the Executive Committee's proposals for breaking into this circle by obtaining more financial assistance to tide over the next three years while the extended services were being developed.

The meeting was in complete agreement with these proposals and went on to discuss the lines along which the Society should be developed. Many suggestions were made which cannot all be referred to in detail here. The general tenor of the discussion appeared to suggest that members felt

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that what was required mostly was an extension of present activities. The main emphasis was on increasing the flow of information particularly between operational research groups, but also to management and students. It was felt that there was need for more meetings in the provinces and for the development of Regional Branches of the Society. Strong views were expressed that library facilities should be developed and that the literature needed to be extended, although it was appreciated that this can only result from an upward flow of papers and information from individual members of the Society and Companies. The provision of speakers, assistance with planning courses at colleges of technology and at universities and the possibility of sponsoring research studentships, were among other subjects discussed.

The meeting went on to discuss the general nature of the Society, in particular whether it should aim towards being a learned society or a more professional society with clear cut qualifications for membership. The membership secretary reported that the total number of members was now about 450, of which nearly half were Associate members, and membership was growing steadily. He explained the qualifications which the Membership Committee required of applicants before admitting them to full membership. While it was felt that some of these qualifications appeared to be slightly arbitrary, and every effort should be made to base admittance more on proved operational research ability, it was agreed that it would be impossible to throw any extra burden on the Membership Committee at the present time. The present selection scheme which limited membership to Full Members or Associates was therefore approved, on the understanding that the whole subject of grades of membership and qualifications would remain under review.

Evening Courses

Two courses of ten lectures each will be given during the 1958–9 session at the Brunel College of Technology, Acton, which may be of interest to operational research workers or aspirants. Dr. F. Gordon Foster (The London School of Economics) will lecture on the *Theory of Queueing and Industrial Applications*, beginning on 7 October. Dr. S. Vajda (Admiralty Research Laboratory) will lecture on *Linear Programming*, beginning on 13 January.

Operational Research Quarterly
VOLUME 9 NUMBER 3 SEPTEMBER 1958

Notes on a Combinatorial Problem of B. O. Koopman

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At the recent International Conference on Operational Research, B. O. Koopman[1] posed an interesting class of problems in the Combinatorial Analysis of Operations.

Koopman defined the class of problem and then considered a simple example of that class, obtaining rough upper and lower limits to the numerical quantity required.

In these notes I consider this simple example and obtain limits which are closer than those obtained by Koopman.

Lastly, I consider the class of problems to which the approximate methods given here may be extended.

1. *The Simple Problem*

Suppose that there are n installations b_i, $(i = 1, 2, \ldots, n)$ and $2n$ stations a_j, $(j = 1, 2, \ldots, 2n)$. At some instant $t = 0$ each installation requires the full service of a single station. Prior to the instant $t = 0$ an arrangement assigning certain stations to the service of certain installations has been made. Thus, suppose that the arrangement (made prior to $t = 0$) assigns to each station a set of k installations, $k \geqslant 2$, and each installation is joined to exactly $2k$ stations. A station is only permitted to provide a service to one of the k installations associated with it.

At some time before the instant at which service is required, but subsequent to the assignment of service facilities, a random process removes some of the stations.

It is supposed that the probability that any given station is removed is p, and that the removals of stations are independent events.

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The problem is to determine the expected value of the number of installations which can be serviced at the instant $t = 0$, the best use being made of the facilities then available, one station giving service to only one installation. The ratio of this expected value to the number of installations present will be called the index of the arrangement and will be denoted by $I(A)$ where A signifies a particular arrangement.

Koopman considers the case $p = \frac{1}{2}$, $k = 2$, $n = 100$ and finds that arrangements A exist for which the following inequality is satisfied

$$0{\cdot}87 \leqslant I(A) \leqslant 0{\cdot}97. \tag{1}$$

In these notes a special study will be made of the case $k = 2$ with n and p arbitrary although for purposes of comparison with the result (1) of Koopman it will be useful to take $n = 100$ and $p = \frac{1}{2}$.

2. *Determination of an Upper Limit to the Index*

Note that when each station may serve only k installations then, since there are n installations and $2n$ stations, each installation is associated with $2k$ distinct stations each of which can provide it with service.

Consider, then, an arbitrary arrangement of service facilities A, which associates the $2k$ stations $a_{i_1}, \ldots, a_{i_{2k}}$ with the installation b_i $(i = 1, 2, \ldots, n)$.

Let $B_{i,m}$ denote the event "b_i loses m of its associated stations", and let $P(B_{i,m})$ be the probability of occurrence of this event.

Then

$$P(B_{i,m}) = (2k)!\, p^m (1-p)^{2k-m} / (2k-m)!\, m! \tag{2}$$

The events $B_{i\,m}$ and $B_{j,s}$ are not independent in general. Thus if $P(B_{j,s} \mid B_{i,m})$ denotes the conditional probability of the occurrence of the event $B_{j,s}$, given that the event $B_{i,m}$ has occurred, then we have

$$P(B_{j,s} \mid B_{i,m}) = P(B_{j,s})$$

only in the case the set of stations $(a_{i_1}, \ldots, a_{i_{2k}}, a_{j_1}, \ldots, a_{j_{2k}})$ contains $4k$ distinct members.

Let $R_{i,m}$ be a random variable which equals unity when b_i loses m of its associated stations and is zero otherwise. Then

$$P(R_{i,m} = 1) = P(B_{i,m})$$

$$P(R_{i,m} = 0) = 1 - P(B_{i,m})$$

and the expected value of $R_{i,m}$, $E(R_{i,m})$ is given by

$$E(R_{i,m}) = P(B_{i,m}) \tag{3}$$

Let

$$R_m = \sum_{i=1}^{n} R_{i,m} \tag{4}$$

then R_m is a random variable whose value is the number of installations which lose m of their associated stations.

The random variables $R_{i,m}$ are not in general independent but it is still the case that the expected value of the sum (4) is the sum of expected values of the form (3). Thus

$$E(R_m) = E\left(\sum_{i=1}^{n} R_{i,m}\right)$$

$$= \sum_{i=1}^{n} E(R_{i,m})$$

$$= \sum_{i=1}^{n} P(B_{i,m})$$

from (3).

Hence using (2) and noting that $P(B_{i,m})$ is independent of i we obtain

$$E(R_m) = n(2k)!\,p^m(1-p)^{2k-m}/(2k-m)!\,m! \tag{5}$$

The random variable R_{2k} is of particular interest. $E(R_{2k})$ is the expected value of the number of installations which are deprived of their associated stations by the random process of removal. Such installations cannot be provided with service whatever assignment is made of the remaining service facilities. The number of installations which can be served is therefore certainly not greater than $n-E(R_{2k})$, and the index of the arrangement $I(A)$ is certainly not greater than $1-E(R_{2k})/n$.
From (5) in the case $m = 2k$ we have

$$E(R_{2k}) = np^{2k}$$

Thus we have the following theorem.

Theorem 1

If A is an arbitrary arrangement its index $I(A)$ satisfies the inequality (6).

$$I(A) \leqslant 1-p^{2k} \tag{6}$$

For example, in the case $p = \frac{1}{2}$ $k = 2$ considered by Koopman this inequality gives

$$I(A) \leqslant 0{\cdot}9375$$

This is a considerable improvement on the upper limit of 0·97 given by (1) and obtained by Koopman.

The inequality (6) may be improved by a consideration of certain patterns of removal which entail that an installation cannot be provided with service facilities.

Thus in the case $k = 2$, if b_i is an installation which has lost three of its associated stations, then its remaining station, a, say, is associated with just

one other installation, $b_{i'}$. If b_i has also lost its other three associated stations, then one of b_i, $b_{i'}$, cannot be given service.

It can be shown that the expected value of the number of distinct pairs of installations with one station in common, and no other station left which can serve either of them, is not less than $2np^6(1-p)$.
Hence

Theorem 2

If A is an arbitrary arrangement, and $k = 2$, then

$$I(A) \leqslant 1 - p^4 - 2p^6(1-p)$$

For example in the case $p = \frac{1}{2}$ this gives

$$I(A) \leqslant \tfrac{59}{64} = 0{\cdot}921875$$

This type of argument may be extended by considering those installations which have lost exactly two of their associated stations. If $k = 2$ each of the remaining stations will be associated with one other installation and if both of these have lost their other three stations then at least one of the three installations must be deprived of service. Proceeding in this way the following theorem can be proved.

Theorem 3

If A is an arrangement in which there are m installations b_i, $i = 1, \ldots, m$ such that no one of these installations has more than one station in common with any other installation and if $k = 2$, then

$$I(A) \leqslant 1 - p^4 - 2p^6(1-p) - 6\frac{m}{n}p^8(1-p)^2$$

For example, in the case $m = n$, p $= \frac{1}{2}$ this inequality gives

$$I(A) \leqslant \tfrac{469}{512} = 0{\cdot}916\ldots$$

Consideration of those installations b_i with three associated stations a_1, a_2, a_3 each with only one other installation with no other station associated with it after the process of removal reduces the right-hand side of the last inequality by about 0·001 in the case $p = \frac{1}{2}$, $m = n$.

3. *Determination of a Lower Limit to the Index*

In this section only the simple problem of Section 1, with $k = 2$, will be considered.

There is an interesting class of arrangements which I call simple. This class is characterized by the following definition. An arrangement is simple

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if there is a set of n stations $a_1, \ldots a_n$ such that a_2 provides service to the installations b_i and $b_{i+1}, i = 1, 2, \ldots n (b_{n+1} \equiv b_1)$.

If, in addition, the arrangement is such that the station a_{n+i} $(i = 1, 2, \ldots n)$ also provides service to b_i, b_{i+1} the arrangement will be called regular. It can be shown that the index of a regular arrangement is given by (7)

$$I(A) = 1 - p^4 - \left[\frac{p^4 q(1-q^{n-2})}{1-q} + p^2 q^{n-1}\right] \tag{7}$$

where $q = 2p(1-p)$

When $n \gg 1$ we have from (7)

$$I(A) \doteq 1 - [p^4/(1-2p+p^2)]$$

For example in the case $p = \frac{1}{2}$ this gives

$$I(A) = 0{\cdot}875$$

This is a slight improvement upon the lower limit given by (1) and obtained by Koopman.

4. *Conclusions*

It has been shown in Section 2 how an upper limit to the index of an arrangement may be obtained in the case of the simple problem of Section 1 with k, n and p arbitrary. It is not difficult to see how the approach to Section 2 can be generalized to more general problems.

For example, consider the case of N stations, with n installations, with each station connected to only k distinct installations. In the case $N = \lambda n$, where λ is an integer not less than 2, we have a straightforward generalization of the simple problem. Each installation is associated with a set of λk stations by a given arrangement and an argument similar to that of Section 2 shows that the index of an arrangement does not exceed $1 - p^{\lambda k}$. This result generalizes that of Theorem 1.

The methods used in Section 2 to improve upon the result of Theorem 1 can be used in more general cases, although it is more complicated to do so, and it does not seem worthwhile to attempt to derive general formulae by this method.

The conclusion of these notes reinforces the feelings expressed by Professor Koopman at the International Conference that with combinatorial problems of the nature treated here, which are difficult to pose and analyse, it is very difficult to obtain even an approximate solution.

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Solution of a Purchase-storage Programme

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PART I

1. *Introduction*

THE problem discussed in this paper was suggested to one of the authors by Dr. J. Bethel, Forestry Department, North Carolina State College. It can be briefly described as follows. A certain commodity is needed in specified amounts in each of a number of time periods. In each of these periods the commodity may be obtained either by purchase, or through storage from earlier periods. At any one time, the commodity may be bought from one or more of several sources, at prices which may vary from one source to another. In each time period, the maximum amount available from each source, the capacity of the storage facilities, and the cost per unit of commodity stored over that period, are known. The cost of storage is presumed to include handling and interest charges as well as rent of the warehouse and losses during storage. The amounts available from each source, their costs, the storage capacity and its cost, can all vary from one period to the next. The time scale is discrete, in that it is assumed that all purchases for a given period are made at the same point in time, the storage costs being measured from that instant to the next purchasing point. It is desired to arrange the programme of purchase and storage in such a way as to minimize total cost and to supply the required amounts of the commodity during the period under consideration, while not exceeding purchase and storage facilities.

This problem can easily be put into linear programming form and, because of its simple nature, a remarkably straightforward and rapid method of numerical solution is possible, which does not need elaborate equipment. The method is also applicable in the more general case where the cost function of the amounts bought and stored is convex rather than being piecewise linear. Formal proofs and a numerical example are given below. In the case where storage capacity is unlimited the solution reduces to that given by Johnson.[2]

2. Notation and Formulation

Let the amounts of the commodity required in time t be denoted by M_t $(t = 1, 2, \ldots, T)$. Let the amounts available at source i at time t be denoted by W_{ti} and the price at which these amounts can be bought by c_{ti} $(i = 1, 2, \ldots, n_t;\ t = 1, 2, \ldots, T)$. There is no loss of generality involved if it is assumed that $n_t = n$ for all periods, since at any time an arbitrary number of sources of zero productive capacity can be added to the known group from whom supplies are obtained. Denote the amount of storage facilities available at time t by S_t and the cost of storage per unit of commodity by d_t $(t = 1, 2, \ldots, T-1)$. Finally, let the amount bought from source i at time t be denoted by w_{ti} and let the amount stored from period t to period $t+1$ be denoted by s_t, and let initial and terminal inventories be given and denoted by S_0 and S_T, respectively. The problem can then be formulated as:
Minimize

$$\sum_{t=1}^{T} \sum_{i=1}^{n} c_{ti} w_{ti} + \sum_{t=1}^{T-1} d_t s_t, \tag{1}$$

subject to

$$0 \leqq w_{ti} \leqq W_{ti} \quad (t = 1, \ldots, T;\ \ i = 1, \ldots, n), \tag{2}$$

$$0 \leqq s_t \leqq S_t \quad (t = 1, \ldots, T-1), \tag{3}$$

$$s_0 = S_0,$$

$$s_T = S_T,$$

$$s_{t-1} + \sum_{i=1}^{n} w_{ti} = M_t + s_t \quad (t = 1, \ldots, T). \tag{4}$$

Conditions (4) ensure that the total amount (bought and carried in storage) available at time t equals the total amount disposed of (used or stored) in the same time. Conditions (3) set an upper limit on the non-negative amounts stored at any one time, along with the (given) amounts of initial and terminal inventory.

Although the problem is already in linear programming form, it pays to simplify it by expressing the s_t in terms of the w_{ti} by means of equations (4). For convenience, denote $\sum_{i=1}^{n} w_{ti}$ by w_t for all t. From (4) we have, on rearranging terms,

$$s_t = s_{t-1} + w_t - M_t,$$

and hence

$$s_1 = s_0 + w_1 - M_1,$$

$$s_2 = s_1 + w_2 - M_2 = s_0 + w_1 + w_2 - M_1 - M_2,$$

and in general

$$s_t = s_0 + \sum_{k=1}^{t} w_k - \sum_{k=1}^{t} M_k \quad (t = 1, 2, \ldots, T). \tag{5}$$

On introducing conditions (3) we obtain

$$0 \leqq S_0 + \sum_{k=1}^{t} w_k - \sum_{k=1}^{t} M_k \leqq S_t \quad (t = 1, \ldots, T-1).$$

$$S_T = S_0 + \sum_{k=1}^{T} w_k - \sum_{k=1}^{T} M_k,$$

the last equation saying that total purchases less total requirements must be equal to the difference between initial and terminal inventories. If we isolate Σw_k in the last inequality and equation we obtain

$$\left.\begin{aligned} \sum_{k=1}^{t} M_k - S_0 \leqq \sum_{k=1}^{t} w_k \leqq \sum_{k=1}^{t} M_k - S_0 + S_t \quad (t = 1, \ldots, T-1), \\ \sum_{k=1}^{T} w_k = \sum_{k=1}^{T} M_k - S_0 + S_T. \end{aligned}\right\} \tag{6}$$

Furthermore, on using (5), the second sum in the minimand (1) becomes

$$\begin{aligned} \sum_{t=1}^{T-1} d_t s_t &= \sum_{t=1}^{T-1} d_t \left(S_0 + \sum_{k=1}^{t} w_k - \sum_{k=1}^{t} M_k \right) \\ &= \sum_{t=1}^{T-1} d_t \left(\sum_{k=1}^{t} w_k \right) + \text{a constant.} \end{aligned}$$

By reversing the order of summation, the last term may be shown to be equal to

$$\sum_{t=1}^{T-1} w_t \left(\sum_{k=t}^{T-1} d_k \right)$$

and hence we can rewrite (1):

Minimize

$$\sum_{t=1}^{T} \sum_{i=1}^{n} c_{ti} w_{ti} + \sum_{t=1}^{T-1} w_t \left(\sum_{k=t}^{T-1} d_k \right),$$

and the reformulated programme becomes:

Minimize

$$\sum_{t=1}^{T} \sum_{i=1}^{n} e_{ti} w_{ti}, \tag{7}$$

subject to $\quad 0 \leqq w_{ti} \leqq W_{ti} \qquad (t = 1, \ldots, T; \quad i = 1, \ldots, n),$

and

$$\left.\begin{aligned} A_t \leqq \sum_{k=1}^{t} \sum_{i=1}^{n} w_{ki} \leqq A_t + S_t \qquad (t = 1, \ldots, T-1), \\ \sum_{k=1}^{T} \sum_{i=1}^{n} w_{ki} = A_T + S_T, \end{aligned}\right\} \tag{8}$$

where

$$A_t = \sum_{k=1}^{t} M_k - S_0 \text{ (from (6))}$$

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and

$$e_{ti} = c_{ti} + \sum_{k=1}^{T-1} d_k \qquad (t = 1, \ldots, T-1),$$

$$e_{Ti} = c_{Ti}.$$

Note that the physical significance of the quantity e_{ti} is the cost of purchasing one unit from source i in time period t and storing it until the end of the operation. The total cost is less than $\Sigma\Sigma e_{ti} w_{ti}$ by an amount representing the fact that storage does not have to be paid on items after they have been used; but the saving from this is a constant, as long as the requirements in each period are constant, and hence it may be suppressed in the expression that has to be minimized.

The significance of conditions (6), or equivalently (8), is that total purchases up to time t must not fall short of total requirements (less initial inventory) up to that time. Also, they must not exceed total requirements up to time t plus storage capacity at that time.

From the nature of the problem we have

$$A_{t-1} \leqq A_t \leqq A_t + S_t,$$

that is, neither storage capacity nor requirements can be negative.

A slightly different version is obtained if initial and terminal inventories are not specified but must be determined as part of the solution. It will then merely be required that the terminal inventory bear a certain relation to the initial inventory, such as equality in the case of a stationary circular flow (or some other). This will be considered in Section 7.

3. *General Formulation*

We now solve the following problem:

Choose non-negative $w_1, \ldots, w_t$ to minimize

$$E = \sum_{t=1}^{T} e_t(w_t)$$

where $e_t(w_t)$ is a continuous convex function, for w_t satisfying the following inequalities

$$A_t \leqq \sum_{k=1}^{t} w_k \leqq A_t + S_t \qquad (t = 1, 2, \ldots, T-1),$$

$$A_T = \sum_{k=1}^{T} w_k.$$

We may assume without loss of generality that the A_t form a non-decreasing sequence, since

$$\sum_{k=1}^{t} w_k \geqq \sum_{k=1}^{t-1} w_k,$$

so that it would be impossible to have

$$\sum_{k=1}^{t} w_k < A_{t-1}$$

even if $A_t < A_{t-1}$.

If we wish to solve such a problem without having

$$\sum_{t=1}^{T} w_t$$

fixed, we can increase T by 1 and define $e_T(w_T) = 0$ for all values of w_T and make A_T arbitrarily large. The final constraint will then be vacuous.

If there are additional constraints of the form $w_t \leqq W_t$, where W_t is a constant, these can be included by writing

$$e_t(w) = N(w - W_t) + e_t(W_t),$$

for $w > W_t$, where N is a very large positive constant.

4. *Method of Solution*

We now require some more definitions. Let $e_t'(w_t)$ be the right-hand derivative of $e_t(w_t)$, i.e.

$$e_t'(w_t) = \lim_{\delta \to 0+} \frac{e_t(w_t + \delta) - e_t(w_t)}{\delta}.$$

It is well known that this limit must exist and that it is a non-decreasing function of w_t if $e_t(w_t)$ is a convex function.

We also define

$$y_0 = 0,$$

$$y_t = \sum_{k=1}^{t} w_k.$$

The constraints can now be written

$$A_t \leqq y_t = y_{t-1} + w_t \leqq A_t + S_t \qquad (t = 1, \ldots, T-1),$$

$$A_T = y_T = y_{T-1} + w_T.$$

The solution to the problem may be built up by introducing the constraints one at a time. The proof that the resulting solution minimizes E is given in Section 5.

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It is clear that w_1 must be at least as large as A_1, and that, for $t>1$, w_t must be at least as large as $A_t-(A_{t-1}+S_{t-1})$.

In the notation of Section 2

$$A_1 = M_1 - S_0,$$

$$A_t - A_{t-1} = M_t \qquad (t = 2, 3, \ldots, T-1),$$

$$A_T - A_{T-1} = M_T + S_T.$$

Hence

$$w_t \geqq M_t - S_{t-1} \qquad (t = 1, \ldots, T-1),$$

$$w_T \geqq M_T + S_T - S_{T-1}.$$

We first put

$$w_t = \max(M_t - S_{t-1}, 0) \qquad (t = 1, \ldots, T-1),$$

$$w_T = \max(M_T + S_T - S_{T-1}, 0).$$

Let us now suppose that we have chosen $w_1, \ldots, w_t$ satisfying

$$A_k \leqq y_k \leqq A_k + S_k \qquad (k = 1, \ldots, t-1),$$

$$A_t = y_t.$$

This is now true for $t = 1$.

We now show how to *increase* one or more of $w_1, \ldots, w_{t+1}$, so that the resulting values satisfy the constraints

$$A_k \leqq y_k \leqq A_k + S_k \qquad (k = 1, \ldots, t),$$

$$A_{t+1} = y_{t+1}.$$

Let us suppose that, for our present set of w_k,

$$y_r = A_r + S_r,$$

and

$$y_k < A_k + S_k \quad \text{for all} \quad r < k \leqq t.$$

We define $r = 0$ if in fact $y_k < A_k + S_k$ for all k. Then the procedure is to increase all w_m for which

$$e'_m(w_m) = \min_{r+1 \leqq k \leqq t+1} e'_k(w_k).$$

If in fact there is more than one such w_m, they must all be increased in such proportions that the values of $e'_m(w_m)$ remain equal to each other. This increase continues until either (a) $y_{t+1} = A_{t+1}$, or (b) $e'_m(w_m)$ has increased so much that it is as least as large as the next lowest value of $e'_k(w_k)$ for $r+1 \leqq k \leqq t+1$, or (c) $y_u = A_u + S_u$ for some $u > r$. In case (a), the present stage is complete and we may proceed to the next value of t.

In cases (b), or (c), we must redefine r, or the set of w_m, or both, on the basis of the present values of the w_k, and then make further increases.

This procedure is followed for $t = 1, \ldots, T-1$. At this stage all the constraints are satisfied, and it is proved in the next section that we have an optimal solution.

5. *Justification for the Method of Solution*

We consider first whether the method of solution outlined above does, in fact, terminate.

There is only a finite number of values of t to consider, so we can confine our attention to the number of halts in the process for a given value of t. As soon as a halt of type (a) occurs, we can proceed to the next stage. There can only be a finite number of halts of type (c), since each such halt increases r by at least one, and r cannot exceed t. If the functions $e'_k(w_k)$ are all continuous, there can only be a finite number of halts of type (b) between halts of type (c), since each such halt introduces an additional w_m and the number of such variables cannot exceed $t-r+1$. A finite number of discontinuities in each $e'_k(w_k)$ obviously cannot affect the termination of the process, though an infinite number of such discontinuities might. But in practice, a function with an infinite number of discontinuities could be replaced to a close approximation by one which had only a finite number of discontinuities.

Our proof, that the method always gives an optimal solution, is not unlike that given by Johnson, but it is desirable to express it more formally in order to make it clear that the storage capacity limitations do not invalidate the method.

We first prove a lemma.

Let $f_1(x), \ldots, f_n(x)$ be convex functions and let their right derivatives be denoted by $f'_1(x), \ldots, f'_n(x)$. These derivatives are non-decreasing functions since the $f_i(x)$ are convex.

For any h, x_i^0 can be found such that

$$x < x_i^0 \quad \text{if} \quad f'_i(x) \leqq h,$$

$$x > x_i^0 \quad \text{if} \quad f'_i(x) \geqq h,$$

where we define $\quad x_i^0 = 0 \quad \text{if} \quad f'_i(0) > h,$

and $\quad x_i^0 = +\infty \quad \text{if} \quad f'_i(x) < h \text{ for all } x.$

Note that if $f'_i(x) = h$ for some range of values of x, x_i^0 is not uniquely defined as a function of h but this is of no consequence in the subsequent argument.

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Lemma. If $x_i^0 < \infty$ for $i = 1, 2, \ldots, n$, the set of non-negative values $x_i = x_i^0, (i = 1, 2, \ldots, n)$, which satisfy the single constraint

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i^0,$$

minimizes

$$\sum_{i=1}^{n} f_i(x_i).$$

In the present problem we only require the lemma for $n = 2$, but since it has other applications, notably in simple programming problems under uncertainty, we prove it for general n.

The proof is trivial. Consider any other set of non-negative values of x_i satisfying the constraint, say $x_i = x_i'$. Let us renumber the variables so that $x_i' \leqq x_i^0$ for $i \leqq r$, $x_i' \geqq x_i^0$ for $i > r$. Then

$$\sum_{i=1}^{n} f_i(x_i') = \sum_{i=1}^{n} f_i(x_i^0) + \sum_{i=1}^{r} \{f_i(x_i') - f_i(x_i^0)\} + \sum_{i=r+1}^{n} \{f_i(x_i') - f_i(x_i^0)\},$$

$$\geqq \sum_{i=1}^{n} f_i(x_i^0) + \sum_{i=1}^{r} (x_i' - x_i^0)\, h + \sum_{i=r+1}^{n} (x_i' - x_i^0)\, h, \quad = \sum_{i=1}^{n} f_i(x_i^0),$$

since

$$\sum_{i=1}^{n} (x_i' - x_i^0) = 0 \text{ from the constraint.}$$

The problem of finding non-negative x_i satisfying

$$\sum_{i=1}^{n} x_i = A$$

to minimize $\Sigma f_i(x_i)$, therefore, reduces to that of finding h such that

$$\sum_{i=1}^{n} x_i^0 = A.$$

This can be done by gradually increasing h, and thereby increasing

$$\sum_{i=1}^{n} x_i^0,$$

until this quantity reaches the required value.

We now prove by induction that our method always gives an optimal solution to the purchase-storage problem.

Let us suppose that the method gives an optimal solution for all possible values of A_t and S_t for $T \leqq p$, and then try to prove that it also gives an optimal solution for $T = p + 1$.

Clearly the method is sound if $T = 1$, so there is no difficulty in starting the induction.

The problem for $T = p+1$ for $p \geqq 1$ can be written as follows: Find the non-negative y_p and w_{p+1} satisfying the constraints

$$A_p \leqq y_p \leqq A_p + S_p,$$

$$A_{p+1} = y_p + w_{p+1},$$

so as to minimize $$E = E_p(y_p) + e_{p+1}(w_{p+1}),$$

where $E_p(y_p)$ denotes the minimum value of

$$\sum_{k=1}^{p} e_k(w_k)$$

for non-negative w_k satisfying

$$A_t \leqq \sum_{k=1}^{t} w_k \leqq A_t + S_t, \ (t = 1, \ldots, p-1), \quad y_p = \sum_{k=1}^{p} w_k.$$

Now, by the induction hypothesis, a set of w_k minimizing $E_p(y_p)$ can be found by the method of Section 3. If we use the notation of Section 3 with $t+1 = p$, it follows that the right derivative of $E_p(y_p)$ is $e'_m(w_m)$. This is a non-decreasing function of w_m, since $e_m(w_m)$ is convex, and w_m is a non-decreasing function of y_p. Hence $E_p(y_p)$ is a convex function.

We could now apply our lemma (with $n = 2$) to the problem of minimizing E, but for the constraints $A_p \leqq y_p \leqq A_p + S_p$.

The most illuminating way of allowing for these is to change variables, writing

$$x_1 = y_p - A_p,$$

$$x_2 = w_{p+1} - \max\{A_{p+1} - (A_p + S_p), 0\}.$$

The choice of non-negative x_1 and x_2 satisfying

$$x_1 + x_2 = A_{p+1} - A_p - \max(A_{p+1} - A_p - S_p, 0),$$

i.e. $$x_1 + x_2 = \min(S_p, A_{p+1} - A_p)$$

is then equivalent to the choice of non-negative y_p and w_{p+1} satisfying all the constraints.

It is clear that the method of solving this problem described immediately after the proof of the lemma is precisely the method advocated in Section 3. This completes the proof.

6. *Computational Details*

The most important computational point not brought out by the preceding sections is the fact that one does not explicitly need the values of the A_t. The algorithm is completely defined by the functions $e'_t(w_t)$ and the constants $M_t = A_t - A_{t-1}$ and S_t.

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When applied to the problem in Section 2 we have

$$e'_t(w_t) = e_{t1} \quad \text{for} \quad w_t < W_{t1},$$
$$e'_t(w_t) = e_{t2} \quad \text{for} \quad W_{t1} \leqq w_t < W_{t1} + W_{t2},$$
$$\ldots$$
$$e'_t(w_t) = e_{ti} \quad \text{for} \quad \sum_{j=1}^{i-1} W_{tj} \leqq w_t < \sum_{j=1}^{i} W_{tj},$$

assuming that the w_{ti} have been numbered so that the c_{ti}, and hence the e_{ti}, form an increasing sequence with increasing i.

It is first necessary to compute e'_t as a function of w_t for all t. (The results can conveniently be recorded in a critical table.) The main working sheet then consists of 4 columns for each value of t, which can be underneath each other.

The first set of columns records purchases, i.e. the present values of the w_t. Initially, these are given by

$$w_t = \max(M_t - S_{t-1}, 0) \quad \text{if} \quad t < T,$$
$$w_T = \max(M_T + S_T - S_{T-1}, 0).$$

The second set of columns records unfulfilled requirements, i.e. $A_t - y_t$. Initially, these are given by

$$A_1 - y_1 = 0,$$
$$A_t - y_t = \min(S_{t-1}, M_t) \text{ if } t > 1.$$

The third set of columns records unused storage, i.e. $A_t + S_t - y_t$. Initially, these are given by

$$S_t \quad \text{for} \quad t < T.$$

Storage at the end of the Tth period is irrelevant.

The fourth set of columns records marginal costs, i.e. $e'_t(w_t)$.

The procedure is, then, to reduce each unfulfilled requirement in turn to zero, by increasing purchases in the period under consideration or else in an earlier period. The rule is that increases must always be made in a period with the smallest marginal costs, but if this is not the same period as that in which the items are required, the unused storage for the period in which the purchases are made and all subsequent periods up to, but not including, the one in which the items are required must be reduced by the amount purchased.

The marginal costs must of course be increased if necessary to correspond with the current values of the w_t, and in addition they must be made infinite if the unused storage for the period in question, or any subsequent period, is reduced to zero.

The computational routine is illustrated in the following numerical example.

TABLE 1

NUMERICAL EXAMPLE OF PURCHASE-STORAGE PROBLEM

	Available (W_{ti})	Cost (c_{ti})	Adjusted Cost (e_{ti})	($\Sigma_t W_{ti}$)
$S_0 = 10$				
January	47	2	15	47
$M_1 = 130$	69	7	20	116
$S_1 = 100$	58	10	23	174
$d_1 = 2$	41	12	25	215
March	8	3	14	8
$M_2 = 68$	50	6	17	58
$S_2 = 100$	39	7	18	97
$d_2 = 1$	97	11	22	194
May	72	4	14	72
$M_3 = 152$	57	5	15	129
$S_3 = 100$	95	13	23	224
$d_3 = 3$	73	15	25	297
July	30	5	12	30
$M_4 = 118$	39	7	14	69
$S_4 = 50$	62	16	23	131
$d_4 = 4$	82	18	25	213
September	78	6	9	78
$M_5 = 260$	44	9	12	122
$S_5 = 100$	77	19	22	199
$d_5 = 3$	73	25	28	272
November	95	3	3	95
$M_6 = 70$	8	5	5	103
$S_6 = 100$	9	12	12	112
$d_6 = 1$	45	14	14	157
$S_T = 10$				

Month	January	March	May	July	September	November
Purchases	Initially,					
$\max(M_t - S_{t-1}, 0)$ if $t < T$,						
$\max(M_T + S_T - S_{T-1}, 0)$ if $t = T$.						
	120	0	52	18	210	0
		68	129	69		80
		91		91		
		140				
		168				
Unfulfilled requirements	Initially,					
$\min(S_{t-1}, M_t)$ if $t > 1$,						
0 if $t = 1$.						
	0	68	100	100	50	80
		0	23	49	22	0
			0	0	0	
Unused storage	Initially S_t.					
	100	100	100	50	100	
		77	51	22		
		23	23	0		
		0				
Marginal costs	$e'_t(w_t)$.					
	23	14	14	12	28	3
		18	23	23		
		22				

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7. Finding Optimal Initial and Terminal Inventories

The problem becomes only slightly more complicated if the initial and final inventories, s_0 and s_T, are not specified, but are required to satisfy some condition, $s_0 = f(s_T)$. Let us assume that $f(s_T)$ has a derivative f'.

It is easy to see that the effect of increasing s_T by x units is the same as the effect of increasing M_T by x. Further, the effect of increasing s_0 by x is the same as that of decreasing M_1 by x, except that this decreases the difference between E and the true cost, by the cost of storing x units from the first period to the end of the operation.

We are therefore led to consider the effect on the optimal solution of changing any M_t.

Suppose first that we decrease some M_t by a small amount ϵ.

We could obviously take advantage of this by reducing w_t by ϵ. But, alternatively, we could reduce w_{t-1} by ϵ and decrease the amount stored over the end of period $t-1$ by ϵ, unless nothing was stored at this time. Another alternative possibility is to reduce w_{t+1} by ϵ and increase the amount stored over the end of period t by ϵ, unless there was no spare storage capacity at this time.

More generally, let

$$s_a = 0,\ s_u > 0 \quad \text{for} \quad a < u < t,$$

where

$$a = 0 \quad \text{if} \quad s_u > 0 \quad \text{for all} \quad u < t.$$

And let $s_u < S_u$ for all $t \leqq u < b$, $s_b = S_b$, where $b = T$ if $s_u < S_u$ for all $u \geqq t$.

Then we can allow for the small decrease in M_t by decreasing any one of

$$w_{a+1}, w_{a+2}, \ldots, w_b,$$

by ϵ, and it is intuitively obvious that we should choose the one giving the greatest reduction in cost, i.e. the one for which the left derivative of e is greatest.

Similarly, if M_t is increased by a small amount ϵ, we can redefine a and b so that

$$s_a = S_a,\ s_u < S_u \quad \text{for all} \quad a < u < t,$$

where

$$a = 0 \quad \text{if} \quad s_u < S_u \quad \text{for all} \quad u < t.$$

$$s_u > 0 \quad \text{for all} \quad t \leqq u < b,\ s_b = 0,$$

where

$$b = T \quad \text{if} \quad s_u > 0 \quad \text{for all} \quad u \geqq t,$$

and we can allow for the small increase in M_t by increasing any one of

$$w_{a+1}, w_{a+2}, \ldots, w_b,$$

by ϵ, and it is intuitively obvious that we should choose the one giving the

smallest increase in cost, i.e. the one for which the right derivative of e is smallest.

We can therefore tell whether s_0 and s_T have optimal values as follows:

Suppose s_T is increased by ϵ. In view of the given relationship between s_0 and s_T this will increase s_0 by $f'\epsilon$.

We then compute the increases in cost due to

(a) Increasing M_T by ϵ,

(b) Storing an extra ϵ over the end of period T,

(c) Storing an extra $f'\epsilon$ from period 1 to period T.

Then a small increase in s_T is profitable only if the cost of decreasing M_1 by $f'\epsilon$ exceeds the sum of the above costs. Similarly, the effect of decreasing s_T may be examined by using a negative ϵ.

It is, therefore, a straightforward matter to find local optimal values of s_0 and s_T. If s_0 is an arbitrary function of s_T, this may well not be a true optimum; but if it is a linear function, so that f' is a constant, the local optimum must be a true optimum. (The proof is an elementary convexity argument.)

This process can be applied to the numerical example considered in Section 6. Let us suppose that circular flow is required, i.e.

$$s_0 = s_T,$$

and that

$$d_6 = 1.$$

Then, starting from the optimal solution with $s_0 = s_T = 10$, found in Section 6, consider the effect of increasing s_T. Here $a = 4$, $b = 6$, so we must increase w_5 or w_6. Our table of marginal costs shows that an increase in w_6 is cheaper, and costs 3 per unit. The extra storage costs (b) and (c) in circular flow are simply the costs of storing throughout the whole operation, i.e. 14 per unit.

An increase in s_T is therefore profitable if a reduction in M_1 saves more than 17 per unit. Here $a = 0$, $b = 2$, so we must decrease either w_1 or w_2. We must consider the marginal costs when the number of units purchased is decreased, i.e. the left derivatives of the $e_i(w_i)$, to see which is more profitable. As it happens, the left and right derivatives of $e_1(120)$ both equal 23, and the left and right derivatives of $e_2(168)$ both equal 22. It is therefore best to decrease w_1 and this produces an overall reduction in cost of 6 per unit. This situation persists until s_T has been increased by 4 units, when $w_1 = 116$ and the saving per unit decrease in w_1 is reduced to 20. A decrease in w_2 then becomes more profitable. The whole computation can be set out as in Table 2.

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TABLE 2

January	March	May	July	September	November
Purchases					
120	168	129	91	210	80
116	157				84
	149				95
					103
Unused storage					
100	0	23	0	100	S_T-10
89					14
81					25
					33
Marginal costs					
23	22	23	23	28	3
					5
					12

Extra storage costs through increasing

$$s_T = \sum_{t=1}^{T} d_t = 14 \text{ per unit.}$$

The optimal solution is $s_0 = s_T = 33$, $w_1 = 116$, $w_2 = 149$, $w_3 = 129$, $w_4 = 91$, $w_5 = 210$ and $w_6 = 103$.

PART II

1. *Transport Model Formulation*

The purchase-storage problem discussed in the first part of this paper can be regarded as a problem in minimum cost transport through space and time. Considering only time period t, there are n sources of the commodity at which the amounts available are W_{ti}. Each source can send at cost c_{ti} to the factory (F_t) or at cost zero to the "dump" (D; independent of t). The requirements are M_t at F_t and

$$\sum_{t=1}^{T} \left(\sum_{i=1}^{n} W_{ti} - M_t \right)$$

(total availability minus total requirements) at D. F_t can also receive the commodity from F_{t-1} up to the amount S_{t-1} at costs d_{t-1} and can send to F_{t+1} up to the amount S_t at cost d_t.

If this transport problem is formulated in the conventional cost matrix, with sources down the left, and destinations along the top, the problem of the limited capacity routes need not be explicitly dealt with in computation.*

* This was pointed out by Mr. Beale.

If Orden's[4] method for dealing with transhipment is applied, not with indefinitely large tonnages at the transhipping points, but rather with the amounts (S_t) specified by the capacities, the computation would proceed exactly as for the standard form of the transport problem.

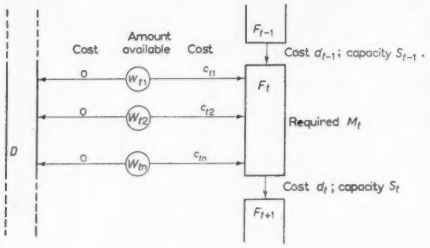

FIGURE 1.

Amount available	Amount required / Destination / Source	$\sum_{t=1}^{T}(\sum_{i=1}^{n} W_{ti} - M_t)$	$M_1 - S_0 + S_1$	$M_2 + S_2$	. . .	$M_{t-1} + S_{t-1}$	$M_t + S_t$	$M_{t+1} + S_{t+1}$	.
		D	F_1	F_2	. . .	F_{t-1}	F_t	F_{t+1}	.
W_{11}	11	0	C_{11}	M	. . .	M	M	M	.
W_{12}	12	0	C_{12}	M	. . .	M	M	M	.
.	.	.	.	.		.	.	.	
W_{1n}	1n	0	C_{1n}	M	. . .	M	M	M	.
.	.	.	.	.		.	.	.	
W_{t1}	t_1	0	M	M	. . .	M	C_{t1}	M	.
W_{t2}	t_2	0	M	M	. . .	M	C_{t2}	M	.
.	.	.	.	.		.	.	.	
W_{tn}	t_n	0	M	M	. . .	M	C_{tn}	M	.
.	.	.	.	.		.	.	.	
S_1	F_1	M	0	d_1	. . .	M	M	M	.
S_2	F_2	M	M	0	. . .	M	M	M	.
.	.	.	.	.		.	.	.	
S_{t-1}	F_{t-1}	M	M	M	. . .	0	d_{t-1}	M	.
S_t	F_t	M	M	M	. . .	M	0	d_t	.
S_{t+1}	F_{t+1}	M	M	M	. . .	M	M	0	.
.	.	.	.	.		.	.	.	

FIGURE 2.

The transport matrix illustrated in Figure 2 is of dimension $(n+1)T-1$ by $T+1$. There are not more than two possible routes in each row, nor

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more than $n+2$ in any column except the first. The only advantage of writing out the problem in the cost-matrix form is that this notation (introduced for computational purposes) has become the standard representation of the transport model. It might be thought that the extension of the problem, which arises when the cost of storage from (e.g.) t to $t+2$ is not simply the sum of d_t plus d_{t+1} (the Caterer problem), could be dealt with by replacing the appropriate M costs by actual costs. But in fact, this method would only be valid if the limitation on storage capacity were a limitation on the store's reception facilities in any month, rather than on the total amount held in store at any moment of time.

As it is not proposed to use the transport model as the computational routine for solving this problem, the cost-matrix form will be abandoned in favour of a discussion in terms of the transport network illustrated in Figure 1. The criterion that this problem has been solved[1, 3] is that dual "prices" ($\bar{W}_{ti}$, $\bar{F}_t$ and $\bar{D}$) can be assigned to each point (source, factory and dump) such that, for every ordinary route which is used, and for every restricted capacity route which is used below capacity, there is zero profit:

$$\bar{D} - \bar{W}_{ti} = 0, \tag{1}$$

$$\bar{F}_t - \bar{W}_{ti} = c_{ti}, \tag{2}$$

$$\bar{F}_{t+1} - \bar{F}_t = d_t. \tag{3}$$

For every route which is not used, profit is negative or zero:

$$\bar{D} - \bar{W}_{ti} \leqq 0, \tag{4}$$

$$\bar{F}_t - \bar{W}_{ti} \leqq c_{ti}, \tag{5}$$

$$\bar{F}_{t+1} - \bar{F}_t \leqq d_t. \tag{6}$$

For every restricted route which is used at capacity, the cost of the route is to be considered as raised sufficiently to cut down the use of it to the allowed level.*

$$\bar{F}_{t+1} - \bar{F}_t = d_t + \gamma_t, \ (\gamma_t \geqq 0). \tag{7}$$

The revised cost of the route $(d_t + \gamma_t)$ can be regarded as the marginal value of an extra unit of storage space.[1, 3]

As in a transport problem the origin of the price set is arbitrary, we can take the price at the dump as zero without loss of generality. Then conditions (1) and (4) become:

* Specifically, the cost is raised by an amount large enough to introduce a neutral circuit into the graph in which S_t is a possible tonnage for the route $F_t \rightarrow F_{t+1}$.

When the (ti)th source to dump route is used:

$$\bar{W}_{ti} = 0, \tag{1'}$$

and when it is not used:

$$\bar{W}_{ti} \geqq 0. \tag{4'}$$

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2. *Optimum Solution*

It is possible to solve this problem sequentially so that for the period up to the one at present being computed, the programme is feasible both for the primal and for the dual solutions.

If the sources in period $t = 1$ are re-ordered so that $c_{11} \leqq c_{12} \leqq \ldots \leqq c_{1n}$, then the requirements for the first period $(M_1 - S_0)$ should be met by taking all the supply from the first source, all from the second, etc. until for some source c_{1k} only part of the supply is required.* This is clearly a feasible programme for the primal problem.

As only part of the supply from the kth source is to be sent to the factory the remainder is to go to the dump (as well as the whole supply for every source where $i > k$).

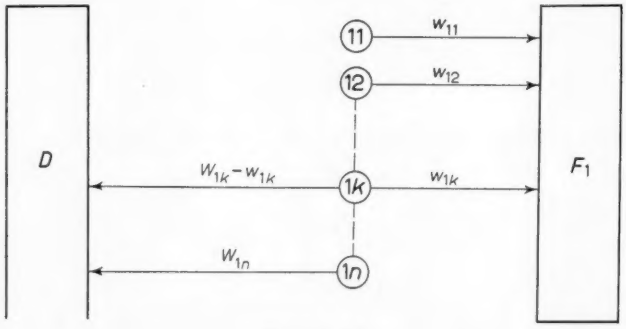

FIGURE 3.

To meet condition (1'), therefore, the price at the kth source is zero, and to meet (2), the price at $F_1 = c_{1k}$. Also, to meet (2), the prices for the sources $1i$ $(i = 1, 2, \ldots, k-1)$ are $c_{1k} - c_{1i}$, which are necessarily non-negative since $c_{1k} \geqq c_{1i}$ $(i = 1, 2, \ldots, k-1)$ and hence satisfy condition (4'). The price at all sources $i = k+1, \ldots, n$ is zero to satisfy condition (1'), therefore

$$\bar{F}_1 - \bar{W}_{1i} = c_{1k}. \quad (i = k+1, \ldots, n)$$

* If it happens that the requirements exactly exhaust the supply from the $(k-1)$th source, the kth source should be regarded as one which supplies the factory, although the amount supplied happens to be zero.

And $c_{1k} \leqq c_{1i}$ $(i = k+1, \ldots, n)$, therefore condition (5) is satisfied. Hence for the first period alone, the solution, so far, is feasible both for the primal and for the dual.

In the second period the whole supply from the cheapest sources should be taken until the requirement M_2 is exactly met. But in this case the sources

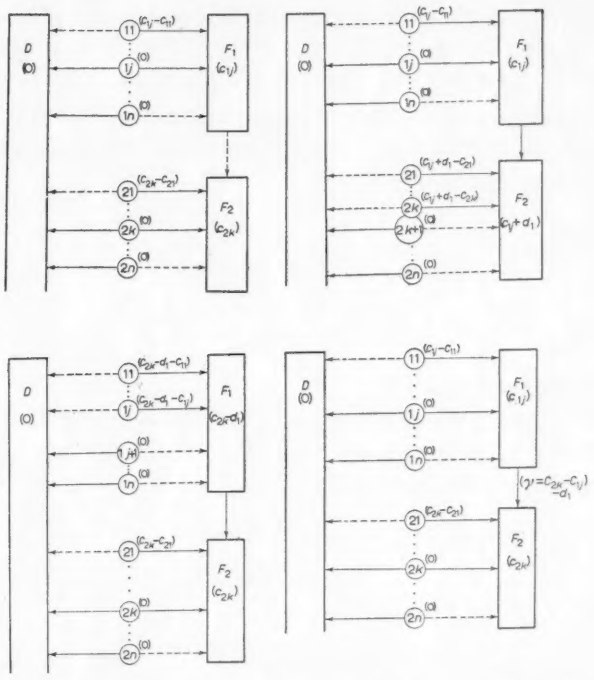

FIGURE 4. *Case* 1, $F_1 \rightarrow F_2$ *not used. Case* 2, $F_1 \rightarrow F_2$ *used below capacity. "Last" source in period* 1. *Case* 3, $F_1 \rightarrow F_2$ *used below capacity. "Last" source in period* 2. *Case* 4, $F_1 \rightarrow F_2$ *used at capacity.*

are not simply those of the second period, but, also, those of the first period which are not fully used in supplying F_1, up to the amount S_1, the maximum capacity of the route $F_1 \rightarrow F_2$. The costs of the first-period sources for supplying F_2 are $c_{1i} + d_1 (i = k, k+1, \ldots, n)$. These sources are to be considered as inserted in the list of second-period sources, and the whole extended list arranged according to cost.

Again, this programme is clearly feasible for the primal problem for the first two periods. Let us consider the four different cases which can arise on the dual solution according to whether the last source whose whole supply is

not taken* belongs to period 1 or period 2, and according to whether the storage route, $F_1 \rightarrow F_2$, is unused, used below capacity, or used at capacity.

The bracketed symbols represent the dual prices at each point, taking the price at the dump as the zero origin. With the sources in each period numbered from the cheapest to the dearest, it can be checked that the unused routes from sources to dump and to factory satisfy conditions (4′) and (5) respectively. Only the important distinguishing features of the four cases will be discussed here.

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Case 1. This can only arise if the requirement of the second period can be met more cheaply from second-period sources alone, than from period one sources with the cost of storage (d_1) added to the original cost (c_{1j}) of the cheapest remaining first-period source.

i.e. $$c_{2k} \leqq c_{1j} + d_1,$$

but $$\bar{F}_2 = c_{2k}$$

and $$\bar{F}_1 = c_{1j},$$

therefore $$\bar{F}_2 - \bar{F}_1 \leqq d_1,$$

which satisfies condition (6).

Case 2. In Case 2 the dual price at F_2 is derived from the origin via F_1. By definition, the last second-period source used is less (not more) expensive than the last first-period source, $1j$.

i.e. $$c_{2k} \leqq c_{1j} + d_1,$$

therefore $$c_{1j} + d_1 - c_{2k} \geqq 0,$$

satisfying condition (4′). Also

$$c_{1j} + d_1 \leqq c_{2q} \qquad (q = k+1, \ldots, n),$$

satisfying condition (5).

Case 3. In this case the dual price at F_1 is derived from the origin via F_2. A similar argument to the above shows that all the conditions of the dual solution are again met.

Case 4. It is here that the difference between this problem and Johnson's[2] occurs. In this case there is a circuit in the graph $(D - 1j - F_1 - F_2 - 2k - D)$. Because of the capacity constraint it has not been possible to take the whole supply from $1j$, although $c_{1j} + d_1 \leqq c_{2k}$. In effect, the dual-price tree is arbitrarily cut off at the route $F_1 \rightarrow F_2$ and separately linked to the origin below this point. Formally γ can be obtained by difference from the independently obtained prices of F_2 and F_1. This makes a neutral circuit in the

* But see footnote on p. 190.

graph since:

$$\bar{D} = 0; \quad \bar{W}_{1j} = \bar{D} - 0 = 0; \quad \bar{F}_1 = \bar{W}_{1j} + c_{1j} = c_{1j};$$

$$\bar{F}_2 = \bar{F}_1 + d_1 + \gamma_1 = c_{1j} + d_1 + \gamma_1 = c_{1j} + d_1 + (c_{2k} - c_{1j} - d_1) = c_{2k};$$

$$\bar{W}_{2k} = \bar{F}_2 - c_{2k} = c_{2k} - c_{2k} = 0; \quad \bar{D} = \bar{W}_{2k} + 0 = 0.$$

$d_1 + \gamma_1$ represents the true value of a marginal unit of storage space as opposed to the accounting value d_1 which has been assigned to it. One extra unit of storage made available at a cost of d_1 would reduce the total cost of the purchase-storage programme by γ_1.

Thus, in all four cases the conditions on the dual solution are met and hence the solution for the first two periods is feasible in respect of both the primal and the dual conditions.

Similarly, the third and successive periods can be solved by meeting requirements from the cheapest remaining sources from the first up to the tth period, within the limits of the storage capacity constraints and adding the successive costs of storage to the original cost on all sources "carried forward" from earlier periods. The conditions of the dual problem are still satisfied. If F_t receives tonnage from F_{t-1}, it is irrelevant whether it is actually purchased in period $t-1$ or carried forward from an earlier period. Thus the four possible states of the dual-pricing tree are still applicable. In cases (1) and (4) F_t and F_{t-1} are independently linked to the origin; in case (2) F_t is linked to the origin via F_{t-1} (and, possibly, F_{t-2}, etc.); and in case (3) F_{t-1} (and, possibly, F_{t-2}, etc.) is linked to the origin via F_t.

Hence, when period T is completed, a solution has been obtained which is feasible both for the primal and for the dual conditions, and which is therefore the minimum cost feasible solution for the primal problem.

3. *Algorithm*

In practice it is unnecessary to consider the dual during the course of the computation, so that the procedure described above can be summarized as follows:

For the first period, sources should be ranked from the cheapest to the dearest. Net requirements $(M_1 - S_0)$ should then be met from the cheapest sources. That is to say, the whole supply should be taken from each successive source until requirements are fulfilled.

For the remaining tonnage at sources in the first period, up to a total not exceeding storage capacity S_1 (still working from the cheapest remaining source, in order of increasing cost) add the cost of storage (d_1) to the cost of

TABLE 1

	Source	Cost at t	Rank	Amount			Cost at $t+1$
				Available	Used in t	Available for $t+1$	
January							
$M_1 = 130(-10)$	**11**	**2**	1	**47**	47		
$S_1 = 100$	**12**	**7**	2	**69**	69		
$d_1 = 2$	**13**	**10**	3	**58**	4	54	12
	14	**12**	4	**41**		41	14
					120	95	
March							
$M_2 = 68$	**21**	**3**	1	**8**	8		
$S_2 = 100$	**22**	**6**	2	**50**	50		
$d_2 = 1$	**23**	**7**	3	**39**	10	29	8
	24	**11**	4	**97**		71	12
	13	12	5	54			
	14	14	6	41			
					68	100	
May							
$M_3 = 152$	**31**	**4**	1	**72**	72		
$S_3 = 100$	**32**	**5**	2	**57**	57		
$d_3 = 3$	**33**	**13**	5	**95**		23	16
	34	**15**	6	**73**			
	23	8	3	29	23	6	11
	24	12	4	71		71	15
					152	100	
July							
$M_4 = 118$	**41**	**5**	1	**30**	30		
$S_4 = 50$	**42**	**7**	2	**39**	39		
$d_4 = 4$	**43**	**16**	5	**62**		22	20
	44	**18**	7	**82**			
	23	11	3	6	6		
	24	15	4	71	43	28	19
	33	16	6	23			
					118	50	
September							
$M_5 = 260$	**51**	**6**	1	**78**	78		
$S_5 = 100$	**52**	**9**	2	**44**	44		
$d_5 = 3$	**53**	**19**	3	**77**	77		
	54	**25**	6	**73**	11	62	28
	24	19	4	28	28		
	43	20	5	22	22		
					260	62	
November							
$M_6 = 70(+10)$	**61**	**3**	1	**95**	80		
	62	**5**	2	**8**			
	63	**12**	3	**9**			
	64	**14**	4	**45**			
	54	28	5	62			
					80		

The primal solution can be read from the column "Amount used in period t".

The dual solution, if it is required, can be obtained by sketching the network from the computed primal solution.

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purchase for each source. This gives an extended list of sources for the second period. Taking this extended list in order from the cheapest to the dearest, fill the second-period requirements and then produce the extended list for the third period, up to the tonnage capacity S_2. Some of the additional sources for the third period may be first-period sources with two storage costs, d_1 and d_2, added to the purchase cost.

Repeat for the third, fourth periods, etc., always filling requirements from the cheapest end of the extended list of sources, and adding storage cost to the cheapest remaining sources up to storage capacity, to produce the extended list of sources for the next period.

The numerical example described in Part I can be computed as shown in Table 1. The figures in bold type are the original data of the problem. The capacity constraint is only effective for storing from the second to the third and from the fourth to the fifth month.

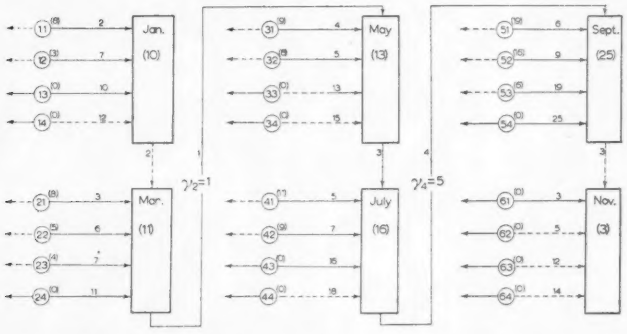

FIGURE 5. *The bracketed figures are the dual prices at the appropriate source or destination.*

4. *Cyclical Programme*

In the case of the stationary circular flow (when the problem is one of programming purchase and storage to meet predictable seasonal fluctuations) the transport network is changed only by the removal of the initial and terminal inventory and the addition of a route with known cost and capacity $F_T \rightarrow F_1$.

If in the numerical example it is further assumed that it is possible to store up to 100 units from period 6 to period 1 at a cost of 1, it will be apparent from Figure 4 that the present solution is not feasible in the dual, since there is a profit on the route $F_6 \rightarrow F_1$ of $10 - 1 - 3 = 6$. The cyclical

TABLE 2

Source or store	Availability or capacity	Primal		Dual		
		Straight	Cyclical	Price	Straight	Cyclical
		Purchased or stored				
S_0	100	10	33	γ_0	0	0
11	47	47	47			
12	69	69	69	$\bar{F}_1$	10	9
13	58	4	0			
14	41	0	0			
S_1	100	0	19	γ_1	0	0
21	8	8	8			
22	50	50	50	$\bar{F}_2$	11	11
23	39	39	39			
24	97	71	52			
S_2	100	100	100	γ_2	1	1
31	72	72	72			
32	57	57	57	$\bar{F}_3$	13	13
33	95	0	0			
34	73	0	0			
S_3	100	77	77	γ_3	0	0
41	30	30	30			
42	39	39	39	$\bar{F}_4$	16	16
43	62	22	22			
44	82	0	0			
S_4	50	50	50	γ_4	5	5
51	78	78	78			
52	44	44	44	$\bar{F}_5$	25	25
53	77	77	77			
54	73	11	11			
S_5	100	0	0	γ_5	0	0
61	95	80	95			
62	8	0	8	$\bar{F}_6$	3	8
63	9	0	0			
64	45	0	0			

Total cost, straight problem, 6716.
Total cost, cyclical problem, 6623 (as compared to 6726 if storage from period 6 to period 1 is arbitrarily set at 10), a reduction of 103 in the total cost of the programme.

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problem can be solved by continuing the computation from period 6 to period 7, which has the same initial data as period 1, and so on, until the pattern begins to repeat.

The two problems can be compared in respect both of their primal and of their dual solutions as shown in Table 2.

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5. *Two or more Commodities sharing Limited Storage Space*

The case where there are several different commodities for which to plan the purchase and storage at minimum total cost is a natural extension of the problem. If the only interdependence between the programmes is that they compete for the limited storage space, then the problem can be represented as separate transport problems, with the additional condition (on the primal) that the sum of the tonnage on all the routes $F_t \rightarrow F_{t+1}$ is to be not greater than S_t. This means that in the dual solution γ_t is to be equal in each commodity programme. The units of the commodities must be defined in terms of the units of storage space which they occupy. The algorithm of Section 3 is no longer applicable, but the problem can still be solved as a capacitated transport model.

Summary

These two papers relate to the scheduling of purchases of a given commodity to meet known future requirements while minimizing total costs. Purchases in advance of requirements are permitted, but storage costs must be considered and storage space is limited.

The problem is essentially one of linear programming, and the two independent approaches both led to what is substantially the same simple algorithm.

ACKNOWLEDGEMENTS

We are grateful to the Admiralty for permission to publish this paper. Our thanks are due to Miss Alison Doig of the London School of Economics for her extensive help and criticism during the preparation of this paper.

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Linear Programming—An Application in an Indian Textile Mill*†

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My task today has been greatly simplified by this talk following Mr. Rice's report.‡ I do not, therefore, have to describe the general organizational structure of the Mills, nor do I have to stress the extremely helpful attitude towards applied research, and towards change and improvement, found throughout the firm. I shall confine myself almost entirely to a statement of the work which was done in introducing linear programming and of some of the consequences of this introduction. It was inevitable that within the linear programming frame, questions of wider implications should arise, some of which will be discussed or, at least, mentioned.

The assignment was twofold: to test the contribution which linear programming could usefully make in production planning; and to train somebody to take over the work after a brief visit of about four weeks. It seemed unlikely that definitive results could be obtained in so short a time, or that the job could be run as a one-man show, and hence training was given equal importance with the main task. It should, therefore, be obvious that success depended very largely on whether suitable assistants could be found; and I was very fortunate in that the two young, but senior, accountants who worked with me throughout my visit were exceedingly able and hardworking men, familiar with the system of accounting in use, with most of the technical aspects of production, with the handling of data, and having some knowledge of mathematics and of basic economic theory. It should not be inferred that without such paragons introduction of linear programming, as a routine operation in a firm, is impossible; but under the actual conditions the high calibre of these two men was most welcome. The specific reason for the secondment of accountants was that it had been decided at the outset that linear programming work was to be carried on in

* Paper read at the meeting of the Operational Research Society, 6 January 1958, by Miss Doig.

† I am indebted to Mr. Gautam Sarabhai for his permission to refer to work done and results obtained at the Calico Mills, Ahmedabad.

‡ For a detailed description see K. A. Rice, *Productivity and Social Organisation—The Ahmedabad Experiment*, Tavistock Publications, London, 1958.

the department of the financial controller, partly because it was to be integrated with costing and budgeting, and also because data, on which production planning—as a managerial function—was based, were already in the hands of that department.* Up to that time, production planning centred on the decisions as to which looms to use for what products and plans for the loom utilization were drawn up half-yearly and occasionally revised. The basis for such decisions was a figure of "profit per loom shift" which took into account realized prices of the various types of cloth and all direct costs.† The translation of the main decisions into operational detail became then the function of the production manager who had taken part in the formulation of the master plan. It was this main set of decisions for which linear programming was intended to become a partial or complete substitute. Making "profit per loom shift" the basis of planning was basically sound; the only criticism, that of initially ignoring a large set of other effective considerations, would make sense only if human beings could, in fact, handle large masses of data simultaneously. In an enterprise of the size of the Calico Mills it would be unreasonable to expect this, and hence, at the initial stages, only a few key figures were taken into account. While such a procedure is eminently practical, I would stress that if there are mechanical means of arriving at decisions within a well-defined framework, logical as well as factual, managers would (or, at any rate, should) welcome the need for less thought and mental effort, together with the added freedom of applying judgement where really needed, which this partial mechanization of thought provides, once it has become an operational routine.

Throughout, the two aims—programming and training—were pursued simultaneously. At the start a sample problem of order four by four was taken and the technique of solution (simplex, inverse basis) discussed, together with the interpretation of the solution and of its dual. Shortly afterwards, it became clear that one part of the mills' production—blankets—could be separated and treated on its own without much loss of realism. The inter-relation between the production of blankets and of other products

* The systematic collection of data both by the Statistics Department and in recent years by the Accounts Department turned out to be extremely valuable. Normally—and especially under Indian conditions—only a small fraction of the required information would be so readily available.

† Monthly loom utilization statements had been prepared and had proved a useful practical guide to an assessment of the current situation and for revision of current plans. It was in fact the realization of their inadequacy which led to the tentative introduction of linear programming.

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is slight. Blanket looms cannot be used for any other type of cloth and the raw material is almost entirely waste which is obtained in fairly constant amounts from the rest of the production line, or bought from outside. Equally, the rest of the equipment required for the making of blankets is specific. Further advantages of separate and early treatment of the blanket programme were that it absorbs only a small number of the firm's looms, and that the number of effective restrictions was sufficiently small so that the formulation and the solution could be readily understood. There were three types of blankets which could be produced in five different activities, since two of the blanket types could be produced on two different looms (60 in. and 72 in. loom width). There were only seven restrictions to be taken into account and the solution was a five by five set. The outcome of this programme was carefully checked for detailed feasibility and sense, and was found to be a slight improvement on existing plans. This minor effort thus provided good training and added to the confidence and enthusiasm of the team.

At the same time attempts were made to come to grips with the main problem. Originally we tried to get a complete stage-by-stage description of the flow of production from raw cotton to finished cloth. It soon became apparent that this method of formulation would lead to an impossibly large table, quite apart from the difficulty of obtaining all the numerical details required. After some discussion one important fact emerged, viz. that there was a comparatively small number of crucial stages which would constitute effective restriction upon output and its composition. Stages between these had enough elbow-room to handle any through-put which could possibly come from the earlier (bottleneck) stage. Thus, instead of planning for the whole sequence of production, it became possible to focus attention only on the "bottlenecks", always with the implicit understanding that if there were any unsuspected infringement of restrictions, other than those incorporated in the programme, they would be added to the model.*

At this stage it might be helpful to those not familiar with linear programming to be given a very short summary of what it is about. It consists of a list of activities or processes, which transform certain inputs, or factors

* This is always possible. When using the inverse basis method, one or more restrictions can be added without much labour. Formally, if the inverse of a matrix is given, the inverse of another matrix with some additional rows and a set of unit columns added to the original one is obtained by matrix multiplication. This inherent simplicity is fortunate: frequently management becomes aware of some restrictions only after they are violated by a proposed solution. At other times such restrictions are so much part of the sub-conscious of managers that the linear programmer has to assume the role of a psycho-analyst before setting out on his main task.

(raw materials, machine hours, men hours), into outputs, which may be intermediate or final commodities. It is assumed that the ratios between all the inputs and outputs remain constant in each process at all levels. With each process there is associated a "profit" which may be positive, negative or zero, and with each input and each output there may be specific amounts of inputs available or of outputs required. The purpose of the programme is to find the levels of processes which will maximize total profits, it being understood that negative levels are meaningless. The solution is usually obtained by an iterative procedure which may take quite some time.

In many respects, the situation at the mills is a linear programmer's paradise. It is our common experience that many of the restrictions (i.e. a list of availabilities of factors and requirements of products) introduced in our models are only rough approximations. Sometimes they are guesses, sometimes, though accurate, they can be altered at relatively short notice. The main restrictions in the present case are, however, quite definite and not easily changed. For example, under existing regulations it is impossible for the firm to increase the number of looms and ring spindles (hence the emphasis on profits per loom shift), the regulations being a consequence of the desire of the Indian government to protect home weavers and to discourage further growth of the textile industry in Bombay State. For similar reasons, the total daily average of production of printed materials is a fixed figure which must not be exceeded. Or, again, the amount of excise payable on certain types of cloth increases discontinuously with the amount sold. While not all of these restrictions are necessarily physical ones, they can be incorporated in the model and treated as such, including the fact of different amounts of excise payable.

In the present model, processes were identified with a selection of final products to be produced. This selection was agreed, after some discussion, as representing a reasonable sample of the range of possible products which extends to about 400 products at any one time. Of these the forty principal ones were included. The differences between some of the 400 products are often only in the actual pattern of the cloth, changes in which can be effected quickly, without altering the production schedule, so that several of these forty could literally represent many others. For each of the forty products, requirements, per unit output, of the various scarce items of equipment were listed.

At this stage the treatment of looms became important. If allowance is made for variations in loom type (ordinary, automatic, Dobby, Jacquard), then a fully realistic picture would require a listing of more than a hundred

different kinds of loom, which would have been clearly impracticable. Hence, looms were sorted into twelve different groups, each group containing those looms which were readily interchangeable in use and similar in operating characteristics. More particularly, if some kind of cloth could be woven by one loom in a group, it could be woven by any other of the same group. It does not follow, however, that any one kind of cloth could be woven on looms of one group only. Thus while ordinary looms and automatic looms of similar widths have widely differing operating characteristics, cloth of suitable width could be woven on either of these—a fact which had to be allowed for. In the listing of technically feasible ways of producing a given kind of cloth, it also became necessary to draw the line somewhere, e.g. not to consider the technically possible method of weaving 30 in. cloth on 60 in. looms.* Since some kinds of cloth could, in fact, be woven by looms of different groups, it became necessary to list them separately as distinct activities corresponding to each loom group. (This explains also why in the blanket programme, mentioned before, three kinds of blankets required five processes.) The same difficulty arose with regard to slubbers, of which two kinds were listed according to the counts of yarn. The two kinds overlapped in the middle range, i.e. counts of thirty to forty could be handled by either type of slubber and, hence, the cloth using such yarn would have to be listed twice. If such cloth could also be woven in looms of three different groups, it would appear six times in all. For this reason the list of activities contained, finally, about 160 columns. Restrictions, represented by rows, numbered twenty. They were: Combers, Slubbers (two kinds), Ring Spindles, Doubling Spindles, Looms (12 groups), Printing, Excise (two ranges according to amounts sold). The profits on each kind of cloth were obtained as the difference between realized sales value and costs, the components of which were: labour, raw materials, fuel, the variable components of maintenance cost of specific items of equipment, and, in special cases, excise. It will be seen that dutiable goods subject to different levels of excise would appear identically in the main body of the table, apart from the entry corresponding to the two rows representing

* It became frequently necessary to temper rigour with practicability. Normally, one would like the profitability of using any kind of loom to weave some kind of cloth to emerge from the procedure. It happened frequently that when one of the managers was asked whether some process was *technically* possible he would reply "No, it wouldn't pay". The purist should object that whether a process would pay should emerge from the computation if based on correct technical data, and, at times, the purist would be right. In practice, though the range used was wider than the production manager was ready to admit at first, a sensible compromise was reached on the basis of which it was seen that the manager was usually right.

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restrictions on the amount sold subject to any one rate of excise; but they would, of course, differ in their profit figure. Requirements of equipment, and amounts of equipment available, were listed in terms of hours. Depreciation and general overheads did not enter the calculation for two reasons: first, depreciation was in fact not being allocated to individual departments, a flat allowance of 5 per cent of sales value being made throughout; and second, this is perfectly correct procedure in any case, since overheads should not be allowed to influence decisions as regards short-term production plans.

It need not be pointed out that most of the data were subject to error. Apart from the legal restrictions which are precise, amounts available of other factors of production may be accurately known but the reductions due to idle time which must be allowed for are "averages"; different allowances were in fact made for different pieces of equipment. Also, the number of hours required in a given stage, for a given type of cloth, is itself based on estimation and averaging. This was unavoidable where loom groups were used, since the operating characteristics of different looms within any one group differ to some extent.

A particularly arbitrary set of data is that of realized prices, which form the basis of profits obtainable in the production of the various types of cloth according to the method of production. Estimates of prices some months ahead cannot be very reliable however familiar with market conditions the sales department may be. But there is a further difficulty. Some of the mills' products are sold in a market economists call perfectly competitive; i.e. the volume of likely sales will not affect the price realized by the firm. With other products, however, the firm has a strongly monopolistic position and, hence, unit prices will largely depend upon the volume sold. (A typical case would be organdie where the firm was, until recently, the sole Indian producer while even now there are only a few Indian firms in the market.) If demand conditions were accurately known, we should, at best, have to introduce quadratic programming—in the case of a straight-line, downward-sloping, demand curve—and that, though still manageable, would introduce additional, computational problems. But, in the absence of detailed market information, it was decided to ignore this effect initially and, if output of such types of cloth under the linear programming plan should greatly differ from production in the past to revise estimated prices according to the best available guesses (this amounts to guessing a few points on the demand curve) until a new solution was obtained. Another possibility discussed was that of putting somewhat

arbitrary limits on the amounts to be produced under the linear programming plan, say not more than 150 per cent of current production. Neither of these methods requires much additional labour. It is well known that a given solution of the problem can be readily revised in order to deal with changes in some of the data. This flexibility can be of great advantage if there are frequent changes in prices and costs during the period of the plan.* It should be pointed out that all these ambiguities and uncertainties are inherent in the management of any enterprise. It is not linear programming which requires more accurate data; linear programming rather has the advantage of showing up the range of ignorance under which actual planning must take place, since the same data will go into any planning whether they are actually used or merely form a vague background to managerial decisions. The only risk is the well-known, psychological one of assuming accurate knowledge merely because a number has been put on paper, a risk against which it is admittedly difficult to guard.

A comparatively minor problem is that of indivisibilities. Linear programming is based on the assumption that any factor can be used in whatever fractional quantities are desired. With looms, the pretence that loom hours can be taken as the ultimate unit is not serious, any rounding off to the nearest loom day being unlikely to do any damage. But in the case of slubbers—even though the unit used in computation was the spindle hour—the problem was serious since scheduling is done on the basis of large, and for practical reasons indivisible, units called preparations; and here a rounding off to the nearest whole preparation can in fact be misleading.†

Another difficulty, generated purely by the linear programming approach, deserves mentioning. The solution of the problem provides also what economists call "marginal value products" of the various scarce factors. They measure, in the present instance, by how much profits would be greater if, say, another loom hour or an additional spindle hour were available. This, in itself, is valuable information obtained at no extra computational cost, but it follows that the total net profit under any linear programming solution is divided up completely among all the scarce resources listed in the computation. Hence, if some kind of equipment is

* There is no inherent reason why production plans should not be checked (and revised) in the light of changing market conditions quarterly or even monthly, or else whenever prices and costs turn out rather different from those assumed in the plan.

† It does not follow that rounding off to the nearest integer up or down in an originally non-integral solution will yield the best integral answer. Theoretically, *all* the integral answers would have to be investigated.

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not restrictive under a given solution, if, e.g. an extra spindle hour could not increase total profits because spindles were not fully used, the apparent contribution of the spinning department to profits appears to be nil. It raises the question how to counteract the psychological effect of partly understood numerical details, or perhaps how to keep certain items of information from middle grade management. This is also related to the difficulty of persuading management that it may be better to keep certain items of equipment idle, rather than aim at full employment of all factors. For instance, it may happen that even though looms in general are scarce, it still pays not to use fully a particular type of loom, since the use of ancillary equipment or of the printing allowance for products other than those made on the particular looms may be more profitable.

A further consequence of a linear programming model with only twenty restrictions is that optimum solutions exist which will not require the production of more than twenty types of cloth (which, it will be recalled, includes already the differentiation according to process used, and hence may have fewer than twenty distinct items). This was already known to management when the model was set up; and it so happened that top management had already been arguing for a reduction in the range of products and a concentration on steady production of fewer products with a possible improvement in quality and reputation for those products. But a specific point, raised by the sales department, was that unless a sufficiently wide range of products was offered to wholesalers some custom might be lost. To account for this argument within a linear programming framework is almost impossible, and, in any case, decisions of this kind depend principally on judgement about longer-term effects. The production of any item not on the final list will cause a drop in "profits" as maximized by linear programming, the extent of which can be easily obtained from the original solution. Hence, if some additional lines are to be produced—for whatever reasons—the cost of this indulgence is known.

At the end of the four weeks the work had not yet been completed, partly because of changes in the selection of commodities. The latest interim solution was however "tried on for size" and it was found that this solution did make sense and that it could be readily translated into the scheduling of production. This caused some mild surprise at the time, particularly when it was found that it did not violate any of the restrictions which had been incorporated into the model in the first place! Meanwhile the work has been going on, the firm having decided to set up a special group to handle linear programming problems and possibly to acquire electronic equipment for

this and other purposes. It is, unfortunately, in the nature of things that no adequate comparison is available on the contribution to increased profits from linear programming methods. When linear programming is already in partial use management decisions will be affected by the knowledge obtained from it and, in the absence of a separate and parallel planning-operation with artificial loss of memory, any comparison will be biased. Apart from effects on profits, which are unlikely to be negative, the principal gain would seem to lie in freeing top management from detailed and prolonged planning sessions, in making clear the assumptions on which planning must be carried out, and in the additional information gained as by-products of setting up adequate models and obtaining numerical solutions from them.

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Operational Research and Advertising: Some Experiments in the Use of Analogies

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Introduction

It has become quite common in attempting to construct a theory to explain observed phenomena, especially where chains of causation have to be explored, to make use of analogies or models. The possible relationships involved are more easily thought about in terms of pictures; they are given shape and definition which facilitate inferences about the probable consequences of changes in the various independent variables. There are dangers in the use of models as Braithwaite[1] has pointed out in that, for example, attributes peculiar to the model may be erroneously imported into the deductive system, but if these dangers can be avoided the method is powerful.

The present example concerns the relationship between the amount of advertising and the response to it; and in the following paper a number of different analogies have been employed to assist in the consideration of this relationship. But it should be emphasized at the outset that the analogies are drawn upon only as sources of ideas as to the kind of mathematical curve which might be used to express the relationship between advertising and response. It is not suggested that the conditions normally present in the analogies systems do really exist or are even approximated to in advertising practice. What we are doing here, statistically, is to fit a smooth curve to some crude observations, and in such an operation it is legitimate to form prior judgement as to the possible shape of that curve. The analogies are used only to this end.

The Stimulus and its Measurement

Promotional activity is designed to direct the thinking of people by the association of ideas in such a way as to lead to the desired response, whether this be recruitment to the Forces, attendance at cinemas or the purchase of consumer goods. Leaving aside for the moment the discussion of the necessity for pre-existing need or motivation, and of the difficulty of establishing the independence of any one spell of advertising, we may first

* The Royal Institution.

consider advertising as an example of stimulation and inquire whether advertising may be meaningfully described in terms of stimulus intensity on the basis of empirical relationships between the "amount" of advertising and the observed response.

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TABLE 1

RELATION BETWEEN ADVERTISING INPUT AND RESPONSE

(1) Type of Product and Advertisement	(2) Period or Spell	(3) Advertising Input (units*)	(4) Direct Response (units*)	(5) $\frac{(4)}{\log_e(3)}$
(A) Vitamin Product (journals and mailed leaflets)	I	1·53	109	256
	II	3·20	123	106
(B) Textbooks (mailed leaflets)	I	220	51	9
	II	477	72	12
	III	1,400	169	23
	IV	2,000	140	18
	V	3,700	315	38
(C) Service Recruiting, Type 1 (newspapers)	I	135	6,500	13×10^{-2}
	II	442	11,455	25×10^{-2}
(D) Service Recruiting, Type 2 (newspapers)	I	816	302	45
	II	1,416	530	73
	III	2,144	1,188	155
	IV	3,555	1,141	140
(E) Radio Equipment (newspapers)	I	240	30	5
	II	330	75	13
	III	540	103	16
	IV	1,250	162	23
	V	3,000	267	33

* Actual units have been concealed by multiplication by constants to prevent identification of amounts. Moreover, *the multipliers are different for different cases*; in particular the two types of service recruiting cannot be compared.

In Table 1 there have been arranged data drawn from five different sets of advertising campaigns:

(A) Advertisement, by way of technical-journal announcements and leaflets mailed to medical practitioners, of a vitamin product.

(B) Mailed leaflets based on the membership lists of learned societies and professional organizations.

(C) Service recruiting by means of newspaper advertisements, Type 1.

(D) Service recruiting by means of newspaper advertisements, Type 2.

(E) Radio equipment advertised in the daily newspapers.

The quantity of advertising related to numbers of announcements, journal issues and numbers of leaflets distributed, is shown in column (3), arbitrary units being used to protect the confidential nature of the information and to prevent comparison of absolute results. Column (4) shows a measure of response in terms of inquiries, again expressed in arbitrary units.

Physiological Analogy

If the analogy of the physiological response to stimulus were to hold good we should expect a linear relationship between the response R, and the natural logarithm of the advertising input A. Column (5) of the table shows the ratio of R to $\log_e A$; the gradually rising value of this ratio with increased stimulus suggests that such a relationship may exist.

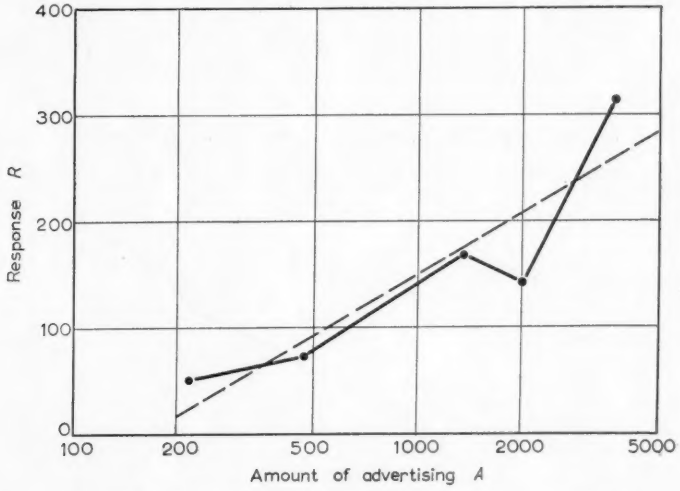

FIGURE 1. *Case (B). Fitted curve is* $R = 84{\cdot}06 \log_e A - 432{\cdot}6$.

In Figures 1, 2 and 3, the response values for cases (B), (D) and (E) have been plotted against the input on a logarithmic scale and linear functions (shown as broken lines) have been fitted by least squares to these points. They seem feasible. It would however be necessary to accumulate a considerable amount of data under stable conditions before reaching any firm conclusions.

Figure 4 shows the same three functions plotted on linear scales. They imply, if the analogy can be pressed thus far, (i) that there is a threshold

value of advertising of a not inconsiderable quantity below which there is no appreciable response and (ii) that there is eventually a state of near saturation in the sense that an inordinate increase in advertising is required to achieve any increase in response.

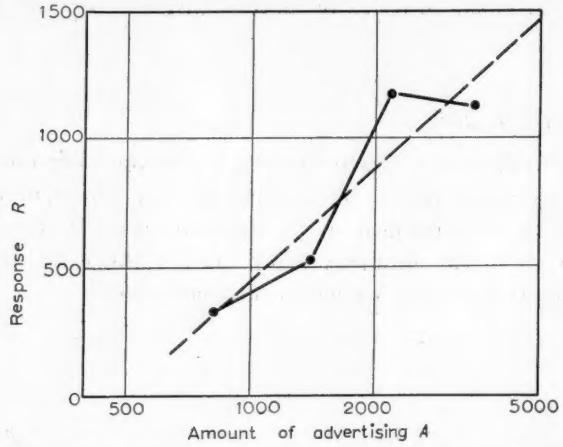

FIGURE 2. *Case (D). Fitted curve is* $R = 654{\cdot}6 \log_e A - 4088{\cdot}4$.

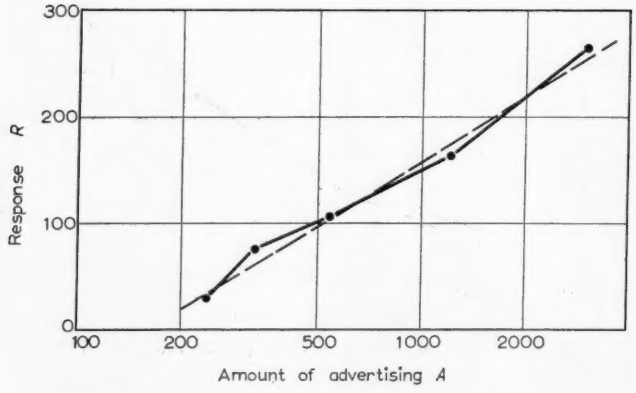

FIGURE 3. *Case (E). Fitted curve is* $R = 87{\cdot}88 \log_e A - 447{\cdot}5$.

The point at which the marginal gain would be too small to justify increasing the scale of advertising would be decided on the basis of advertising costs and profit margins, but given the data and this criterion it would be predictable from such a curve as is illustrated in Figure 4.

If the relationship between response R and advertising amount A is

$$R = a \log_e A + b$$

then in the increment in R, (ΔR), resulting from an increment in A, (ΔA), will be

$$\Delta R = a \log_e\left(\frac{A+\Delta A}{A}\right).$$

If the profit margin is r per unit then the criterion for increasing advertising by x per cent will be

$$\left[r.a.\log_e\left(1+\frac{x}{100}\right)-\frac{Ax}{100}\right] \geqslant K$$

where A is expressed in terms of cost and K is a prescribed minimum increase in revenue.

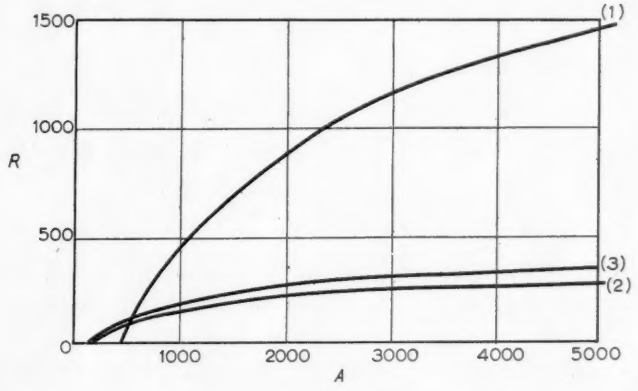

FIGURE 4. (1) ... $R = 654{\cdot}6 \log_e A - 4088{\cdot}4$. (2) ... $R = 84{\cdot}06 \log_e A - 432{\cdot}6$. (3) ... $R = 87{\cdot}88 \log_e A - 447{\cdot}5$.

The application of the logarithmic relationship for response to stimulus, viz., in the notation used above, $R = a \log_e A + b$, to message diffusion has been tested (De Fleur[2]) with encouraging results. In a number of towns in the United States, of similar size and characteristics, leaflets were dropped at different intensities (leaflets per head of population). The leaflet required the finder to note and pass on a message. Subsequent surveys were carried out to estimate the proportion of the population who were message-knowers. The figures in Table 2 have been extracted from the published results.

The fit of the curve is quite strikingly good having regard to extraneous sources of irregularity. A similar fit was obtained for the proportion of actual leaflet-finders who complied with an instruction to mail a postcard to a particular centre.

TABLE 2

VARIATION OF RESPONSE WITH INTENSITY OF STIMULUS

Town	Leaflet Ratio (i.e. intensity of stimulus)	Observed Proportion knowing Message (per cent)	Expected Proportion based on Logarithmic Relationship (per cent)
A	1/4	25·2	25·2
B	1/2	37·4	34·0
C	1/1	30·1	42·9
D	2/1	44·1	51·7
E	4/1	63·4	60·6
F	8/1	71·8	69·4
G	16/1	82·4	78·3
H	32/1	87·9	87·1

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Law of Diminishing Returns

Zentler and Ryde[3] have been impressed with the logical attraction of the law of diminishing returns and have proposed an empirical and generalized form:

$$R = R' \frac{c_2 x^2 + c_3 x^3 + \ldots c_n x^n}{1 + d_1 x + d_2 x^2 + \ldots d_n x^n}$$

Where R is response, x is real expenditure on promotion, R' is a limiting value that R approaches as x becomes infinitely large, and $d_1 \geqslant 0$, $c_n = d_n$, $c_s \leqslant d_s$ for $2 < s < n$.

For practical purposes the number of constants can be very restricted and it is possible to suggest a basic form:

$$R = R' \frac{(x/b)^2}{1 + (x/b) + (x/b)^2}$$

where b = the level of expenditure at which the average response is equal to the marginal response (the point of contact with the tangent from zero expenditure before the rising curve of response flattens off).

Case D above, for example, yields

$$R = 1800 \frac{(x/1200)^2}{1 + (x/1200) + (x/1200)^2}$$

and the comparison with the logarithmic equation is

	x	1,000	2,000	3,000	5,000
R	Logarithmic	433	887	1,152	1,487
	Zentler and Ryde	493	920	1,154	1,388

The Analogy of Biological Assay

So far the analysis has been in terms of absolute response, i.e. the numbers of persons responding to an advertising appeal. A different situation is created if we are concerned with a closed population of fixed size, to every member of which the appeal is extended (or an approach to such a model). In this event we are concerned with the proportion responding and to changes in that proportion as a result of changes in the intensity of advertising. The analogy is with the problem of biological assay—what increase in the proportion of subjects affected will result from a prescribed increase in the dosage of a particular stimulant?

The data would be similar to those shown in Table 2 but each line of the Table would refer to the same fixed community.

Let $f(x)$ represent the ordinate of the distribution of response to stimulus x at the point x, i.e. it represents the additional proportion responding to an infinitesimal increase in stimulus. Since all those who react to stimulus x_1 will react to smaller stimuli, the proportion responding to x_1 will be:

$$P_R = \int_{-\infty}^{x_1} f(x)\,dx$$

and if $f(x)\,dx$ is Normal {i.e. $f(x) = [1/\sqrt{(2\pi)}]\exp(-x^2/2)$}, P_R will be a curve of characteristic sigmoid shape, rising slowly at first, then more rapidly, and finally flattening off to a value of unity (all of the population affected).

If $f(x)dx$ is not Normal then the variable is transformed, e.g. to $\log x$ to produce such a distribution. (The attraction of the Normal distribution in this context is partly that the distribution values are tabulated and well known but mainly that the corresponding sigmoid curve for the cumulative function is almost linear over the greater part of the central position.)

The method adopted in the analogous situation in biological assay is to first transform the scale of dosage until the proportional response distribution is Normal, then the abscissae or probits corresponding to specified proportions of the normal distribution which is assumed to represent the response–dosage relationship are derived; and by fitting a straight line to these probits the sigmoid curve representing the cumulative response function is simply obtained by transformation.

Table 3 was compiled from consideration of the data of Table 2 (see Figure 5) as referring to the same population. The parameters of the Normal curve here are $\mu = 2{\cdot}607$ and $\sigma = 2{\cdot}556$. So that 50 per cent of the population will react at an intensity of $\log^{-1} 0{\cdot}607$ or 1·84 pamphlets per

head. For an intensity of 10 per head we have

$$\frac{(2+\log 10)-2{\cdot}607}{2{\cdot}556} = 0{\cdot}663$$

and the proportionate area under the Normal curve to the left of such a standardized deviate = 74·6 per cent.

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TABLE 3

PROBIT ANALYSIS

Dose	Dosage (x) (log+2)	Proportion Responding P_R	Actual Probit	Linear Fit (0·3912x+3·98)	Expected Proportion
0·25	0·614	0·252	4·336	4·220	0·217
0·50	1·307	0·374	4·678	4·491	0·305
1·0	2·000	0·301	4·482	4·762	0·406
2·0	2·693	0·441	4·852	5·034	0·512
4·0	3·386	0·634	5·342	5·305	0·618
8·0	4·079	0·718	5·574	5·576	0·719
16·0	4·773	0·824	5·932	5·847	0·802
32·0	5·466	0·879	6·157	6·118	0·868

Improved values may be obtained by a maximum likelihood fit, viz. 0·3818x+4·004 and these are

$$\mu = 2{\cdot}609$$

$$\sigma = 2{\cdot}619$$

but this refinement makes very little difference to the expected response P_R. Bearing in mind the quality of the data likely to be available, one would expect quite crude methods to suffice.

Physical Analogy

Vidale and Wolfe[4] in a comprehensive paper have introduced the concept of a saturation level M as the practical limit of sales that can be generated as promotional activity is increased in intensity, together with a response constant r defined as the sales generated per advertising expenditure unit at the point when sales S are zero; and have expressed the sales generated per advertising expenditure unit at level S as

$$\frac{r(M-S)}{M}$$

or in our terminology

$$\frac{dR}{dA} \propto R_M - R$$

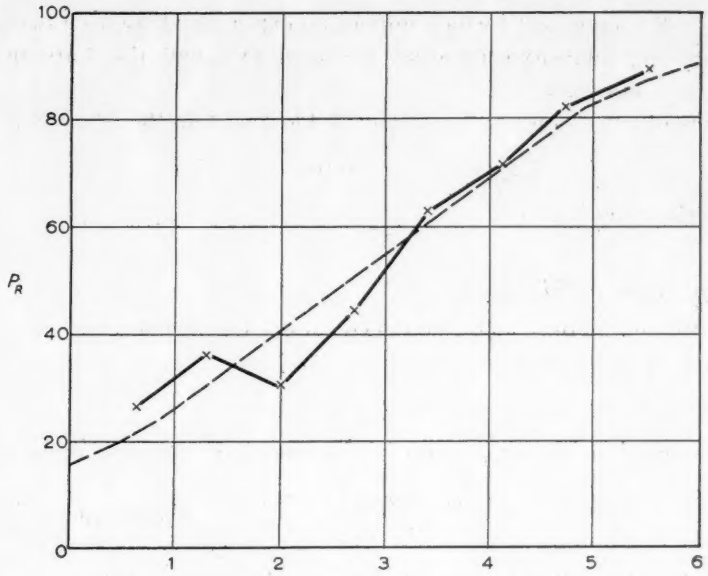

FIGURE 5. ——— *crude data.* — — *probit.*

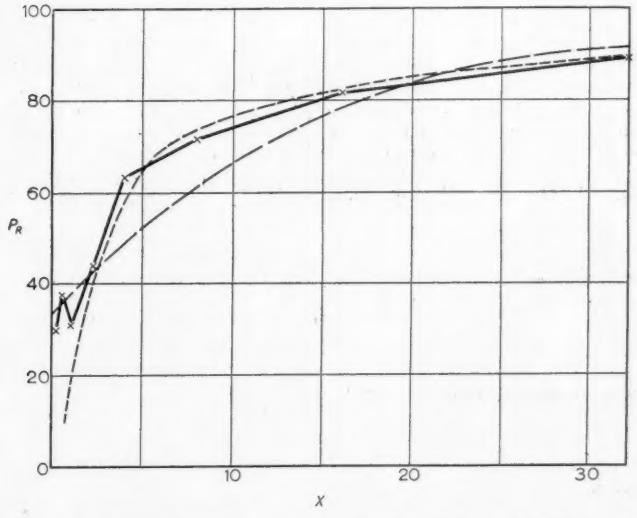

FIGURE 6. ——— *crude data.* — — $\log_e(1-P_R) = -0{\cdot}0688x - 0{\cdot}429.$

– – – $P_R = \dfrac{0{\cdot}8276x^2}{1+1{\cdot}788x+0{\cdot}8845x^2}$.

i.e. $R_M - R = \text{const.}\, e^{-kA}$ which implies an exponential rise to a maximum as advertising intensity is increased (the analogy is with the charging of an electrical condenser).

Referring once more to the data of Table 2 we find that the equation of fit is

$$\log_e(1 - P_R) = -0{\cdot}0688x - 0{\cdot}429.$$

This is shown in Figure 6.

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Comparison of Methods

Table 4 compares the results obtained from the data of Table 2 by the various methods discussed in this paper.

TABLE 4

COMPARISON OF PROPORTION RESPONDING ACCORDING TO DIFFERENT ASSUMPTIONS

Stimulus (x)	Actual Response P_R	Logarithmic Law		Probit Analysis		Exponential		Zentler and Ryde*	
			Deviation from Actual		Deviation from Actual		Deviation from Actual		Deviation from Actual
	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)
0·25	25·2	25·2	—	21·7	− 3·5	35·9	+10·7	3·3	−21·9
0·50	37·4	34·0	− 3·4	30·5	− 6·9	37·1	− 0·3	9·8	−27·6
1·00	30·1	42·9	+12·8	40·6	+10·5	39·2	+ 9·1	22·5	− 7·6
2·00	44·1	51·7	+ 7·6	51·2	+ 7·1	43·2	− 0·9	40·8	− 3·3
4·00	63·4	60·6	− 2·8	61·8	− 1·6	50·5	−12·9	59·4	− 4·0
8·00	71·8	69·4	− 2·4	71·9	+ 0·1	62·4	− 9·4	73·7	+ 1·9
16·00	82·4	78·3	− 4·1	80·2	− 2·2	78·3	− 4·1	82·7	+ 0·3
32·00	87·9	87·1	− 0·6	86·8	− 1·1	92·8	+ 4·9	87·9	—
Total deviations regardless of sign			33·7		33·0		52·3		66·6
x_7^2			57·09		59·41		105·08		

* Using a more general form $P_R = 100\left(\frac{ax^2}{1+bx+cx^2}\right)$.

Discussion

It can be seen clearly that there is little difference in the closeness of fit obtained by the application of the logarithmic law and by probit analysis. The advantage of the logarithmic law lies in the much smaller amount of

computational work involved. The large values of χ^2 (for these two curves) are mainly associated with the irregularity at $x = 1{\cdot}00$ which produces large deviations at this value and adjacent values of x, for the five remaining points the value of χ^2 is for the logarithmic law only 4·33, a reasonably close fit. The simplest form of Zentler and Ryde's curve is not satisfactory here; the more complex expression used is a bad fit at the bottom of the range but is excellent at the more important higher values of stimuli. On the scale of Figure 6 it does not look at all bad. Doubtless the addition of further constants would achieve a better all round fit but by the same token this would render the expression inconvenient compared with alternative expressions.

ACKNOWLEDGEMENTS

We wish to record our thanks for the interest and assistance shown to us during these investigations by: The Board of Admiralty, The Air Council, The Directors of Ferranti Ltd., Glaxo Laboratories Ltd., Parke, Davis & Co. Ltd., and E.M.I. Institutes Ltd.

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Rapid Statistical Techniques for Operational Research Workers

W. E. DUCKWORTH and J. K. WYATT
Glacier Metal Co. Ltd.

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THERE are occasions in the lives of operational research workers when rapid evaluation of the significance of a set of data is necessary or desirable. For example it may not be possible for data which are being discussed at a meeting to be taken away and analysed either because of the time taken or because of their confidential nature. It is also useful, before handing the results of an investigation to a computer, to be able to extract the meat of the information rapidly while all the circumstances are fresh in one's mind. A quick method of checking the correctness of the analysis when it is completed is also valuable.

This paper describes quick tests for the population mean, the comparison of two population means, and for the existence of trends, association and other non-randomness. A rapid method for analysis of variance is also described. Ways of predicting the number of trials required in certain circumstances are given. No tables are required except for the analysis of variance. Unless otherwise stated the 5 per cent significance level is being used, i.e. a real deviation from the hypothesis being tested is assumed to have occurred if a chance deviation of this size would happen on less than 5 per cent of occasions.

It must be emphasized that these tests give only approximate indications of probability levels and usually err on the conservative side. That is to say that where the data contain a significant effect these tests will be successful in detecting it on about 90 per cent of the occasions on which the appropriate classical test would. They may best be described as statistical first aid—to be used only when proper analytical facilities are not available and to be followed as soon as possible by correct analysis. As with medical first aid the results may be deadly if they are used by unskilled people.

The Sign Test

This has been known for a long time and no novelty is claimed. When it is quoted in the text-books, however, tables are usually referred to and they are not really necessary. This test can be used whenever the statistical problem can be resolved into one of comparing pluses and minuses.

Example 1. Could the following numbers have been drawn from a population with mean 10?

13, 12, 11, 9, 12, 8, 13, 12, 11, 11, 12, 10, 13, 11, 10, 14, 10, 10, 9, 11, 11.

We merely count the numbers which are greater than 10 (plus) and the numbers which are less than 10 (minus) and take the difference.

There are 14 numbers more than 10 and 3 less than 10, so the difference is 11. The 5 per cent probability level of this difference is $2\sqrt{N}$ where N is the total number different from 10.

In this case $N = 17$

$$2\sqrt{N} = 2\sqrt{17} = 8{\cdot}2\,;$$

11 is greater than 8·2, therefore it is very unlikely that the above sample could have been drawn from a population with mean 10.

Application of the *t*-test gives the result that the sample mean differs from 10 at the 1 per cent level of significance.

When two samples are compared, in order to test the difference between their means, it is not even necessary to compute the means. The samples are paired off in random order and the number of times one sample result is greater than the corresponding result in the other sample is compared with the number of times the reverse event occurs.

The following example relates to comb weights of two groups of chicks treated with different hormones and is taken from G. Snedecor, *Statistical Methods*, p. 75.

Example 2.

A	*B*	Sign of difference $A - B$
57	89	−
120	30	+
101	82	+
137	50	+
119	39	+
117	22	+
104	57	+
73	32	+
53	96	−
68	31	+
118	88	+

B is greater than *A* 9 times and less twice. The difference between pluses and minuses is therefore 7. $N = 11$, $2\sqrt{11} = 6{\cdot}6$ and so the difference between the groups is significant.

This result is confirmed by the *t*-test which indicates a difference significant at the 1 per cent level.

An unfortunate pairing of the readings can cause the sign test and the *t*-test to produce differing results, but occasional differences are to be expected as the former makes less use of the available data than does the latter. The example below, from a paper by Watkins[1], illustrates this point.

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Example 3.

A	*B*	*B–A*
9·5	9·4	–
9·1	9·6	+
9·0	9·2	+
9·5	8·0	–
12·2	8·9	–
10·0	8·5	–
10·7	9·7	–
10·4	11·1	+
11·5	9·4	–
11·2	11·4	+
11·6	10·9	–
10·2	11·3	+
11·8	9·8	–
12·0	9·3	–

There are 9 minuses and 5 pluses so the difference is only 4 and it should be $2\sqrt{14}$, i.e. 7 to be significant. The *t*-test does give a significant result however and the sign test has failed, owing to the unfortunate pairing of the numbers.

An unpublished test due to Tukey can be used here.* It will be noticed that the highest and lowest numbers of Group *A*, 12·2 and 9·0 respectively, are both greater than the highest and lowest numbers of Group *B*, to wit 11·4 and 8·0. We then count how many numbers in Group *A* are higher than 11·4 and how many numbers in Group *B* are lower than 9·0. The results are 5 and 3, the total being 8. Tukey has shown that if this total is more than 7 the difference between the groups is significant. The Tukey test is useful when the sign test is not directly applicable to the hypothesis being examined and is extremely quick when the data are arranged in order of magnitude. Its applicability is not restricted to the case of equal sample sizes, but for badly unbalanced samples the significance total should be taken as 8 or 9.

If the readings have been paired off in such a way that experimental conditions for different pairs are not the same, the sign test is appropriate.

* This test will be published in *Technometrics* No. 1 (1959).

It should be used to test whether the algebraic differences between paired observations come from a population whose mean differs significantly from zero. The Tukey test is then not applicable.

Trend

The sign test can be used to test for trend in a way suggested in a remark by Cox[2]. The figures are divided into three equal groups, the first group is then paired off with the third group and the usual comparison of pluses and minuses carried out.

Example 4. Does the following sequence exhibit a trend?

20, 21, 37, 25, 39, 46, 29, 10, 49, 45, 15, 7, 57, 44, 23, 28, 59, 50, 40, 31, 41, 24, 47, 51, 42, 47, 55, 47.

There are 28 numbers so the first nine are compared with the final nine as follows:

A	20, 21, 37, 25, 39, 46, 29, 10, 49
B	31, 41, 24, 47, 51, 42, 47, 55, 47
B–A	+ + − + + − + + −

There are 6 pluses and 3 minuses. The difference is 3, for significance it should be $2\sqrt{9} = 6$, so there is clearly no adequate evidence of a trend.

The above test is designed to detect what is usually meant by trend, viz. that if successive readings are assumed to come from a population whose parameters are changing, then the population mean is either increasing or decreasing monotonically.

A modification of the sign test has been suggested[3] to give a test which is not restricted in this way, but which will detect more general systematic variations of the population mean.

In a sample of size N, there are $N-1$ occasions on which consecutive readings will either both be on the same side of the sampled median or will be on opposite sides. Let F and G be the respective frequencies of occurrence of these events. Then the null hypothesis of random sampling from the same population will be rejected if

$$|F-G| \geqslant 2\sqrt{(N-1)}$$

i.e. if

$$|N-2G-1| \geqslant 2\sqrt{(N-1)}.$$

Example 5. Are the following numbers drawn randomly from the same population?

24, 27, 26, 28, 31, 35, 33, 37, 36, 37, 34, 32, 32, 29, 28, 28, 31, 28, 26, 25.

Here the sample median is $29\frac{1}{2}$, $N = 20$ and $G = 4$.

Thus $|N-2G-1| = 11$ and $2\sqrt{(N-1)} = 8{\cdot}7$, showing that the numbers are not randomly drawn from the same population.

Association

The test for association is as follows.

The number of pairs in which *both* observations are higher than the respective group medians is added to the number of pairs in which *both* observations are lower than the respective group medians. This total is then compared with the number of pairs in which one observation is below its median and the other above. The difference between the two totals is then compared with $2\sqrt{N}$ where N is the total number of pairs considered.

The following example is taken from Snedecor, p. 142.

Example 6. Is there an association between the following groups of numbers?

I 73, 41, 83, 71, 39, 60, 51, 41, 85, 88, 44, 71, 52, 74, 50, 42, 85, 53, 85

II 29, 24, 34, 27, 24, 26, 35, 18, 33, 39, 27, 35, 25, 29, 13, 13, 40, 23, 40
44, 66, 60, 33, 43, 76, 51, 57, 35, 40, 76.
22, 25, 21, 26, 19, 29, 25, 19, 17, 17, 35.

The two sample medians may be taken as 55 and $25\frac{1}{2}$. There are 22 pairs in which both numbers are higher or both lower than the respective median and 8 in which this is not the case. The difference is 14 which exceeds $2\sqrt{30}$, thus showing that an association exists. In fact the correlation coefficient is 0·774 which for this sample size is highly significant.

This association test may be adapted for use as a test for trend. A trend is diagnosed if an association is found to exist between the N readings given and the numbers $1, 2, 3, \ldots, N$. This test is not as powerful, however, as the one suggested by Cox[2].

Predicting the Number of Tests required

If the sign test fails to show significance in a given set of data and more information is readily available then the amount of extra information necessary to establish significance, if in fact the data do warrant further examination, can be shown to be

$$\left(\frac{2N}{D}\right)^2 - N$$

where N is the size of the initial sample and D is the difference between the pluses and minuses observed in the initial sample. It is clear that when $D = 2\sqrt{N}$ no extra information is necessary.

When extra information is not readily available and the sign test has been supplemented by the more accurate *t*-test, but still without establishing a significant result, then a more accurate evaluation of the amount of extra information required is necessary. This can be done by the usual procedure of stating the difference 'δ' which it is required to detect and calculating the standard deviation of the existing samples. The total information then needed to establish δ with a known value of σ is then given by

$$R = 11\left(\frac{\sigma}{\delta}\right)^2 + 3$$

when a single group of observations is being considered, and

$$R = 21{\cdot}5\left(\frac{\sigma}{\delta}\right)^2 + 3$$

when two samples are being compared. R is the number of observations required in each sample. The last formula can be expanded into the general case for the number of replications required in a 2^n factorial experiment as follows

$$R = 43\,\frac{\left(\frac{\sigma}{\delta}\right)^2 + 6}{2^n}.$$

The Analysis of Variance

A great reduction in the computational labour in the analysis of variance can be achieved by the methods to be developed, but it has not been found practicable to dispense with tables in this case. The method of approach is that used by Patnaik[4], Hartley[5] and David[6].

TABLE 1

Replication No.	Treatment No.			
	1	2		K
1	X_{11}	X_{21}	... ...	X_{k1}
2	X_{12}	X_{22}	... ...	X_{k2}
⋮	⋮	⋮		⋮
72	X_{1n}	X_{2n}	... ...	X_{kn}
Total	$\Sigma_i x_{1i}$	$\Sigma_i x_{2i}$	... ...	$\Sigma_i x_{ki}$
Range	${}_1Wn$	${}_2Wn$	... ...	${}_kWn$

The table represents the case of a simple classification in which k treatments each have an equal cell frequency of n. Let ${}_1Wn, {}_2Wn, \ldots, {}_kWn$ denote

the ranges of the sets of n values associated with the k treatments. Let the average of these k ranges be denoted by $\bar{W}_n$. Then an unbiased estimate of the within-treatment standard deviation is

$$\frac{\bar{W}_n}{d_n} \tag{1}$$

using the usual notation.

Let the largest and smallest values of the treatment totals be denoted by (Σx) max and (Σx) min respectively. Then

$$\frac{(\Sigma x)\,\max - (\Sigma x)\,\min}{d_k}$$

can be used to estimate the standard deviation of the treatment totals and hence

$$\frac{(\Sigma x)\,\max - (\Sigma x)\,\min}{\sqrt{(n)}.d_k} \tag{2}$$

gives the between-treatments standard deviation. To test the null hypothesis that the treatments do not have any effect, the variance ratio $(2)^2/(1)^2$ is calculated and referred to the F-table as if it had been obtained in the orthodox manner. Of course, the distribution of $(2)^2/(1)^2$ corresponds only approximately to the distribution of F. Thus the expression for the variance ratio is

$$\frac{1}{n}\left\{\frac{(\Sigma x)\,\max - (\Sigma x)\,\min}{\bar{W}_n}.\frac{d_n}{d_k}\right\}^2$$

But, for $x \leqslant 10$, $d_x \doteq \sqrt{x}$ and the criterion becomes

$$F \equiv \frac{1}{k}\left\{\frac{(\Sigma x)\,\max - (\Sigma x)\,\min}{\bar{W}_n}\right\}^2$$

which should be referred to the distribution of $F_{k-1}: k(n-1)$.

Example 7. Table 2, which shows measurements of tensile strength made on six specimens of rubber randomly selected from each of five different batches, is taken from the Biometrika Tables, Volume I, 1954, p. 38.
Here $k = 5$, $n = 6$, $(\Sigma x)\,\max = 1166$, $(\Sigma x)\,\min = 950$, $\bar{W}_6 = 49{\cdot}2$.
Applying the formula, $F = 3{\cdot}86$, giving a significance level between 2·5 per cent and 1 per cent. The actual F value is 3·47.

When dealing with the case of a double classification without replication, a modified method is necessary to obtain the error variance. If there are k treatments and n blocks, the treatment mean will be subtracted from each of the individual readings under each treatment, leaving the "treatment residuals". The ranges of these residuals taken over the blocks will be

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averaged to give $\bar{W}_k$. $\bar{W}_k/d_k$ is used to estimate the error standard deviation. The between-treatments standard deviation, as before, is

$$\frac{(\Sigma x)\max - (\Sigma x)\min}{\sqrt{n}.d_k}$$

and the variance ratio becomes

$$\frac{1}{n}\left(\frac{(\Sigma x)\max - (\Sigma x)\min}{\bar{W}_k}\right)^2$$

which should again be referred to the distribution of $F_{k-1;\,k(n-1)}$.

TABLE 2

Specimen No.	Batch No.				
	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$n = 1$	177	116	170	181	177
$n = 2$	172	179	156	190	186
$n = 3$	137	182	188	210	199
$n = 4$	196	143	212	173	202
$n = 5$	145	156	164	172	204
$n = 6$	168	174	184	187	198
Total	995	950	1,074	1,113	1,166
Range	59	66	56	38	27

Treatment residuals have been used in finding the error standard deviation because the treatment means will normally have been calculated, but block residuals could have been used. If n exceeds k, the latter procedure leads to a more precise estimate of the error standard deviation

viz. $$\frac{\bar{W}_n}{d_n} \qquad (3)$$

The expression for variance ratio is now $(2)^2/(3)^2$

viz. $$F \equiv \frac{1}{k}\left(\frac{(\Sigma x)\max - (\Sigma x)\min}{\bar{W}_n}\right)^2$$

This is the expression previously found to apply to a simple classification, and it should again be referred to the distribution of $F_{k-1;\,k(n-1)}$.

If it is desired to test for the significance of the between-blocks effect, the expressions for F are identical to those derived for the between-treatments effect for error standard deviation based on treatment residuals and block residuals respectively. This appearance of symmetry is a consequence of the assumption $d_x \doteq \sqrt{x}$.

Example 8. Table 3 gives the yields of four strains of wheat planted in five randomized blocks. It is taken from Snedecor, p. 266.

TABLE 3

Block	Strain			
	A	*B*	*C*	*D*
1	32·3	33·3	30·8	29·3
2	34·0	33·0	34·3	26·0
3	34·3	36·3	35·3	29·8
4	35·0	36·8	32·3	28·0
5	36·5	34·5	35·8	28·8
Total	172·1	173·9	168·5	141·9

The strain residuals needed to estimate the error standard deviation are shown in Table 4.

TABLE 4

Block	Strain				Range
	A	*B*	*C*	*D*	
1	−2·1	−1·5	−2·9	0·9	3·8
2	−0·4	−1·8	0·6	−2·4	3·0
3	−0·1	1·5	1·6	1·4	1·7
4	0·6	2·0	−1·4	−0·4	3·4
5	2·1	−0·3	2·1	0·4	2·4
			Mean range		2·86

The appropriate F to use in the case of the between-strains effect is calculated by the use of

$$n = 5,\ k = 4,\ \bar{W}_4 = 2{\cdot}86,\ (\Sigma_x)\max = 173{\cdot}9,\ (\Sigma_x)\min = 141{\cdot}9,$$

$$F = \frac{1}{5}\left(\frac{173{\cdot}9 - 141{\cdot}9}{2{\cdot}86}\right)^2 \doteq 25,$$

a highly significant value. The actual F value is 20·5.

Consider now the case of a double classification in which factor A has k levels and factor B has n levels and in which the number of replications is r. This case differs from the above in two ways. Firstly, the error standard deviation can now be found from the ranges of replications in the same cell and, secondly, the interaction between the factors can now be tested. If the

average range of the replicates in a cell is $\bar{W}_r$, the error standard deviation can be taken as

$$\frac{\bar{W}_r}{d_r} \tag{4}$$

The interaction standard deviation may be found by considering residuals to either the treatment A means or the treatment B means. Suppose that the former course is being followed and that the mean range $\bar{W}_k$ has been calculated which will relate to k values each of which is the sum of r observations. The estimate of the interaction standard deviation is

$$\frac{\bar{W}_k}{d_k\sqrt{r}} \tag{5}$$

The F to be used in testing the interaction is thus $(5)^2/(4)^2$ or

$$\frac{1}{r}\left(\frac{\bar{W}_k}{\bar{W}_r}\cdot\frac{d_r}{d_k}\right)^2$$

or approx.

$$\frac{1}{k}\left(\frac{\bar{W}_k}{\bar{W}_r}\right)^2$$

which should be referred to the distribution of $F_{(k-1)\ (n-1):\,kn(r-1)}$.

The normal estimator for between-treatments standard deviation is modified slightly to allow for the effect of the replications. The standard deviation associated with factor A is

$$\frac{(\Sigma x)\max - (\Sigma x)\min}{\sqrt{(nr)}\,.\,d_k} \tag{6}$$

(the Σ's being summed over each level of A)
giving a variance ratio of $(6)^2/(4)^2$ or

$$F \equiv \frac{1}{nk}\left(\frac{(\Sigma x)\max - (\Sigma x)\min}{\bar{W}_r}\right)^2$$

after simplification.

The corresponding expression relating to factor B is identical with this but the summations are now over each level of B. The variance ratios for A and B should be referred to the distributions of $F_{k-1;\,kn(r-1)}$ and $F_{n-1;\,kn(r-1)}$ respectively.

Example 9. The data for this example come from O. L. Davies, *Statistical Methods in Research and Production*, p. 91. The analysis is somewhat similar to that used by David[6] on the same example.

Three "gaugers" prepared twelve samples of cement each, four of which were tested for compressive strength by each of three "breakers".

TABLE 5

TOTALS OF FOUR REPLICATES

Gauger \ Breaker	1	2	3	Total
1	136	−4	−74	58
2	18	138	−194	−38
3	270	166	−33	403
Total	424	300	−301	423

TABLE 6

RANGES OF FOUR REPLICATES

Gauger \ Breaker	1	2	3
1	104	186	116
2	116	132	62
3	80	96	114

TABLE 7

BREAKER RESIDUALS (TOTALS)

Gauger \ Breaker	1	2	3	Range
1	−5	−104	26	130
2	−123	38	−94	161
3	129	66	67	63
Total				354

In this example $k = 3$, $n = 3$, and $r = 4$.
Table 6 gives a mean range value of 111·8.
Table 7 gives a mean range value of 118·0.
For testing the interaction,

$$F \equiv \frac{1}{3}\left(\frac{118}{111{\cdot}8}\right)^2 \doteqdot 0{\cdot}45 \quad \text{—not significant.}$$

For testing the "breaker" effect,

$$F = \frac{1}{9}\left(\frac{725}{111{\cdot}8}\right)^2 \doteqdot 4{\cdot}7 \quad \text{—significant at 5 per cent level.}$$

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For testing the "gauger" effect,

$$F=\frac{1}{9}\left(\frac{441}{111{\cdot}8}\right)^2 \doteqdot 1{\cdot}7 \qquad \text{—not significant.}$$

These results agree with those of Davies and David.

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Conclusion

This paper has described only methods which have been found useful by the authors and does not claim to be exhaustive. Ranking methods, for example, have not been treated. It will be found, however, that there are relatively few tests in existence which meet the requirements of not depending on the use of tables and of keeping computing and graphical work to an absolute minimum. Interest in the field is growing, however, and this need will no doubt soon be satisfied.

Appendix I

Derivation of the $2\sqrt{N}$ *criterion*

Suppose that n trials take place, at each of which success and failure are equally likely.

Then, if u is defined as the number of successes occurring,

$$\Pr\,(u = r) = \binom{n}{r}(\tfrac{1}{2})^n \qquad (r = 0, 1, 2, \ldots, n).$$

These probabilities are, of course, the successive terms of the expansion of

$$(\tfrac{1}{2}+\tfrac{1}{2})^n.$$

An approximation to the distribution of u is provided by the Normal distribution

$$N(\tfrac{1}{2}n, \tfrac{1}{2}\sqrt{n}).$$

The closeness of the approximation depends on the magnitude of n.

The null hypothesis to be tested is that the probabilities of success and failure are, in fact, equal. Using the Normal approximation, the 5 per cent significance limits may be taken as

$$u = \tfrac{1}{2}n \pm \frac{1{\cdot}96}{2}\sqrt{n},$$

or

$$u = \tfrac{1}{2}n \pm \sqrt{n}$$

for practical purposes.

If v is now defined as the number of failures occurring, the 5 per cent significance limits may be stated in the alternative form

$$v = \tfrac{1}{2}n \pm \sqrt{n}.$$

At both 5 per cent significance limits

$$|u-v| = 2\sqrt{n}.$$

Thus the null hypothesis that the probabilities of success and failure are equal is rejected if

$$|u-v| \geqslant 2\sqrt{n}.$$

This model does not allow for the possibility that neither success nor failure will occur. Observations falling into this category should be excluded from consideration and the value of n should be correspondingly reduced.

The applications of this criterion which have been given have as a common feature the need to test whether the observed difference between the number of "successes" and the number of "failures" is sufficient to discredit the null hypothesis of their occurring with equal probabilities.

Appendix II

Derivation of Formulae for Number of Observations required

1. *For the sign test.*

After N observations in a sign test suppose

u = number of successes

v = number of failures

where $|u-v| = D < 2\sqrt{N}$.

Suppose the ratio of successes to failures in the population is equal to the ratio found in the sample, and that this ratio will continue to be found in further sampling.

Suppose that the sample size is now increased to N'. The difference between the number of successes and the number of failures, since it is proportional to sample size on the assumptions made, becomes

$$\frac{N'}{N}|u-v| \quad \text{or} \quad \frac{N'D}{N}$$

For significance we require

$$\frac{N'D}{N} \geqslant 2\sqrt{N'}$$

i.e.

$$N' \geqslant \left(\frac{2N}{D}\right)^2$$

Thus the number of further observations required is

$$\left(\frac{2N}{D}\right)^2 - N.$$

2. *For a* 2^n *factorial experiment.*

Let n = number of factors

R = number of replications

δ = difference to be detected

σ = error standard deviation.

Testing at the 5 per cent significance level is being considered and a power of 95 per cent is required.

For a particular factor, the standard deviation of the mean of the observations at either of its levels is

$$\frac{\sigma}{\sqrt{(2^{n-1}R)}}.$$

For simplicity we wish to use the unit Normal distribution instead of the t-distribution. By a useful empirical rule, this can be done if the following adjusted standard deviation is substituted for the one just obtained:

$$\frac{\sigma}{\sqrt{(2^{n-1}R-3)}}.$$

The difference between the means at the two levels will have the standard deviation

$$\sigma' = \frac{\sigma\sqrt{2}}{\sqrt{(2^{n-1}R-3)}}.$$

The distributions of this difference, (a) under the null hypothesis in which the difference is zero and (b) when the difference is δ, are shown in Figure 1. The condition under which a difference of δ would be detected is

$$\frac{\delta}{\sigma'} = 3{\cdot}29$$

i.e.
$$\frac{\delta\sqrt{(2^{n-1}R-3)}}{\sigma\sqrt{2}} = 3{\cdot}29$$

i.e.
$$R = \frac{21{\cdot}5(\sigma/\delta)^2+3}{2^{n-1}}.$$

The expression for R when two means are being compared is obtained by taking $n = 1$ in the above. The formula $R = 11(\sigma/\delta)^2+3$ which applies to the test of a population mean can be obtained by very similar reasoning to the above.

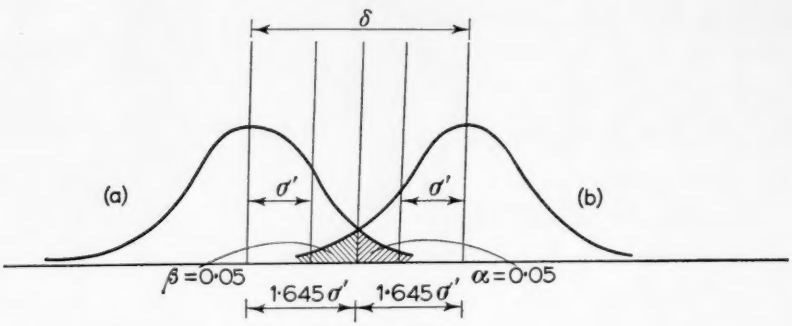

FIGURE 1.

Appendix III

Significance Levels

The formulae given in the paper are all for the 5 per cent significance level. For those who prefer the 1 per cent or 0·1 per cent or even the 10 per cent significance levels the appropriate formulae are given in Table 8.

TABLE 8

Significance Level	10 per cent	5 per cent	1 per cent	0·1 per cent
Sign Test	$1{\cdot}6\sqrt{N}$	$2\sqrt{N}$	$2{\cdot}6\sqrt{N}$	$3{\cdot}3\sqrt{N}$
Tukey Test	4	7	10	14
No. of Replications (single sample)	$R = 9(\sigma/\delta)^2+3$	$R = 11(\sigma/\delta)^2+3$	$R = 16(\sigma/\delta)^2+3$	$R = 22(\sigma/\delta)^2+3$
No. of Replications (2^n factorial)	$R = \frac{34(\sigma/\delta)^2+6}{2^n}$	$R = \frac{43(\sigma/\delta)^2+6}{2^n}$	$R = \frac{63(\sigma/\delta)^2+6}{2^n}$	$R = \frac{90(\sigma/\delta)^2+6}{2^n}$

ACKNOWLEDGEMENT

The authors wish to thank the Glacier Metal Company Limited for permission to publish this paper.

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Two Studies in Airport Congestion*

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Introduction

THE Operational Research Branch of B.O.A.C. has recently carried out two distinct investigations both of which concern the congestion of ground services at airports. They illustrate the way in which apparently similar problems may sometimes lead to the use of very different techniques if a practical solution is to be achieved.

The first problem concerns the provision of coach transport to carry passengers across the tarmac at London Airport; the other concerns the performance of an integrated programme of servicing for aircraft in transit through Karachi. Both problems consisted of the choice of an adequate level of serving capacity in queueing situations characterized by constant service time and a fluctuating input of aircraft movements, but whereas the first problem led to the application of some almost (but not quite) classical queue theory, the second had to be solved by rather cruder techniques.

After describing the two studies in turn, this paper concludes with a critical comparison of them, in an attempt to justify the divergence in method.

The London Coach Problem

Whenever an aircraft of B.O.A.C., or of certain other long-distance airlines, arrives or departs at London Airport (North), B.O.A.C. is responsible for transporting the passengers across the tarmac between the aircraft and the airport passenger buildings. If the aircraft happens to be drawn up sufficiently close to the building and the weather is fine, it may be convenient for the passengers to walk, but otherwise a B.O.A.C. motor coach, or coaches, are provided. The capacity of a coach is 35, so two coaches are required whenever an aircraft load exceeds 35 passengers.

Under the current system, the direction of coach operations on the tarmac is integrated with that of the coach service that B.O.A.C. provides between the Airport and Central London, the same coach being used to

* Paper given to the Operational Research Society, London, 10 December, 1957.

transport passengers on both stages of the journey wherever possible. As this system suffers from a number of disadvantages, it was recently decided that a new system should be introduced whereby the tarmac operation is controlled separately, minimizing the amount of supervision required by maintaining a "pool" of coaches standing by on the tarmac ready to transport passengers to or from their aircraft at a moment's notice.

Because of the element of uncertainty in aircraft arrival and departure times, it was realized that tarmac operation under the new system would inevitably involve some risk of demand for coaches occasionally exceeding the supply in the pool, and the problem which the Operational Research Branch was asked to investigate was:

> "What is the minimum number of coaches that would require to be allocated to the pool at different times of day to keep the risk of demand exceeding supply, with consequent delay to passengers and possibly to aircraft, at an acceptable level?"

It was agreed that the possibility of dispensing with coach service in dry weather would not be considered, this being a matter for current operation rather than advance planning in the unpredictable British climate.

The first aspect of the situation to be investigated was the timing of the coach operation. Whether an aircraft arrival or a departure is being served, the basic pattern of operation is always the same; passengers board at *A*, the coach drives off to *B*, the passengers disembark and then the coach repositions itself to *C*. A sample of 76 such operations was timed, giving a mean of 7·5 minutes and a standard deviation of 1·7 minutes. In the light of the relatively small ratio of standard deviation to mean and of the invariable basic pattern of the operation, it was decided that an assumption of a constant service time of $7\frac{1}{2}$ minutes would be realistic.

The situation appeared to have the characteristics of a queueing problem but, as is frequently the case in the air-transport field, a fluctuating rate of input into the queue presented an obstacle to the application of classical queueing theory. The rate of aircraft movements—arrivals and departures combined—was found to vary from nil to about ten per hour according to time of day. All movements were governed to some extent by a schedule, but irregularity of movements (particularly of arrivals) often made unreliable the estimation of the movement rate, during any given hour, from the scheduled number of movements within that hour. However, a multiple regression analysis showed that prediction could be much improved by computing a weighted average of the number scheduled within the hour with the number scheduled in the previous hour and the subsequent hour.

Simplification of the regression equation gave the following rough formula for estimating the actual movement rate m_r during the rth hour from the scheduled rate s_r:

$$\bar{m}_r = \tfrac{1}{4}(s_{r-1} + 2s_r + s_{r+1}).$$

A sample of a week's operations showed the error term $(m_r - \bar{m}_r)^2/\bar{m}_r$ to be distributed as χ^2 with 1 degree of freedom, suggesting that the distribution of m_r about the variable mean $\bar{m}_r$ was Poissonian, so that the input, within any particular hour, was apparently similar to that given by a random flow with mean $\bar{m}_r$. This hypothesis was borne out by the fact that, during periods of reasonably constant activity, the intervals between successive movements gave an encouragingly good fit to the exponential distribution. It was therefore decided to assume that, within any particular hour, a steady-state queueing situation existed with random input of mean rate $\bar{m}_r$. Relative to the $7\frac{1}{2}$-minute service time, an hour was considered a long enough period for the "steady state" assumption to have reasonable validity.

If, during a given hour, the estimated movement rate per unit service time (i.e. per $7\frac{1}{2}$ minutes) is m, then the probability of i movements in a unit $7\frac{1}{2}$-minute interval can be represented by the Poisson formula:

$$q_i = \frac{m^i}{i!} e^{-m}.$$

The actual input of customers into the queue is, however, rather more complicated than this. A proportion of aircraft movements, it may be remembered, require two coaches and so, defining a "customer" as a batch of passengers capable of being accommodated in one coach, aircraft movements generate customers either singly or in pairs. If the frequency ratio of these two types of movements is c': c'' $(c' + c'' = 1)$, then the input can be considered as two separate Poisson inputs; one of single customers with mean rate $c' \cdot m$ and one of pairs of customers with rate $c'' \cdot m$. The probabilities of i customers arriving in unit time from the respective inputs are:

$$p'_i = \frac{(c'm)^i}{i!} e^{-c' \cdot m}$$

$$p''_i = \frac{(c''m)^{\frac{1}{2}i}}{(\frac{1}{2}i)!} e^{-c'' \cdot m} \quad (i \text{ even}),\ p''_i = 0 \ (i \text{ odd})$$

and the joint probability of n customers from either input is:

$$p_n = p'_n \cdot p''_o + p_{n-2} \cdot p''_2 + p_{n-4} \cdot p''_4 + \ldots.$$

In practice, a steady proportion of about 80 per cent of aircraft movements require two coaches, so that $c' = 0{\cdot}2$ and $c'' = 0{\cdot}8$. Table I, giving p'_n,

p''_n and p_n for $m = 1$ aircraft movement per unit time, shows the type of distribution that arises in these circumstances.

TABLE I

PROBABILITIES OF n CUSTOMERS ARRIVING PER UNIT TIME

n	p'_n	p''_n	p_n
0	·8187	·4493	·3679
1	·1637	—	·0736
2	·0164	·3595	·3016
3	·0011	—	·0594
4	·0001	·1438	·1236
5	—	—	·0239
6	—	·0383	·0338
7	—	—	·0064
8	—	·0077	·0070
9	—	—	·0013
10	—	·0012	·0011
11	—	—	·0002
12	—	·0002	·0002

Table 1 makes clear the bias towards even numbers of customer-arrivals per unit time, which is a natural consequence of the requirement for two coaches to serve the majority of aircraft movements.

The basic queueing equations describing the system can now be written down, expressing the probabilities $P_i(T+1)$ of different numbers of customers being served or waiting at time $T+1$ in terms of the corresponding probabilities $P_i(T)$ unit time ($7\frac{1}{2}$ minutes) earlier, and of the probabilities p_i of different numbers of arrivals in the intervening unit interval. It was found convenient to express the equations in a modified form involving cumulative probabilities $S_i(T)$, where $S_i(T) = P_0(T)+P_1(T)+\ldots+P_i(T)$ (the chance of i or less customers being served or waiting), instead of individual probabilities $P_i(T)$. The equations are then:

$$S_0(T+1) = p_0 S_N(T)$$

$$S_1(T+1) = p_0 S_{N+1}(T)+p_1 S_N(T)$$

$$S_2(T+1) = p_0 S_{N+2}(T)+P_1 S_{N+1}(T)+p_0 S_N(T)$$

. . . . . .

Solution of these equations in their steady state form, where

$$S_i(T+1) = S_i(T),$$

is a formidable task even in the simpler case where the p_i are Poisson-distributed. The alternative equations based on an exponential instead of a

TABLE 2(a)

FIRST TWO APPROXIMATIONS TO SOLUTION FOR $N=4$, $m=1$, $c':c''=\cdot20:\cdot80$

First Approximation	·3679 p_0	·0736 p_1	·3016 p_2	·0594 p_3	·1236 p_4	·0239 p_5	·0338 p_6	·0064 p_7	·0070 p_8	·0013 p_9	·0011 p_{10}	·0002 p_{11}	·0002 p_{12}
·3679 S_0													
·4415 S_1													
·7431 S_2													
·8025 S_3													
·9261 S_4	·3407	·0682	·2793	·0550	·1145	·0221	·0313	·0059	·0065	·0012	·0010	·0002	·0002
·9500 S_5	·3495	·0699	·2865	·0564	·1174	·0227	·0321	·0061	·0067	·0012	·0010	…	…
·9838 S_6	·3619	·0724	·2967	·0584	·1216	·0235	·0333	·0063	·0069	·0013	·0011	…	…
·9902 S_7	·3643	·0729	·2986	·0588	·1224	·0237	·0335	·0063	·0069	…	…	…	…
·9972 S_8	·3669	·0734	·3008	·0592	·1233	·0238	·0337	·0064	·0070	…	…	…	…
·9985 S_9	·3673	·0735	·3011	·0593	·1234	·0239	·0337	…	…	…	…	…	…
·9996 S_{10}	·3678	·0736	·3015	·0594	·1236	…	·0338	…	…	…	…	…	…
·9998 S_{11}	·3678	…	·3015	…	…	…	…	…	…	…	…	…	…
1·0000 S_{12}	·3679	…	·3016	…	…	…	…	…	…	…	…	…	…
Second Approximation													
·3407 S_0													
·4177 S_1													
·7111 S_2													
·7782 S_3													
·9074 S_4	·3338	·0668	·2737	·0539	·1122	·0217	·0307	·0058	·0064	·0012	·0010	·0002	·0002
·9372 S_5	·3448	·0690	·2827	·0557	·1158	·0224	·0317	·0060	·0066	·0012	·0010	…	…
·9765 S_6	·3593	·0719	·2945	·0580	·1207	·0233	·0330	·0062	·0068	·0013	·0011	…	…
·9856 S_7	·3626	·0725	·2973	·0585	·1218	·0236	·0333	·0063	·0069	…	…	…	…
·9952 S_8	·3661	·0732	·3002	·0591	·1230	·0238	·0336	·0064	·0070	…	…	…	…
·9973 S_9	·3669	·0734	·3008	·0592	·1233	·0238	·0337	…	…	…	…	…	…
·9991 S_{10}	·3676	·0735	·3013	·0593	·1235	·0239	·0338	…	…	…	…	…	…
·9995 S_{11}	·3677	·0736	·3014	·0594	·1235	…	…	…	…	…	…	…	…
1·0000 S_{12}	·3679	…	·3016	…	·1236	…	…	…	…	…	…	…	…

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constant service time, which yield a simple solution in the Poisson input case, promised no easy escape in this instance of a compound Poisson input. It was therefore decided, while investigating the above equations analytically, to embark simultaneously on a Monte Carlo approach as an insurance against the failure of analytical methods. This subsequently proved unnecessary since it was found that, for any particular set of p_i, the steady state solution of the equations could be obtained—in most instances quickly—by successive approximations from an initial assumption that the flow of arrivals began instantaneously at time $t = 0$. Then, at time $t = 1$, the number of customers in the system would be equivalent to the number of arrivals in unit time, so that $P_i(1) = p_i$, and

$$S_i(1) = p_0 + p_1 + \ldots + p_i.$$

A simple computing routine based on the set of equations above then gives the $S_i(2)$ in terms of the $S_i(1)$, then the $S_i(3)$, $S_i(4)$, ... until a stable solution is obtained.

The computing procedure is illustrated in Table 2(a) for the case $m = 1, N = 4$ and $c' : c'' = 0{\cdot}20 : 0{\cdot}80$. The p_i are written horizontally along the top of the page and then cumulatively down the left-hand side to represent the initial set of S_i. A multiplication table is then constructed, multiplying each p_i by $S_4, S_5, S_6, \ldots$ starting with S_4 because $N = 4$ in this example. To save indefinite repetition, each column is terminated as soon as the full value of p_i is reached, correct to the number of decimal places in use. To give the next set of S_i, the products are then summed along the diagonals, starting in the top left-hand corner of the Table. This gives, in this example,

$$S_0 = 0{\cdot}3407, \quad S_1 = 0{\cdot}0682 + 0{\cdot}3495, \quad S_2 = 0{\cdot}2793 + 0{\cdot}0699 + 0{\cdot}3619, \ldots$$

up to $S_{12} = 1{\cdot}0000$. A new multiplication table is then constructed with the new S_i and the original p_i, and the process is repeated. Successive sets of S_i given by repetition of this routine are monotonically decreasing (herein lies the advantage of dealing with the S_i rather than the P_i) and, provided the system is not operating above saturation level, convergent. Thus, a stage in the calculations will occur when, to the number of decimal places in use, the same set of S_i recurs. In this example, 8 repetitions were required. The final set of S_i, representing the exact steady state solution of the equations correct to four decimal places, is given in Table 2(b).

The solutions obtained in this way for $N = 3$ upwards are compared in Figure 1. It will be noticed that, for $N = 6$ and over, they are almost identical; there is scarcely any queueing and the P_i differ little from the p_i.

For lower N, the increasing importance of the queueing element, reflected by the increased chances of 4, 5, ... customers being in the system, is evident. Against each N, the number of approximations that were necessary is quoted. This is to some extent a function of the degree of accuracy selected but nevertheless gives an interesting indication of the time required for the system to settle down from an instantaneous start, each approximation representing a time lapse of $7\frac{1}{2}$ minutes. The case $N = 2$ is not shown in Figure 1; customers would then be arriving at 90 per cent of the saturation rate and the number of approximations to reach a steady state would be prohibitive. The method appears to be practicable, however, so long as saturation is of the order of 50 per cent or less, and it is not visualized that coaches will ever be required to operate on the tarmac at saturation (i.e. utilization) levels in excess of this.

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TABLE 2(b)

FINAL SOLUTION AFTER EIGHT APPROXIMATIONS

S_0 ·3299	S_1 ·4081	S_2 ·6961	S_3 ·7667	S_4 ·8966	S_5 ·9299	S_6 ·9712
S_7 ·9823	S_8 ·9933	S_9 ·9961	S_{10} ·9986	S_{11} ·9994	S_{12} 1·0000	

Having obtained solutions of this type for a range of m and N, their interpretation was necessary. A means had to be devised for management to select the number of coaches to be allotted to the pool at different times of day to keep the risk of congestion at an acceptable level. Among the possible ways of defining this "acceptable level" were:

(i) the proportion of coachloads delayed shall not average more than x per cent;

(ii) the proportion of aircraft inadequately served, whether by lack of one coach or two, shall not average more than y per cent;

(iii) the average delay per coach operation shall not exceed z minutes.

After consultation with management, (ii) was selected as the best definition; if an aircraft is not served promptly, it matters relatively little whether it is for lack of one coach or two, or how long the delay may be (in practice, rarely more than five minutes).

For a randomly-chosen aircraft arriving or waiting to depart, the chance $Q(N)$ that it will be inadequately served, with a total of N coaches in circulation, is the chance that more than $N-1$ aircraft will be already being

served or waiting if it requires one coach, or more than $N-2$ if it requires two. This can be expressed as:

$$Q(N) = 1 - c' S_{N-1} - c'' S_{N-2}.$$

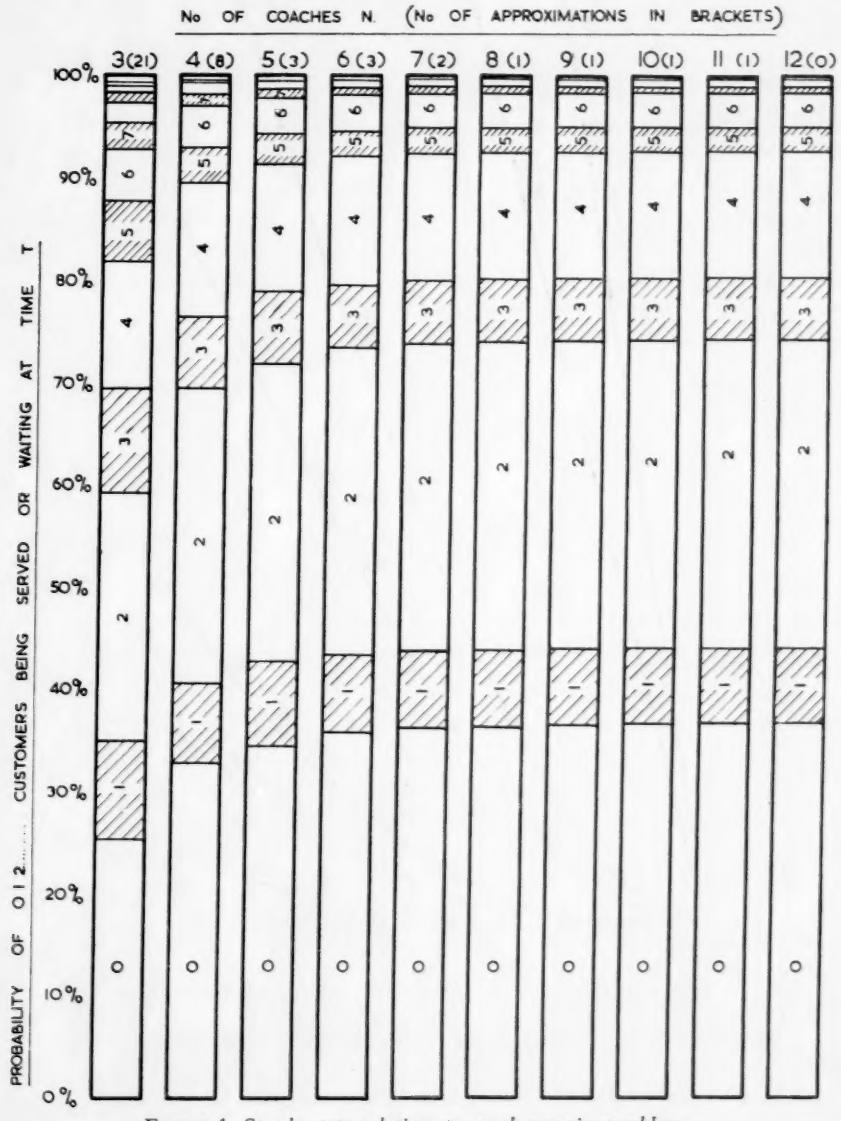

FIGURE 1. *Steady-state solutions to coach queueing problem. For* $m = 1$, $c' : c'' = 0{\cdot}20 : 0{\cdot}80$ *and* $N = 3$ *to* 12.

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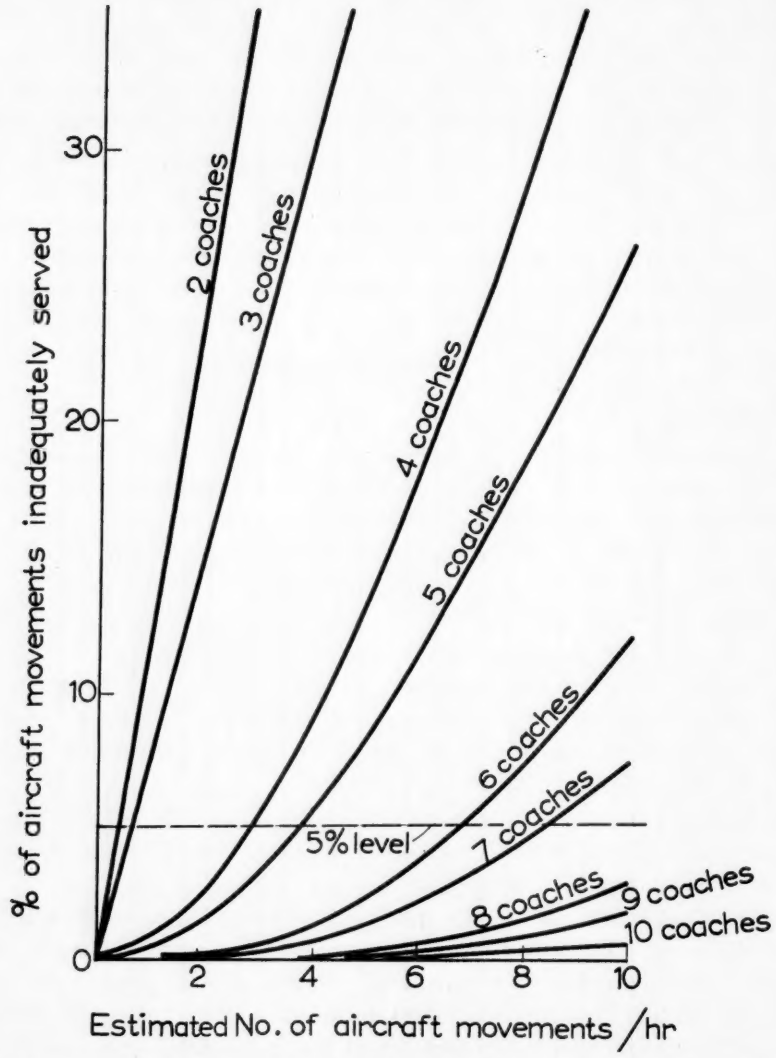

FIGURE 2. *Chart for assessing coach requirements* (80 *per cent of aircraft requiring two coaches*).

In Figure 2, $Q(N)$ is plotted against values of m from 0 to 1·25 (0 to 10 movements per hour) for the ratio $c' : c'' = 0{\cdot}20 : 0{\cdot}80$ and for $N = 2$ upwards. The tendency for the curves to lie in pairs indicates that the provision of an even number of coaches will give a better return for investment than provision of an odd number, as might be expected when 80 per cent of aircraft require two coaches. This suggested a selection policy based on even numbers only. Thus, if a fixed level of risk of $Q = 5$ per cent were selected (i.e. 19 out of 20 aircraft to be adequately served) an estimated movement rate of 8 aircraft per hour would give a point between the 6 and 7 coach curves and 6 coaches would be allotted, while 9 movements per hour would give a point between the 7 and 8 coach curves and 8 coaches would be allotted. On the average, this "even-number" policy gives a slight bias to over-insurance.

The Karachi Transit Problem

An analogous problem arose shortly before that of the London Airport coaches, during a study of ground servicing at Karachi, an important transit station on the air routes to the East. Here, a local team of B.O.A.C. staff is responsible for carrying out an intensive programme of refuelling, mechanical checks, interior cleaning and other activities whenever a B.O.A.C. or Australian Qantas aircraft arrives. The standard time for transit is 90 minutes, and the various activities of the different members of the team are all geared accordingly.

When two aircraft arrive within 90 minutes of each other, congestion will result unless a standby team is available, and the problem which was posed to the Operational Research Branch was:

> "Given the weekly schedules for future operations, how many teams should be available at different times of day and night to keep the risks of congestion within acceptable bounds?"

Among the different viewpoints quoted were two diametrically opposed ones: the first that all aircraft should be assumed to arrive on schedule; the second that unpunctuality created such uncertainty that duplication and even triplication of teams was essential during most of the day if serious delays were to be avoided. It was hoped that a scientific appraisal of the chances of congestion would provide a more objective basis for decision.

As in the London coach investigation, the "service time" was effectively constant and the basic input was a schedule of aircraft movements (arrivals only being relevant in this case) with random deviations from schedule.

At an early stage, however, it was decided that any attempt to solve this problem by the techniques of queueing theory would be unrealistic.

With an average of only four arrivals scheduled per day (mostly in the evening), the loss of information in considering them as the product of a random input would override any advantages gained in ease of analysis. Attention was therefore turned to a more realistic, albeit more cumbersome, model of a scheduled arrival time for each individual Flight, with a probability distribution of deviations from schedule superimposed. The form of this distribution of deviations from schedule, as obtained from a sample spread throughout a year's operations, was basically similar for all Flights, eastbound and westbound, and is shown in Figure 3(a). The mode of the distribution occurred at zero, but the occasional occurrence of unforeseen delays, owing to such rare contingencies as engine changes, gave an extremely attenuated "tail" to the right. This extreme skewness made it impossible to represent the whole distribution effectively by any of the standard Pearson or other curves, but the discovery that the "tail" could be very closely represented by a lognormal curve led to the further discovery that an extremely accurate representation of the whole curve was given by fitting the following combination of a normal and a lognormal component:

Excess of actual over scheduled arrival time $= X + e^Y$ minutes, where X (mean -15, standard deviation 20) and Y (mean 4, standard deviation 20) are independent normal variates.

This formula not only fitted the actual distribution closely for both early and late arrivals, but suggested a practical interpretation which is brought out in Figure 3(b) and 3(c). The normal element can be visualized as representing variations in the flight time from the previous station, while the lognormal element can be visualized as the accumulation of miscellaneous hold-ups built up along the route beforehand; this accumulation is normally insignificant but includes the possibility of long mechanical and other delays. The prediction of delays with the aid of mathematical formulae is of considerable interest in B.O.A.C. and it is hoped that this type of formula will prove capable of more general application.

Initially, it was decided to measure congestion at Karachi in terms of the probability of "clashes"; i.e. of two aircraft landing within 90 minutes. From these initial considerations, however, a more concrete measure was evolved: the distribution of "average overlap" throughout the week. The theory was briefly as follows. Suppose, as illustrated in Figure 5(a), two aircraft A and B arrive at instants a and b less than 90 minutes apart. If two servicing teams are available, congestion will be avoided and the aircraft

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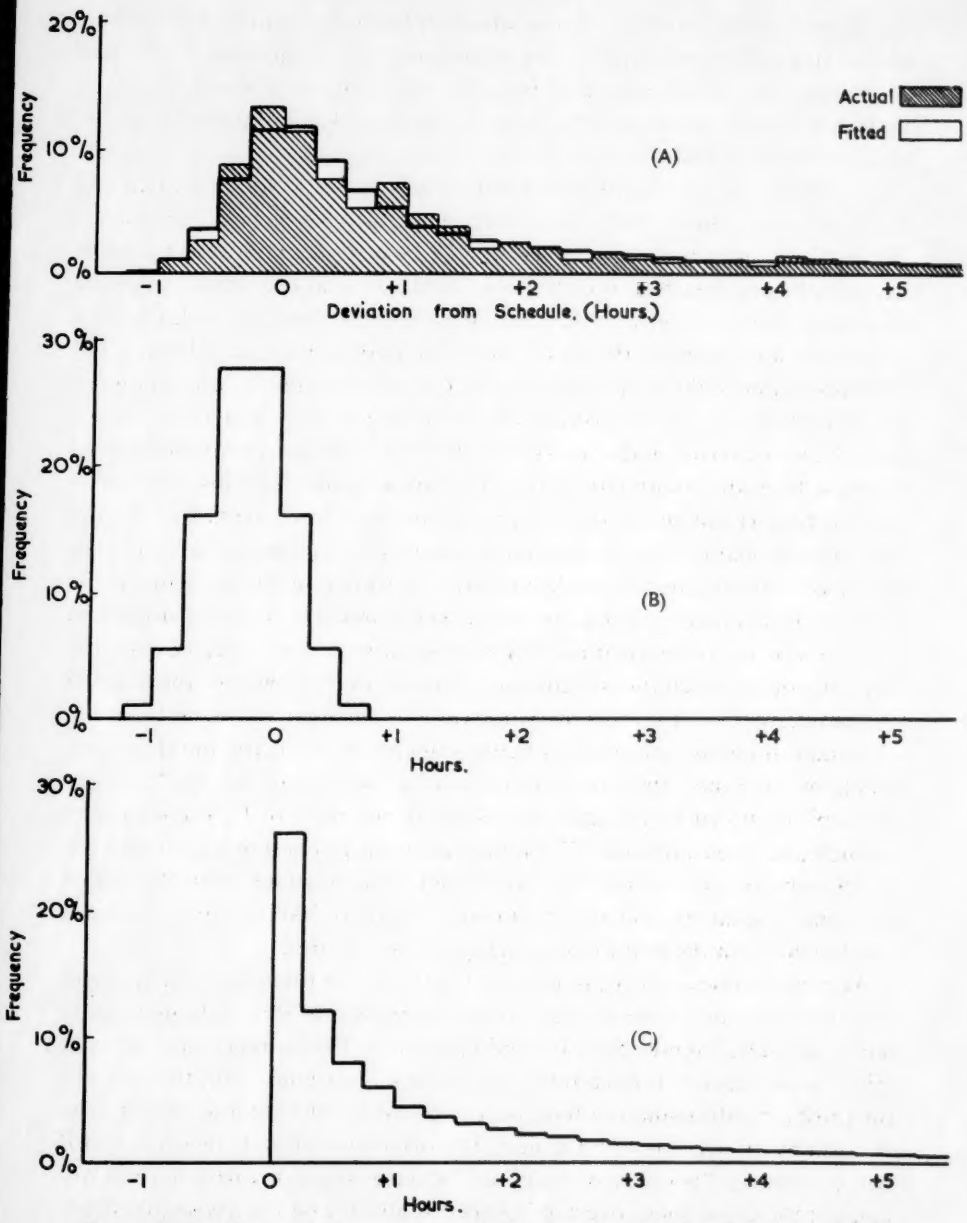

FIGURE 3. *Distribution of deviations from schedule for arrivals at Karachi. (a) Actual and fitted distributions. (b) Fitted distribution: normal (flight time variation) component. (c) Fitted distribution: lognormal (accumulated delay) component.*

will be ready to depart at instants a' and b', 90 minutes from their respective arrival times. If, however, only one team is on duty, then assuming a "first come first served" policy, B cannot be served until A departs and its departure will therefore be delayed by a time (b', b'') equal to the "overlap" (b, a') between A and B.

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For every pair of Flights scheduled to arrive within less than (say) 12 hours of each other, there is a more than negligible chance of "overlapping"; the duration of the overlap may be anywhere between 0 and 90 minutes. For each such pair of Flights, scheduled in at the same times week after week, an "average overlap time" per week can be calculated. Figure 4 shows, as an example, the calculation for two arrivals scheduled in at 1500 hours and 1600 hours respectively. For each Flight, the probability of the aircraft being on the ground and requiring attention at time t is the chance that it arrived in the interval $(t-90, t)$, which can quickly be derived from the basic distribution of deviations from schedule. This distribution of ground time is shown for the 1500 hour and 1600 hour arrivals in Figure 4(a). The probability of both aircraft requiring simultaneous attention at time t is then obtained by multiplication, as shown in Figure 4(b). If this probability averages p per cent over a small interval $(t, t+\delta t)$, then this interval will be "overlap time" for the two aircraft on p per cent of the weeks through which the schedule operates or, in other words, the average overlap per week within this time interval δt is p per cent of δt. In Figure 4, 15-minute intervals are used as being sufficiently small for the degree of accuracy required; the conversion from a percentage to an "average overlap" is shown by an additional scale to the right of Figure 4(b). For example, between 1700 and 1715 hours, the average overlap is 10·6 per cent of 15 minutes, = 1·6 minutes. Adding all such overlaps over the whole potential overlap period (approximately 1500 to 2000 hours), the total overlap between the two Flights averages 16·6 minutes.

At certain times on certain days of the week, the timetables for Karachi revealed an appreciable chance of clashes involving three (though rarely more) aircraft. Figures 5(b), (c) and (d) show different types of clash that might arise between three aircraft A, B and C. In Figure 5(b), the delay to the third aircraft would not have occurred had not the second aircraft been delayed by an overlap with the first. The original overlap between A and B will be called a "primary overlap" and the ensuing delay to B a "primary delay"; the consequent overlap between B and C and the associated delay to C are then both "secondary". In the second example (Figure 5 c), C is subject both to a primary delay (c', c'') arising from the primary overlap

(c, b') and a secondary delay (c'', c''') stemming from the secondary overlap (b', b''). In the last example (Figure 5 d) where all three aircraft arrive within 90 minutes, a careful study will reveal that the total primary delay (to *B* and *C*) is equal to the sum of the three separate primary overlaps (i.e. *A* with *B*, *A* with *C* and *B* with *C*) while the total secondary delay is

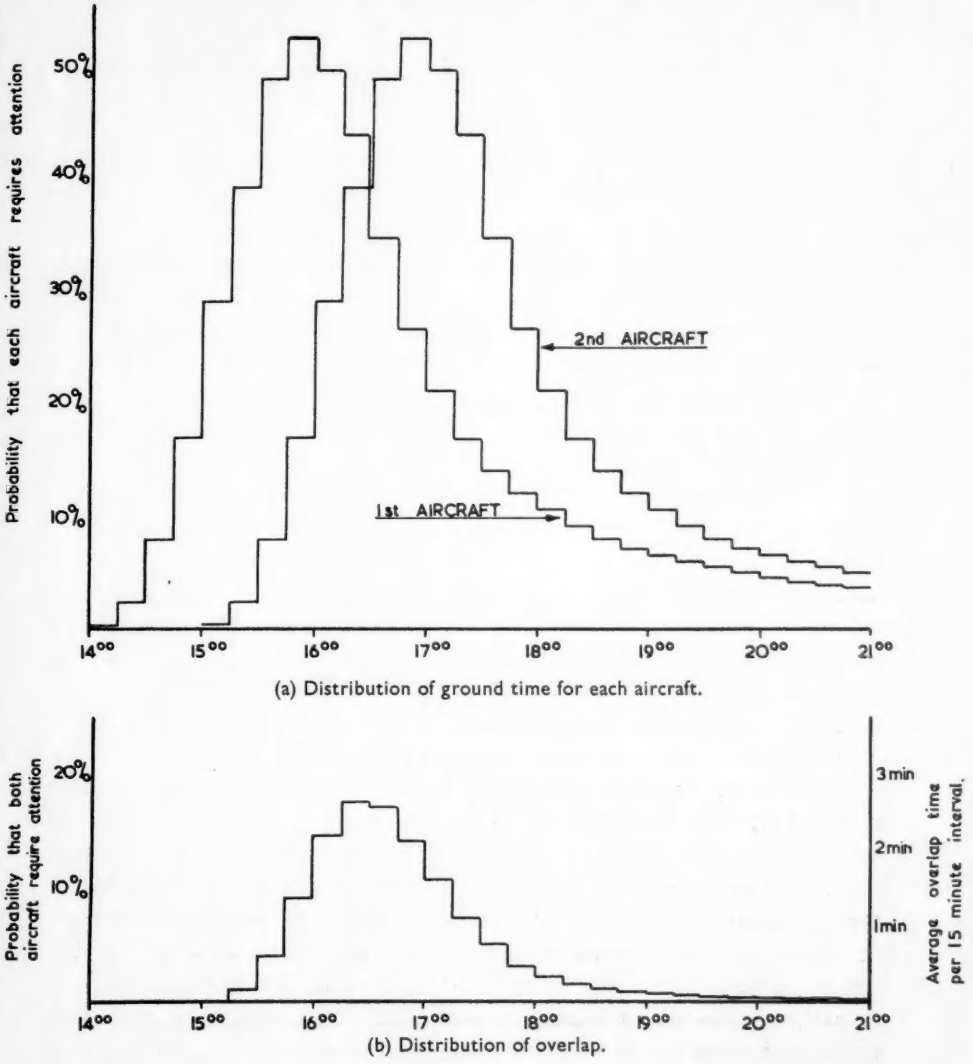

FIGURE 4. *Calculation of average overlap for two arrivals scheduled at* 1500 *and* 1600 *hr.*

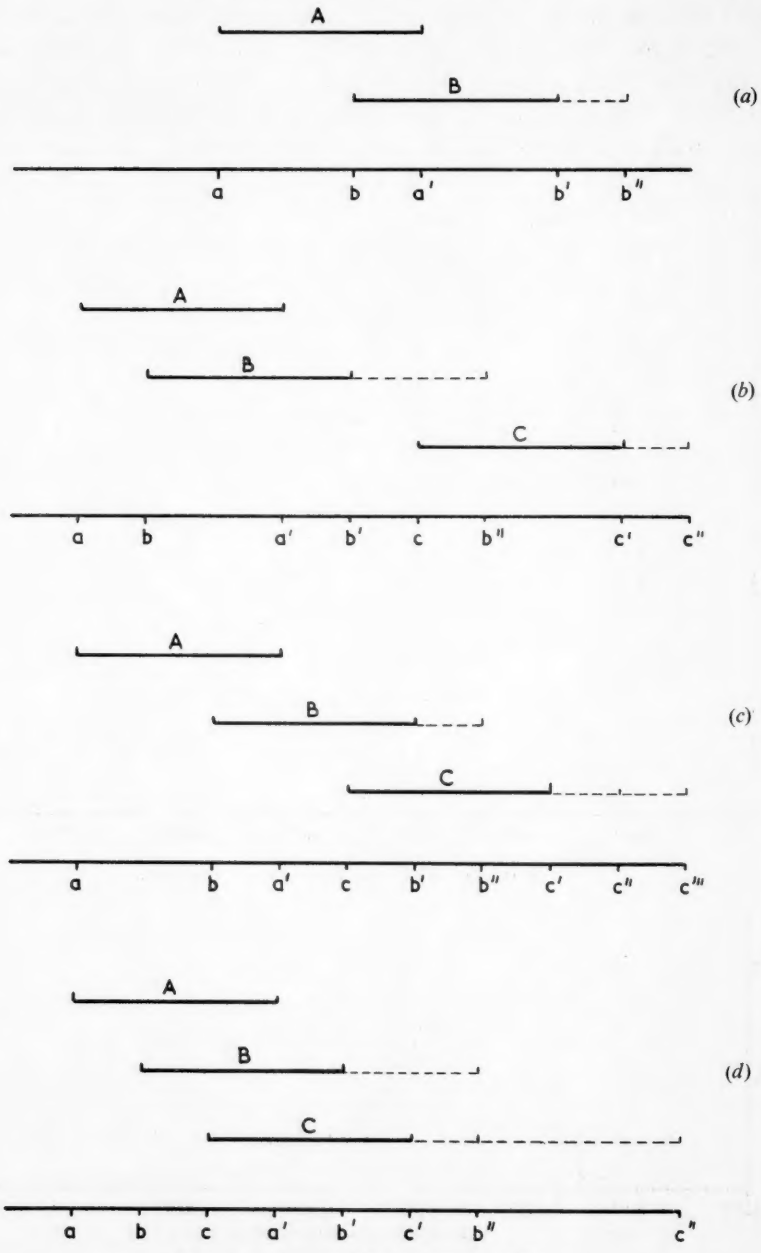

FIGURE 5. *Two- and three-aircraft overlaps.*

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equivalent to the total period (t, t') such that A and B are both in during (t, t') and C is in during the corresponding period $(t+90, t'+90)$ 90 minutes later.

These rules apply generally to all situations involving two or three Flights. In the rare event of more than three being involved, consideration of secondary delay becomes more complex and "tertiary" and higher delays must be defined, but the additive quality of primary delay remains valid. This is of great importance as it signifies that the total primary delay expected per week can be built up by considering overlaps between pairs only.

The technique was first applied to a prediction of congestion during the winter season 1957–8, from schedules issued a year in advance. Summation of the primary overlaps deriving from all possible combinations of pairs of Flights scheduled less than 12 hours apart gave a prediction of 4 hours 27 minutes for the average primary delay per week, assuming only one team available throughout. With two teams available, the average primary delay would be greatly reduced and delay would only occur when three aircraft were claiming service simultaneously, as during the period (c, a') in Figure 5(d). The probability of this occurrence, and the average primary overlap resulting, were obtained in the same way as in Figure 4, from multiplication of three instead of two probabilities. This gave a total of 22 minutes primary delay per week. Thus, of the total potential of 4 hours 27 minutes of primary delay, 4 hours 5 minutes (91·8 per cent) could be saved by employment of a second team.

The method of calculation of primary overlap, by 15-minute intervals, made it simple to break the total down on an hour-by-hour basis. Figure 6 shows the hourly distribution of the 4 hours 27 minutes of primary delay with only one team, and of the 22 minutes with two teams. It shows, for instance, that provision of a double shift for 8 hours only, from 1900 to 0300 hours, would eliminate 3 hours 44 minutes of the total weekly primary overlap, or 91·4 per cent of the 4 hours 5 minutes which could be eliminated by operating double shifts round the clock.

In this way, the relative efficiency of alternative shift systems in eliminating primary delays can be readily compared. A similar technique was evolved for assessing the distribution of secondary overlap through the week, but this was a rather more formidable prospect for practical operation. However, the total delay-time concerned (48 minutes per week) was not great, and so it was decided to sacrifice completeness to convenience and plan shift requirements on the basis of primary delay only. It was reasoned that, as

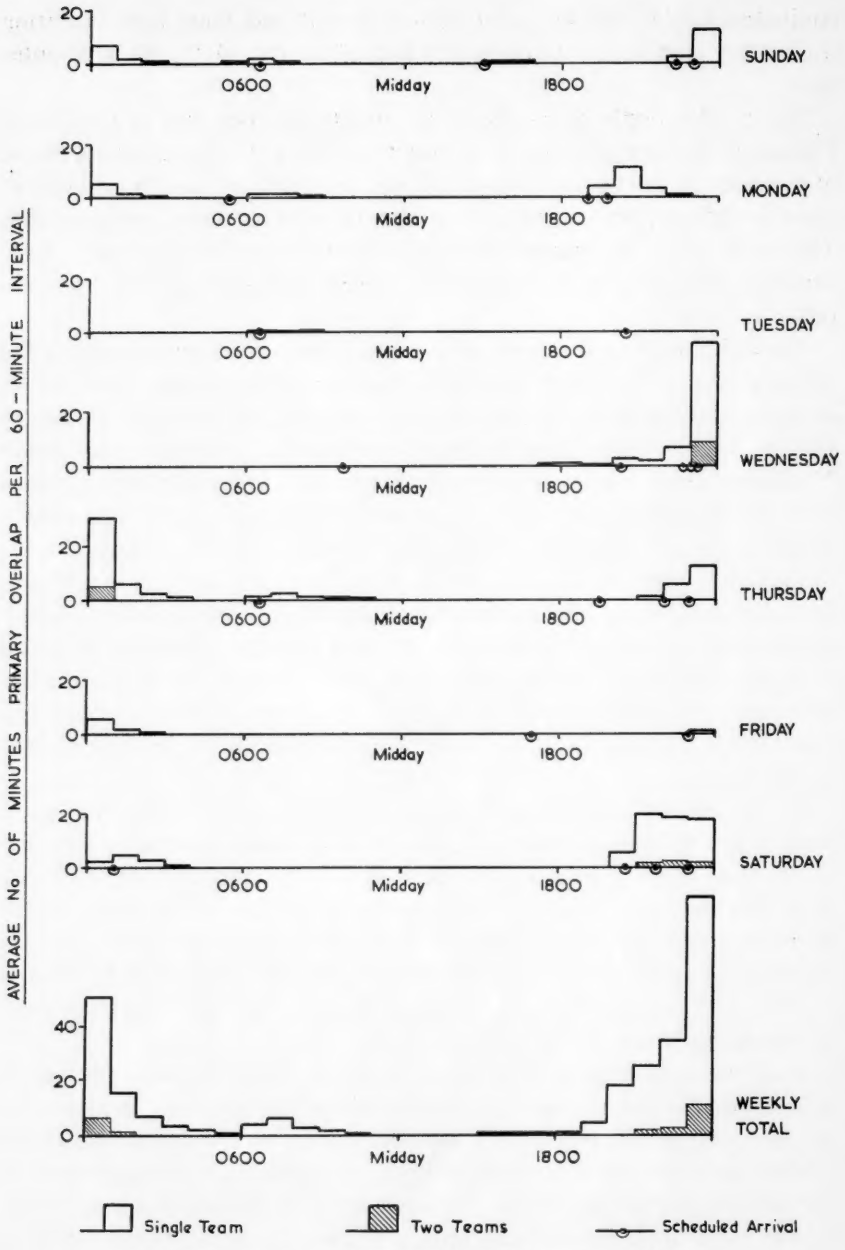

FIGURE 6. *Prediction of distribution of overlap for winter* 1957/58.

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secondary delays could never occur except as a consequence of primary delays, the deployment of extra manpower so as to cover the maximum amount of primary delay at acceptable cost would also provide satisfactory cover of secondary delay.

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A Comparison of the Two Studies

The very different methods adopted for the solution of these two rather similar congestion problems suggested that a "post-mortem" comparison between them would be of value in order to investigate more closely the reasons, conscious and unconscious, that led to the divergence in method and to establish whether a more unified approach would have been possible.

The main features of the two studies are set out side by side in Table 3. The first section of the Table compares the basic situations, while the second lists some approximate parameters describing their relative dimensions. Although the standard service time is 12 times greater in the Karachi problem, it will be noticed that the mean separation between customers is some 30 times greater and so the traffic intensities are not widely different; at London it is $2\frac{1}{2}$ times greater than at Karachi. The same ratio applies to the higher intensities during the daily peak periods in the morning and late evening respectively. Another parameter compared in the Table is the standard deviation σ of the deviation from schedule (extreme delays being excluded). This is a measure of irregularity and was approximately 50 minutes in each case.

The third section of Table 3 compares the mathematical models devised and their methods of solution. It was clear that the reasons for the divergence in method would have to be sought in the relative dimensions rather than in the basic situations, the latter being completely equivalent apart from one consideration—the possibility of aircraft movements at London, but not at Karachi, generating more than one customer—which was in no way fundamental to the choice of method.

In introducing the Karachi study, it was stated that the deciding factor in rejecting a random input model at Karachi was the low rate of movements, averaging one per six hours. This should be viewed in relation to the irregularity measure σ. Dr. G. E. Bell[1] discussed an input of aircraft scheduled with nominal separation τ and varying degrees of irregularity σ. It was established as a working rule that flow is effectively random unless σ is small compared with τ. This criterion gives support for the choice of a

random input model at London ($\sigma = 4\tau$) and its rejection at Karachi ($\sigma = \frac{1}{7}\tau$).

TABLE 3

THE TWO STUDIES COMPARED

	London Coach Study	Karachi Transit Study
Basic Situation		
(*a*) Customers	Batches of up to 35 passengers	Aircraft in transit
(*b*) Input	Aircraft arriving or departing generate 1 or 2 customers	Aircraft arriving represent one customer each
(*c*) Input rate	Variable; morning peak	Variable; evening peak
(*d*) Service	Transport between aircraft and passenger lounge	Engineering, cleaning, refuelling, etc.
(*e*) Servers	Coaches	Teams of ground staff
(*f*) Problem	How many servers to provide adequate service?	How many servers to provide adequate service?
Approximate Dimensions		
(*a*) Standard service time	$7\frac{1}{2}$ minutes	90 minutes
(*b*) Mean separation between customers	$12\frac{1}{2}$ min = 1·7 × service time	6 hr = 4 × service time
(*c*) Duration of peak intensity	0900–1200 hours	2000–2400 hours
(*d*) Per cent of all movements in peak period	35 per cent	55 per cent
(*e*) Mean separation (peak period)	4 min = 0·5 × service time	2 hr = 1·3 × service time
(*f*) σ for deviation from schedule	50 min = 4 × mean separation	50 min = $\frac{1}{7}$ × mean separation
Mathematical Model		
(*a*) Service time	Constant	Constant
(*b*) Input distribution	Random input of customers singly or in pairs; variable mean rate	Scheduled input + random deviations from schedule
(*c*) Solution	Successive approximations to "steady state" queueing solution	Evaluation of average overlaps for pairs of aircraft
(*d*) Criterion of "adequate service"	Not more than x per cent of aircraft movements involve wait for coaches	Weekly average of not more than x minutes of delay to be permitted

With a model based on a scheduled input, as adopted at Karachi, the construction of general queueing equations is impossible, as the probabilities p_i of i arrivals per unit service interval not only change appreciably from one unit interval to the next, but are not independent of how many, and which, aircraft have already arrived during previous intervals. It might be interesting to construct a generalized "algebra of overlap" on the basis of the rules established at Karachi for two- and three-aircraft clashes, but it is

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doubtful whether such an algebraic approach would bring any simplification of the problem.

Thus, the London approach has been shown to be inapplicable to Karachi; the Karachi approach would, it is believed, have been equally impracticable at London, where the combination of a higher traffic intensity with a very much higher σ/τ ratio makes it necessary to allow for up to 6 or 8 customers requiring attention at one time instead of only 2 or 3. The complexity of undertaking any overlap analysis on the Karachi principle, without assuming random input, can readily be imagined.

The two studies differed not only in method but also in the form of criterion finally presented to management to enable decisions to be made on the number and deployment of the servers. The system devised for allotting coaches at London entails an hour-by-hour assessment of coach requirements to maintain a given standard of prompt service throughout the day, while that devised for Karachi is based on planning a daily shift system so as to minimize the total time lost through congestion. These different criteria were to some degree the outcome of the different methods of analysis, but it would have been perfectly feasible to have evolved the opposite form of criterion in either case. However, the considerations in evolving a criterion for decision are not only theoretical but practical, calling for close consultation with management to obtain the local perspective. The perspective differed in these two situations in that, at London, failure to provide a coach at the right instant must be regarded as undesirable as an incident, while in transits through Karachi, time wasted is the crucial factor, and the gravity of a transit delay is roughly proportional to its duration.

REFERENCE

[1] G. E. Bell, "Theoretical Studies into Traffic Congestion, Part II", *Ministry of Civil Aviation Report* ORS/MCA, No. 3 (1949).

Letters to the Editors

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New Methods of Solving Linear Programmes

M. A. ACZEL AND A. H. RUSSELL, in their paper, "New Methods of Solving Linear Programmes" in the December issue of the *Quarterly*, state that the simplex method for linear programming "is certainly not the quickest possible procedure for obtaining a solution". In support of this view they present on the one hand what is considered to be an alternative procedure which they feel will prove more efficient, and on the other hand describe in outline R. Frisch's "Double gradient method".

The authors' "alternative procedure" is, in fact, an optional minor variation on the use of the simplex method.

It should first be noted that the "standard" simplex method, as presented for expository purposes in the cited article by Dantzig, in which the transformations of the method are executed upon the entire tableau of coefficients, has not been used on large-scale computers for many years. Rather, several versions of the "basis inverse" method have been relied upon, in part for the reasons mentioned by the authors in their description of it.

The particular variation proposed consists in withholding, successively, certain groups of columns of the original matrix until that part of the problem under consideration has been solved. The point of this is that if one can guess correctly which columns will not be used by the solution of the problem, he will have solved it without having to read those columns more than once. Now this procedure does not constitute an alternative to the simplex method; it is simply the application of the simplex method to a "nest" of linear programming problems. The ability easily to make such an application has been a feature of linear programming codes in the U.S. since 1954. Using codes written for the IBM 701 and 704, it is only necessary to insert especially punched cards between those blocks of the matrix which are to be so treated. This is known as *partitioning* the matrix of the problem.

Despite the familiarity of many users of linear programming with the authors' device, it has not proved to be very generally useful. The reasons for this will appear in the discussion below of the two advantages claimed

for it on pp. 212–13 of their article under the headings (1) *Reduction of number of iterations* and (3) *Accuracy and storage*. In order to understand these two paragraphs, we must decide what is intended by the word "iteration". The basic step of the simplex method is the change-of-basis, which is the specification of one formerly free variable as basic and the dropping of a basic variable, together with the revision of the basis inverse involved in this change. Since the change-of-basis introduces only one variable at a time into the basis, and requires always approximately the same amount of computation, it appears that each "iteration" of (1) includes the several changes of basis required between successive readings of portions of their matrix, and might be better called a "tape pass".

Now it seems certain that partitioning can, if properly used, greatly reduce the number of tape passes; and Dr. Vajda's proposal of introducing the first variable with a positive price may reduce this number even farther. This feature of partitioning is an important observation in connexion with problems as extraordinary as the $13 \times 3{,}000$ L.P. cited. If, however, one has not been able to guess 13 activities from the 3,000 at hand which will occur in a solution, it is most probable that partitioning will considerably increase the number of changes of basis required for solution, owing to the machine's being required to solve irrelevant intermediate problems.

Now it is the individual change-of-basis, rather than the tape pass, in which "a certain amount of accuracy is lost" (3). It follows that accuracy of the procedure cannot be improved by a reduction in the number of passes, but only by a reduction of the number of basis changes, which the authors in no way indicate is likely to occur. The maintenance of accuracy in the inverse of the basis is important, of course, for purely computational reasons, regardless of the state of the input data of the problem. For this reason the present writer has found the partitioning device to be of quite limited usefulness, and recommends against its use unless the formulator of the problem has quite good knowledge of its structure, or desires the intermediate solutions for their own sake.

It is not clear whether the authors present the "double gradient method" as an alternative to the simplex method, in view of their comment that at least one of its features "makes the possibility of using this method on a computer problematical". We are not aware of any evidence, other than predictions of the sort made by the authors, regarding the computational efficiency of the double gradient method.

This writer, feeling that the efficiency of a computational proposal is established only after it has been tried, believes that the simplex method is

at present by far the best procedure for solving linear programming problems.

PHILIP WOLFE

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Dr. Wolfe's letter contains a discussion of only one concrete question, on the basis of which he rejects a method we had suggested. This conclusion is based on what we think is a misapprehension of a technical nature, due possibly to the rather condensed way we expressed ourselves in paras. (2) and (3) (p. 213).

Suppose, for the sake of concreteness, that four new candidates are introduced every time in our "restricted linear programme". It may be that a great number of steps are needed to find the maximum of the "restricted linear programme", as Dr. Wolfe suggests. The net effect is that at most four variables are introduced into the basis. If these are noted one can, instead of taking a matrix out of the final simple tableau and pre-multiplying the inverse of the basis by this, introduce the four new variables into the basis either (a) by four simplex steps or (b) by inverting a 4×4 matrix as exemplified in formula (8) (p. 209). The last operation can be performed to a lavish degree of accuracy. It follows that each "tape pass" can involve at most the loss of accuracy consequent on four simplex steps, and probably less. Dr. Wolfe grants us that the number of "tape passes" will probably be greatly reduced by the suggested method: it remains to be seen by experiment whether the net effect of the method will give an increase in accuracy, as we had suggested; the number of candidates in the restricted linear programme will have a strong bearing on the matter. It is thus shown that Dr. Wolfe's objection to the method is not a valid one.

A very important point of principle is raised by Dr. Wolfe when he makes the reservation "unless the formulator of the problem has quite good knowledge of its structure". It is our contention that in operational research work, the formulator should not have put the problem on a computer before having at least some knowledge of its structure, otherwise he would not be able to interpret meaningfully the nature of the answer. Both the methods we mentioned use the formulator's knowledge to good effect.

We are sure that the attitude represented by Dr. Wolfe's letter will hardly impede progress in the important field of linear programming.

M. A. ACZEL, A. H. RUSSELL

Operational Research as a Staff Service for Management

B. M. BROUGH's paper is so interesting and valuable that it is sure to be widely quoted as a reference for those interested in the subject; it is for this reason that I would like to challenge his contention that the Operational Research Section should be responsible independently to the executive at the head of the organization it is intended to serve.

To have the goodwill of the chief executive is absolutely essential, but in a company of any size this is as much as can be expected, or is necessary. The objective of an operational research team should be to reduce the demands on this executive's time, not to add to them. I would go further and say that where operational research is being introduced from outside there are obvious risks in placing in such a position a man who is new to the Company and who has an outlook which may well be unfamiliar.

In the larger organizations there is frequently a grouping of functions which are at the service of the whole organization—such departments as those concerned with staff, legal activities, public relations and so on, and it may well be that an operational research team could be located here, being responsible to the executive in charge of the group. In this way the non-partisan nature of the team is preserved and at the same time access to top management and its problems is maintained. These are the vital principles and providing they are satisfied we need not be too concerned about labels and levels.

H. R. W. WATKINS

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On the Comparison of Two Production Processes and the Rule of Dualism.
H. STEINHAUS and S. ZUBRZYCKI.
Colloquium Mathematicum, 1957, **5** (1), 103–115, reprint.
Using the model of the counting of defects in two ribbons, to be evaluated as a ratio of defectiveness by a sampling process, the rule of dualism for a Poisson process is formulated and a proof is supplied, using Bayes' rule. The problem is then translated into its equivalent in the process of drawing balls (of 2 colours) from a container and proofs are again supplied. Pearson's tables of incomplete beta-function can then be applied for the calculation of probabilities in such comparisons.

The 1955 Sample Survey of Domestic Consumers.
The Electricity Council, Utilization Research Report. No. 7, *Jan.* 1958, 23 *pp.*
A survey of the electrical appliances possessed by the 12×10^6 domestic consumers, from a 1 per 1,000 sample, was carried out and is here summarized. The extent to which these were used (e.g., immersion heaters or space heaters), and a comparison of appliances between "North" and "South" regions was also estimated. Various correlations are shown between types of appliance used, between variations of consumption with total kWh used, and between seasonal consumptions. Rural and other conditions are also separately treated. Figures for 1946 are compared, and it is expected that a firm basis for discerning trends in consumption has been laid.

Control Engineering Manual.
B. K. LEDGERWOOD (Editor).
McGraw-Hill Book Co., London, 1956, viii+189 *pp.* 56*s.* 6*d.*
This manual is composed of papers which have been previously published in *Control Engineering*, and an attempt has been made by the editorial staff to present them in a logical order.

Control Engineering is defined (apart from a tautologous definition in Section I) by means of ten case studies presented in Chapters 1 and 3. Apart from one study on "Controlling a Business Process", the papers are extrapolations in modern technology of Watt's steam-governor, and are

of doubtful value to anyone presented with a different process or problem. Chapters 2, 4, 5 and 6 are similar, inasmuch as they describe, somewhat non-technically, the tools available in process control. Chapter 6 explains computers, what they are and how to use them. The information in these chapters is already available in any elementary book on servo-mechanisms, and no great service is given by repeating it. Chapter 7 treats non-linearities in control mechanisms in a "theoretical" manner. The absence of non-linear systems in the case-studies is evident.

The bibliography is extensive and non-selective. Under "Control in Industry" we find "The Shape of Things to Come" and "Brave New World".

The manual may be of some interest to newcomers to the field of system engineering, but is unlikely to be of value to the operational research worker unless to give him an inkling of what industrial processes have already been controlled. The publishers, as usual, have presented the contents in an attractive form.

P. J. O'D.

Tecnica y Teoria Estadistica en el Diseño de Experimentos.
D. J. FINNEY.
Consejo Superior de Investigaciones Cientificas, Madrid. Monografias de Ciencia Moderna, No. 58, 1957. 163 *pp.* 75 pesetas.

This book is the Spanish text of a series of ten lectures delivered in English, and given in Madrid in 1954. It is intended as an introduction to the subject for mathematical statisticians who have some knowledge of the theory of analysis of variance but little experience of the complications of design now used in applied statistics. It does not contain any original material, but is a review of the subject aiming to interest the theoretical statistician by its variety rather than to teach him in full detail. After an introductory chapter, the basic ideas of randomized blocks and of Latin and other orthogonal squares are presented. The three following chapters summarize the theory of factorial designs, fractional replication and confounding; then come two chapters on incomplete block designs. The next chapter gives a brief summary of sequential schemes of research. Chapter IX is a discussion of the broad principles of planning experimentation and choosing designs. The last two chapters illustrate some of the ideas previously encountered with reference to two specific types of experiment, each of which has some unusual complications; these are biological assay and experimentation for estimating the mean of a normally distributed tolerance. (Author's abstract.)

Experimental Designs (second edition).
W. G. COCHRAN and G. M. COX.
John Wiley and Sons, New York; Chapman and Hall, London. 611 *pp.* $10.25; 82*s.*

By the end of the war the main principles and techniques of the design of experiments in agricultural and biological research had been well established and the resulting body of knowledge was authoritatively set out in 1950 in the first edition of this excellent book. Since then, however, there have been extensive developments in other fields of application such as industrial, social and medical research. The attempt to cover these developments as well as elaborations of previous material has led to a size increase of about a third compared with the first edition.

Much of the new material should be of particular interest to those concerned with the technological applications of statistics. For example, of the two new chapters, the first, dealing with fractional replication, has particular value for certain types of exploratory research in which a quick indication is required of which factors are likely to be of greatest importance. The second of the new chapters describes the important recent work of Box and others on the study of response surfaces. A response such as output is regarded as a regression function of input factors. The problem is to allocate the input combinations at which the experiment is to be conducted so as to give economic estimation of the regression function. Designs and analyses are given for linear and quadratic functions. Further developments for locating the input combination yielding maximum response are also described.

A minor criticism from the technologist's point of view is that most of the designs described were constructed so as to allow all treatments to be compared. In practice, however, what we often want to do is pick out the best treatment. Once we have done this we are not very interested in comparisons between the remainder. It is a pity that recent work on this problem by Somerville, Bechofer and others was not mentioned. J. D.

Linear Programming and Economic Analysis.
R. DORFMAN, P. A. SAMUELSON and R. M. SOLOW.
McGraw-Hill, New York, Toronto and London. 525 *pp.* 77*s.* 6*d.*

Most operational research workers feel they should know more about linear programming; many have tried to use it with more or less success. Many economists, too, have similar difficulties and doubts in relating linear programming and other forms of linear economics to the theories with

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which they are familiar. It is for them, primarily, that this book has been written but since it comes from the Rand Corporation, the "birthplace" of linear programming, it carries authority and as, to this, is added clarity of expression it will be useful to many readers of this Quarterly as well.

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After an historical introduction the early chapters give the main ideas of linear programming covering both applications and methods of solution. Here, and indeed throughout the book, more than usual stress is placed on the economic meaning of the dual problem of valuation or the calculation of "efficiency prices" corresponding to the optimum allocation of the scarce resources. The values of the resources emerge, of course, in the course of determining the optimal programme but it is doubtful whether they have been fully used as an aid in reporting and implementing operational research results. The fact that linear programming provides a useful model of a competitive economy in action suggests that a firm can improve the efficient allocation of its resources, without an elaborate planning control, by according to "value" more of the attention which is frequently concentrated on "cost".

Chapters 9 to 14 deal with linear economics in general, including Leontief's input–output analysis and use linear programming to elucidate the basic theorems of welfare economics. This part of the book is difficult reading for one without pre-knowledge of economic theory but readers versed in linear programming concepts may find this avenue of approach to economics a logical one.

The final two chapters deal with theory of games. Chapter 15 gives the basic concepts and their application to economic problems and discusses some methods of practical solution. Chapter 16 explores the mathematical connections between games theory and linear programming.

Fortunately for the reader, the book is not made the occasion for a display of mathematical erudition. Matrices are used sparingly and there is an appendix which can be turned to for help where matrix algebra is used.

Whilst this book cannot be regarded as an operational research text, the serious student of linear programming will wish to give it shelfroom beside the classic "Activity Analysis of Production and Allocation" (Wiley, 1951). He would be unwise to approach it, however, unless he had previous knowledge either of linear programming or of the corresponding parts of economic theory.

The name of the publishers is a sufficient guarantee of the book's standard of production.

J. S.

Statistics for the Social Services (second edition).
T. G. CONNOLY and W. SLUCKIN.
Cleaver-Hume Press, London, 1957. 166 *pp.* 16*s.*

This book is intended to introduce students of the traditionally descriptive sciences, like sociology, medicine, education and psychology, to the disciplines of quantification. It was such a student who asked David Hume by what criteria he should judge a useful book; Hume replied, "Sir, take the book and scan its pages. Seek thereon for any mention of measure, and, if finding none, commit it to the flames." Connoly and Sluckin should have had David Hume to write their foreword, for it is to enable these students to follow Hume's advice that their book is written. The first half of the book is devoted to the primary concepts of quantifying, namely, order and classification, the reliability of samples and the validity of inferences drawn from comparisons of samples. The treatment is simple and lucid; the book is an alphabet of statistics in which the commonest letters are drawn in the largest type. The second half of the book explains what is meant by a Null Hypothesis and goes on to show, in terms of such useful tests as Chi-square and Student's-*t*, how to set one up and test it. It also has two sections on correlation and the analysis of variance; that on correlation should be valuable for those who may soon find themselves having to carry the responsibility for selecting children for higher education. There are references to less known methods, like biserial, ranking and tetrachoric correlation; all of these should be helpful to teachers who have to interpret examination results. The final chapter, on the analysis of variance, is more of a challenge to the beginner to pursue his studies beyond the level that the work aims at than it is (or can be) an organic part of the book, a discrete raising of the curtain for an instant so that the recent convert can see a little of the mystery that is yet to be explored. But the temptation to say too much too soon is overcome and the authors are content to keep to their simple text. In so short a book something must, of course, be left out; the social sciences throw up some very odd distributions, looking nothing like those discussed at length in the book and though some are intractable others, like the negative exponential, can be handled. Another and more serious omission is the lack of references, not so much to more advanced statistical treatises, but to the literature of the social sciences themselves where these statistical methods have been used. But this is academic carping. A book of this level is badly needed in the field of management, to induce managers to measure and work upon the information and statistics that are to them what lead

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is to the plumber and timber to the joiner. A treatise as simple and clear as this, but written *ad hoc*, might be a good start.

R. R.

Queues, Inventories and Maintenance.

P. M. MORSE.

Operations Research Society of America: Publications in Operational Research—Number 1. *John Wiley and Sons, New York; Chapman and Hall, London.* 52*s.*

This book is the first in a series of monographs sponsored by the Operations Research Society of America. The choice of this as the first book is appropriate since a book on the theory of queues was overdue.

The author makes no claim that this is an exhaustive treatise on the subject and, indeed, two or more further books on specific aspects are promised. Except for one short chapter on transient solutions, the analysis is confined to "steady state" solutions.

Non-exponential inter-arrival times and service times are represented either by the Erland device of phases to give χ^2_{2m} distributions or by parallel channels for hyper-exponential distributions.

The greater part of the book is concerned with the basic queueing models for single and multiple channels. The effect of queue disciplines other than first come, first served is considered in a separate chapter devoted mainly to priority problems. The last two chapters deal with inventory control and maintenance. The section of the last chapter which deals with machine interference problems does not follow the European practice of presenting the results in terms of the servicing factor, i.e., the ratio of the average attention time to the average running time between calls for service. This is unfortunate, since the reader who wishes to study this aspect of the subject in greater detail will find that most of the new work appears in European journals.

There are three appendices giving a glossary of symbols, definitions of functions and tables of these functions. The solutions of the problems considered are presented in terms of the tabulated functions so that numerical results are easily obtained.

The author has performed a useful service in writing this book, not least in his repeated assertion that it is not possible to provide ready-made formulae to fit all situations. The reviewer would have liked to have seen this point stressed by a much longer bibliography or, at least, an indication of the extent of the literature.

F. B.

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Erratum

Operat. Res. Quart. **9**, 2, 129 (June 1958).

B. M. BROUGH: Operational Research as a Staff Service for Management

The sentence beginning on line 15 of the second paragraph should read "It proved so successful that a very strong sense of co-operation rapidly developed between the military commanders and their scientific advisers at the highest levels of decision making".

Operational Research Quarterly
VOLUME 9 NUMBER 4 DECEMBER 1958

The Teaching of Operational Research

PRESIDENTIAL ADDRESS TO THE OPERATIONAL RESEARCH SOCIETY ON 23 OCTOBER, 1958

M. G. KENDALL

Introduction

THE Operational Research Society has reached a critical point in its career. Its membership during this last year has increased by 25 per cent; and, if the growth curve of the society is anything like the ordinary logistic, it will be approaching the thousand mark in about seven years' time at the most. We have begun to set up a small permanent staff to relieve the honorary officers of the mass of work which has been thrown upon them in recent years. Very shortly, I hope, we shall have permanent offices and can begin to build up a library. The final shape of things to come is by no means yet apparent but we are obviously in a phase of rapid development which may make the society of the future almost unrecognizable to its founders.

This movement has, however, entailed one development which may be an advantage to the society but is a definite disadvantage to me: it has been decided that in future the President of the Society shall, at the beginning of his term of office, give a Presidential Address. It was further decided, towards the end of a long committee meeting which I unfortunately had to leave early, that the present President should begin this series halfway through his term of office. Being in favour of the general principle, I could hardly object to this particular application. But it is a delicate occasion because anything I say or do can be cited in precedent by later incumbents of the office; and I should like to disengage myself from position by offering, in the first instance, a few remarks on what I conceive to be the scope and function of the Presidential Address. This will at least give later presidents an opportunity of seeing exactly where they can take a different line.

In the first place, papers presented to this Society are always discussed. This I regard as a most valuable practice which should be jealously

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preserved. But incoming Presidents are sometimes offered immunity from discussion, on the grounds that they ought not to be exposed to the adverse criticism which inevitably emerges in it. This is especially true of Presidents who are distinguished men, whose assumption of office is a definite advantage to the Society, but who are not particularly expert in the Society's field and might therefore, to use an operational expression, drop a brick by way of public utterance. It seems to me prudent to establish the rule that the Presidential Address should not be discussed unless the President wishes otherwise. On this particular occasion I opt for discussion.

Secondly, it seems to me that the Presidential Address should be the occasion, not merely for discussing our subject and our affairs among ourselves, but for considering them in relation to the world in which they are immersed. Up to the present we have lacked a method of considering ourselves in the round. This has not necessarily been a drawback in the past, but there might be occasions in the future when we want to place some aspect of our work before a wider audience. Without suggesting then, that presidential comment should acquire an *ex cathedra* quality, I think it may be useful if the address is written with a view to being read outside our immediate membership; and it would be useful if the discussion could be conducted in the same spirit.

Finally, it seems to follow logically that the address should be printed in the Society's journal.

These suggestions are, as you will see, the very reverse of revolutionary. They follow the existing practice of most comparable societies and organizations. But I thought it well to put them on record, for they affect not only future presidents but the general shape of my own remarks. I now proceed to the subject of this address.

The Content of Operational Research

All operational research workers are aware of the difficulty we have in defining our subject. Wherever two or three operational research men are gathered together, it is always possible to inject vigour into a flagging conversation by mooting the question as to what operational research really is. Opinions about what it ought to be are even more diversified. This is unfortunate and causes a good deal of confusion, especially among management. I shall not stay to try to find a sentence which might be inserted in a dictionary by way of definition. In my opinion subjects like ours are not to be defined by definitions. But if we are to discuss the teaching

of the subject we must have at least a rough idea of the domain of study. It would be convenient if we could be systematic and discuss, in turn, the scope and content of operational research, the appropriate methods of teaching it, the scope and content of courses for the purpose and finally the action which we need to bring those courses into being. Unfortunately in the present state of things it seems impossible to draw up a formal development on such lines.

Operational research may be regarded as a branch of philosophy; as an attitude of mind towards the relation between man and environment; and as a body of methods for the solution of problems which arise in that relationship. It is not unlikely that the historian of the future may detect in the western world at the present time a broad movement against the nineteenth-century attitude towards objectivity in science; a reaction similar to, but I hope more sensible than that of Rousseau and the Stürmer and Dränger against eighteenth-century rationalism; a feeling that some closer link must be established between scientific achievement and human values. The carrying over of the spirit of scientific inquiry into social fields is one aspect of this movement. Perhaps, in the realm of philosophy itself, the existentialist movement is another. One can spend interesting but rather unprofitable hours speculating about the effects of the same approach say, on the physical architecture of our buildings or the abstract architecture of our business organizations. I shall not wander into this wilderness. The point of raising the matter at all at this stage is to make it plain that, by whatever name we choose to call it, the neo-scientific approach constitutes a very general setting for operational research and from the broad point of view may be identified with it; so that if we are to consider not only the teaching of operational research techniques but the teaching of operational research methods and attitudes our horizon becomes very wide indeed. For the purposes of this address I propose to abandon any attempt to survey the whole domain. Instead I adopt a more operational approach by worming my way into the subject; and I begin with a list of courses on industrial operational research which are at present in existence; recognizing that the industrial side, though the most highly developed, may be only one facet of the whole subject.

The Education Committee* of this Society has recently reviewed the teaching of courses on operational research and I borrow the following

* The Committee for 1957–8 consisted of Mr. R. S. Gander, Chairman (United Steel Cos Ltd), Dr. F. G. Foster (London School of Economics), Dr. K. Pennycuick (I.C.I. Ltd) and Mr. B. H. P. Rivett (National Coal Board).

summary from their work. The list does not claim to be exhaustive, but it includes all the courses known to us to be actually in existence.

Birmingham University: The University (Department of Engineering Production) announces a twelve months' M.Sc. (operational research) course beginning in October, 1958.

The Institute for Engineering Production is already running a two-weeks' residential course on operational research. The course is given about four times a year.

Ancillary courses on subjects such as "Queueing theory and practice", "Quality control", "Organization and methods", etc. are given at the same place.

Birmingham: College of Technology. The College runs a four-year course for its Diploma in Technology. Towards the end of the course instruction is given in "Industrial mathematics" which has an operational research content.

The College also runs a short series of evening lectures on operational research techniques.

Cranfield: College of Aeronautical Engineering—Work Study School. In its work study courses the College is reported to be offering lectures on operational research.

Durham University. A two-week management administration course given in September, 1958 included six operational research lectures. In the Production Engineering Honours Option in the final year of its Engineering First Degree, operational research lectures are being introduced in 1958–9 and are expected to increase in number over future years.

Glasgow: Royal College of Science and Technology. The College offers an evening class course with a specialist seminar in operational research.

The College also gives a one-session postgraduate course in works management which includes 12 lectures on operational research; and offers a one-week operational research appreciation course.

Leicester: College of Technology and Commerce. The Department of Mathematics offered a course on "Introduction to operational research for engineers" consisting of nine lectures in the first quarter of 1958.

London (Acton): Brunel College of Technology. The college arranged for a series of ten lectures on operational research for 1958–9 under the guidance of Dr. Pennycuick.

London: London School of Economics. A fortnightly course will be given in operational research in the Easter vacation, 1959. L.S.E. is considering more extended courses.

L.S.E. also runs a fortnightly operational research seminar (about 15 seminars a year) for senior industrial executives and senior academic staff.

London: Northampton College of Advanced Technology. In 1957–8 the College offered several sets of evening lectures, both on techniques and with orientation, e.g. operational research and accounting.

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London: Woolwich Polytechnic. The Polytechnic has started a series of lectures introducing operational research for 1958–9.

Manchester: College of Science and Technology. The Department of Industrial Administration offered a series of 20 lectures on operational research in the winter, 1958–9.

Sheffield: College of Commerce and Technology. The College ran a course of eight afternoon lectures on operational research in 1957–8 for local executives from industry.

Courses of various lengths have, I understand, been mooted in a number of other centres, and in particular at Bristol, Glasgow, Huddersfield and Birkbeck College (London). Some industrial concerns with operational research groups lay on internal courses. Mention should be made of less regular courses, such as that arranged by the British Institute of Management and the Institution of Production Engineers.

A glance over the above courses makes several points clear. First, many of them are short courses either lasting only a week or two or consisting of eight or ten lectures. Secondly, none of them (with the possible exception of the projected L.S.E. course) is for undergraduates. Thirdly, they are industrially or commercially orientated and are regarded as a part of a more general subject such as industrial management, not as constituting a full subject in themselves. Finally (though this does not appear from the list), a good deal of the actual instruction is given by members of this Society specially imported for the purpose. It is the demand which is eliciting the teaching; the teaching is not growing spontaneously and teaching centres do not, as a rule, include among their permanent staff individuals with a specialist operational research knowledge.

It is significant for the present discussion that nearly all of these courses are postgraduate in fact or in effect; that is to say, they draw their students from persons who have already been trained in other disciplines and are already at work. There is no indication of the inclusion of operational research in degree courses, although a few items like linear programming exist among the syllabuses for some degrees now being given in various universities. If we take the more general view of operational research as the introduction of scientific methods into behavioral science, I suppose that we can also include certain courses on Logic and Scientific Method; but this may be stretching things rather far.

Let us now look at some of the syllabuses. I shall not list them all. The most comprehensive, of course, is the Birmingham M.Sc. course, which has the following syllabus:

Principles of Engineering Production: evolution and structure of modern industry, types of production, management functions and their co-ordination, historical development of "scientific management", of work study and of operational research, scope and limitations of scientific method in management, human relations in industry.

Mathematical Statistics: statistical inference, parametric and non-parametric tests of significance, estimation—confidence and fiducial intervals, design of experiments, multiple regression, theory of probability—recurrent events—birth and death processes, information theory.

Operational Research Techniques: theory of queues, mathematical programming, theory of games, stock control models, Monte Carlo methods and operational gaming, renewal theory, cybernetics, case studies.

Work Study: analysis of work flow, multiple operation analysis—labour and machine utilization, ergonomics, work measurement—time study, production study, activity ratio study, motion–time standards, memomotion study, analytical estimation, output patterns, job evaluation and incentive schemes.

Analogue and Digital Computers: use of analogues to perform basic arithmetic operations, relative advantages and disadvantages of mechanical, electrical and hydraulic systems, survey of digital computers, programming.

Productivity Measurement: production economics, measurement and control of productive efficiency, cost standards and cost control, budgeting for research, development and production.

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The syllabuses in use elsewhere vary a good deal and I should not like to describe any one as typical. For illustration only I give two which, in a sense, bracket the range of industrial interest, one the introductory Leicester course given by the Department of Mathematics for engineers, the other the Brunel College of Technology course by the Department of Management and Production Engineering for managers.

The Leicester Syllabus is as follows:

Operational research as a scientific aid for management.

Fundamental ideas and background.

The role of operational research in industry and engineering.

Conducting an enquiry: inception and planning, collection of information, analysis of data, consultation and report, follow-up of recommendations.

Organization, records and equipment.

Principal techniques used in operational research: probability in industry, statistics and the engineer, industrial experimentation, quality control, congestion and machine interference, simulation methods, production planning, other methods.

How can the non-specialist make use of operational research ? Guide to literature, services, etc.

The course, which does not presuppose mathematical knowledge, should prove useful to production engineers, technical executives, and industrial administrators. Case histories will be discussed extensively and students will be encouraged to discuss those problems arising in their work which offer scope for an operational research approach. The aim of the course is to inculcate operational research "thinking" rather than details of the various methods that have been found useful. If there is sufficient interest, a continuation course will be held, in which particular methods will be studied more fully.

The Brunel course has the following contents:

Introduction: historical development of operational research, some classic cases.

How an operational researcher looks at a firm: finding the aspects of a firm's activity where operational research is most likely to help.

Human problems of operational research: distinguishing between technical and human difficulties.

Stock and production planning problems: why stocks?

Techniques of determining optimum stocks.

Some cases demonstrating the determination of stock levels.

Distribution: depots and routes.

Methods of transport.

Control in a firm.

How to start operational research training: recruitment for operational research.

Helping an operational research team.

We now have enough evidence, I think, to enable us to consider the problems and to formulate the sort of questions we wish to answer. It is plain that these courses are at very different levels and are addressed to very different types of people. Some of them are not courses of instruction in the methods of operational research in the sense that they purport to teach students how to use those methods; rather are they directed to explaining what operational research is about to people who might be expected to want operational research done for them. We shall have to distinguish between at least four types of course:

(a) those which inculcate the general attitudes and principles of operational research;

(b) those which train people to become operational research workers in particular fields. We may think of this group as the one related to the standards we should expect of persons admitted to full membership of this Society;

(c) those which train people in other disciplines to recognize a situation requiring operational research techniques and perhaps to begin to learn them in the course of practical application. This grade may be regarded as corresponding to associate membership;

(d) those which merely (may I say merely?) inform lay persons what operational research is and what it is capable of.

Most of the courses listed above are in categories (c) and (d).

Perhaps we should add a hybrid category:

(e) parts of courses (e.g., queueing theory as part of a degree in statistics, or quality control as part of a degree in engineering, or even mathematics itself) which can be regarded as among the canonical operational research syllabuses.

The class (a) is almost empty at the present time. It may even seem unrealistic to begin with the nearly empty class (b). But for purpose of presentation it is necessary to do so in order to define the scope of our subject. Let us suppose that we were designing a course, perhaps of full university status requiring three years full time, with the object of training an operational research worker from scratch. For the moment I do this without prejudice to the question whether operational research can be or should be taught in universities or, if in universities, at the undergraduate level. If we can agree about a syllabus of such a course in general terms we have, at least, defined the field of knowledge which we expect the fully qualified operational research man to cover. How he is to acquire that knowledge is another matter.

A Possible Comprehensive Syllabus

The following pre-requisites may be posed:

Mathematics to A level of G.C.E.

Two other A level subjects.

This sounds innocuous enough. The university entry requirements for most scientific subjects amount to at least two and more, usually three A levels. But we no sooner begin to write something down than we run into doubts and difficulties.

(a) Ought the pre-requisite levels to include a natural science? It is highly desirable that some acquaintanceship with experimental techniques should exist or at least that there should be engendered in the newcomer some sympathy towards the experimental approach. But I am doubtful about making an A level in a physical science obligatory, and for a

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practical reason. At most schools physics and chemistry or physics and biology go together, often with mathematics. To make one obligatory would have the effect of making them all obligatory. It might shut off a useful source of recruitment from pupils reading mathematics and economics or mathematics and some other subject.

(b) Are our A level mathematics really necessary? Mathematics as taught in this country is sometimes a poor preparation for industrial mathematics. A student may spend half his time doing projective geometry, which is fascinating but not of much practical use, and the so-called "applied mathematics", which is a misnomer for pure mathematics in a peculiar language borrowed from the natural sciences. On the other hand he may be deficient in numerical analysis, or co-ordinate geometry. There is a good deal to be said for a special kind of A level mathematics suitable for those intending to be statisticians or operational research workers. For practical reasons, however, I think we shall have to accept the G.C.E. as it stands for some time to come. But if so, we must be prepared to see some pure mathematics included in the final syllabus, to fill the gaps and strengthen the theoretical side of the work.

The final course could be on the following lines:

Mathematics: differential and integral calculus, co-ordinate geometry, algebra of determinants and matrices, solution of equations, numerical analysis, elementary diffusion and differential equations, curve fitting.

Computational Methods: tabulation, interpolation, graduation and quadrature, mechanical methods, elements of programming for computers, simulation.

Statistics: distribution theory, sampling theory, estimation and testing of hypotheses, quality control, time-series, experimental design.

Optimization Theory: mathematical programming, routing problems, general theory of maxima and minima, including discontinuous variation.

Probability theory: elements of combinatorial theory, probability in a continuum, stochastic processes, queueing, congestion and inventory theory.

Scientific Method: the scientific approach, scientific techniques in the experimental and non-experimental sciences.

Accountancy: elements, cost accountancy, social and national accounts, input–output analysis, cost accountancy in optimization studies.

Economics: the quantities and relationships of economic theory, economic structure and analysis, the nature and working of economic institutions.

Business administration and industrial decisions: the organization of management, centralization and decentralization, industrial planning, organization of material for managerial decisions, role of an operational research unit in relation to management.

Case Studies: A number of case studies up to and including decisions based on operational research and their consequences.

Before leaving the syllabus let us also notice a few important features of it.

(a) It straddles several distinct disciplines, ranging from pure mathematics to management. There are not many teaching institutions competent to provide instruction on all these subjects. Even if we regard topics such as economics as dispensable refinements, we can hardly take the same view of cost accounting and management.

(b) The subjects divide broadly into two groups, those dealing with industrial or commercial mathematics (including statistics and accountancy); and those dealing with economic organization and industrial management. For the practising operational research man the first is certainly necessary, though the relative intensity with which he studies the various parts of the syllabus may be subject to argument. The second group is, perhaps, not necessary for industrial practice in the narrower sense but I myself should be reluctant to see it excluded from the broad education of an operational research man. The danger of including both types within one syllabus is that if ability to master one is not correlated with ability to master the other (as may well be the case) the syllabus, considered as a course of study, may impose too great a strain except on really first-class students.

(c) There are very few good text-books which a student could read by himself. Until this gap is filled the basic instruction would have to be by lecturing, except in those instances when a recruit to an operational research unit may pick up the subject from his colleagues. One does not make a good comprehensive course merely by throwing together a number of individual courses, however good each may be in itself. Some integration is necessary; and I should say that the problem of integration at the present stage of our development is about as severe as any we have to face.

Methods of Teaching

But to press on, let us suppose that we have some broad agreement about the scope of the syllabus which would best meet current needs. The problems then arise: can such a syllabus be covered by a teachable course; and if so who teaches it and where, and to whom? And if not, where and how does a man acquire this knowledge?

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Up to the present the main burden of such teaching as is carried on in this country has been borne either by a relatively few devoted members of this Society, or by local individuals who have become interested in operational research through their own disciplines and are self-educated in operational research itself. In this direction the Society has done much and perhaps could do more. The Education Committee under Mr. Gander's chairmanship spends quite a lot of time advising about syllabuses and trying to find lecturers to give courses. For the short and occasional course or for the odd conference this seems to me to be properly part of our functions, invidious as may be the task of criticizing syllabuses and onerous as may be the task of finding people who can cover them. But if we look ahead, this system will clearly not meet all the needs which may arise. A panel of didactic evangelists is all very well, but its members have their own jobs to do and we are in sight of the point when demand may far exceed supply. And of course, the system is quite inappropriate to a comprehensive course. I may remark in passing that the efficient teaching of operational research, like the teaching of any subject, is not an ability to be taken for granted. Those who lay immense emphasis on the importance of practical experience in operational research may care to be reminded that practical experience in teaching is also not without its value.

I suppose that another important factor during the formative stages of the subject was the willingness of some organizations to employ people who learnt while doing. A few organizations, in fact, are big enough to be able to lay on courses for their own personnel as part of their normal function. This provides and may well continue to provide a very useful adjunct to the training in operational research which is carried out in this country; but it is an uncontrollable variable, so far as we are concerned, and I do not think the existence of private tutoring will solve the problem of providing public teaching. Assuming, as I hope we may, that there is likely to continue a demand from individuals for instruction in operational research and a demand from industry for people with basic operational research training, where are we likely to see these demands met?

Let us list some of the possibilities:

(a) A systematic course could be provided at a university.

(b) Teaching institutions could collaborate to produce a set of courses which would cover the whole field.

(c) Industry could collaborate with teaching institutions for the provision of comprehensive courses.

(d) This Society could arrange for a comprehensive course to be taught by its members on its premises.

(e) Various institutions might take responsibility for particular courses and a student might cover them all by moving from one institution to another.

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This is a fairly short list. A corresponding list of objections would be far longer and I shall not attempt it. The problems of getting different universities to collaborate in joint instruction are not perhaps insuperable but they are very formidable. If this Society were to become a teaching institution our financial position, organization, and indeed our whole outlook, not to mention our tax position, would have to be very different. If in some way the work were to be parcelled out over a geographical area we should also have to look at the position from the point of view of the student, whether he is an undergraduate or a business executive sent by his firm on a training course. Students object to continual change of environment and I think they are probably right.

It seems to me that in present circumstances, if we want operational research taught up and down the country, there is only one really feasible solution to the problem. A few universities are sufficiently oriented towards business, technology and mathematics to be able to cover the syllabus if they chose. Most of them are in urban centres where technical institutes are also available. What I have in mind is that the basic instructional unit is not necessarily the university or the technical college, but the city. In each centre it ought to be possible by agreement among the teaching units—presumably under the auspices of the university if there is one—to provide a fairly comprehensive course by sharing out the work. The situation is not ideal but it seems to me to be workable. Each centre, if it preferred, could have its own qualifications; if universities were involved it would have to have.

A system like this could be made to overcome our other difficulty, that of providing practical instruction. A good many operational research practitioners in industry are insistent that true expertise is gained only in actual practice. I think myself that these claims can be exaggerated but there is no doubt that some practical experience is essential before an operational research man can be regarded as fully qualified. Given co-operation between the teaching centres and business organizations there is no reason why such experience should not be integrated into a teaching course. Conversely, there would be no reason why people already engaged in industry should not go to the teaching centres for instruction. It is felt

in some quarters, I know, that operational research is not a suitable university discipline *per se*; that men should be left to graduate in other disciplines as they do at present, go out into industry and then, if they develop the flair, return to the teaching centres for postgraduate training. This view also, in my opinion, can be overstressed. A university ought to teach what people want to learn, and if a young man of 18 wants to study operational research there seems no logical reason why he should not be allowed to go through a theoretical course on the subject. However, under the system I am advocating these difficulties or differences are resolved. If the teaching centres provide the instruction and the industrial centres provide the practising grounds it should be possible to meet the requirements both of the student who wants the practice and the practitioner who wants the study. I know that this envisages a greater degree of interflow between university and industry than we have yet achieved. But we seem to be approaching the time when it at last dawns on the English that university education need not stop at the age of twenty-three.

There is another way of achieving the basic object. It might be possible, at least in the earlier stages of development, to discourage attempts to teach operational research on an extensive scale in many different centres, and to concentrate resources in two or three conurbations. There are many attractions in this approach, especially to anyone like myself who already lives in one of the conurbations which would certainly be chosen. But we should have to recognize that under such a system men who wanted to do an operational research course would have to go and live for a time in the chosen centres. Students at many universities would be debarred from taking the course; and postgraduates would have to be prepared to leave home for a time. On the whole I do not think this is a very serious objection in a small country like ours; and it is thus worth considering whether we should not be wiser to concentrate instruction in one or two of the more progressive localities and leave it until later before we try to set up comprehensive courses up and down the country.

Even if such systems as I have described are practicable we are not yet out of the wood. Who pays for the work? The sort of teaching we want requires experienced and competent people who, however dedicated, cannot remain insensitive to the salaries which operational research men can command in industry. It is not to be done part-time by an assistant lecturer in mathematics or business administration. Moreover the fact, to which I have drawn attention earlier, that our subject straddles a number of disciplines, does not make things any easier. If senior teaching posts

were to be created in a university, under which faculty should they be? One has to have worked in a university to appreciate the tremendous gravity of such issues.

You have probably been expecting that at this point I should remark that if industry and commerce wants more operational research workers, industry and commerce should take some active steps to provide the funds. It is indeed true that industry could do a great deal to accelerate the growth of teaching operational research by agreeing with the teaching centres to finance it. But I think that this subject is not quite so simple as it looks. Demands on industry for money are increasing quite rapidly and it is not entirely fair to expect industry to support a particular branch of teaching even if the benefits are fairly plain and direct. Help on a particular project, as some firms have recently helped our Society in a critical phase of development, may reasonably be expected. That industry should permanently assume the burden of certain kinds of teaching seems to me to raise some very awkward issues of principle and I am far from sure that it would command academic approval.

A secondary problem which would give trouble in practice concerns the kind of diploma which should be conferred on those who have successfully been through the course. A university degree in operational research seems very far away. On the other hand, a man who successfully reads any subject for a substantial period at a university is entitled to a certificate of proficiency. We may then have to contemplate that for a time the subject should not be an undergraduate one at all. It could, however, have its university diploma; and for those who did not wish to submit to university examinations perhaps full membership of this Society would be adequate. The more one thinks about it, however, the more difficult does this problem of conferring an academic certificate of achievement appear. My present feeling is that we should try to produce the properly trained men and let paper qualifications look after themselves.

These then are our problems in the teaching of operational research. To decide what should be taught and how it should be taught; to decide which institutions should do the teaching and how the cost should be met; to decide what class of student should be taught and at what stage of his development he should receive the instruction. We have, I hope, advanced far enough to be able to ask the right kind of question. It will be some years before we find the answers, and we may have to find them by trial and error. This is, in fact, very much a field where operational research begins at home.

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Operational Research Today and Tomorrow*

A. W. SWAN

THIS is essentially a plea for simplicity, and for depth and breadth of thinking.

Simplicity

To illustrate the simple approach, I would like to begin with an investigation which was made during the War before I had heard of operational research, one which has been used by Sir Charles Goodeve in at least one lecture on this subject. I was sent by the Ministry of Supply to install statistical quality control in a factory making cartridge cases. The management seemed sceptical, but on my asking if they had any problems that worried them (my standard approach ever since, by the way) one of the managers said that he was puzzled as to the meaning, if any, which he could attach to the thousands of results relating to hardness logged by the inspectors. I asked for one of the log books and took a random sample of 100 consecutive readings. I set these out as a distribution, and what I found is shown in Figure 1. It is to be noted that, in order to pass an inspection, the hardness had to be at least 80, and they were rejected at 79 or lower. You will observe that there is a "cliff" at 80 hardness.

I told the manager that I had very bad news; that this distribution shape is impossible except when there is cheating, and that the inspectors were marking cartridge cases which were in reality of hardness 77, 78 and 79 as 80, 81, or more. He said that this was a very serious statement to make and asked if I stood behind it. I said "Yes"; investigations were made, and suitable disciplinary action was taken. This incident so impressed the management that it was an easy matter thereafter to go ahead and install statistical quality control.

The above example illustrates that the simple approach can be effective. It illustrates, too, a particular form of simple examination of numerical information, which I have used very extensively over the years, and which

* Paper given to the Operational Research Society on 24 April 1958.

still seems to me one of the first steps which should be taken by operational research workers in looking at numerical information, namely to set it out as a distribution curve and look at its shape. I have found, for instance, in connexion with time-study work, that it has sometimes been possible to

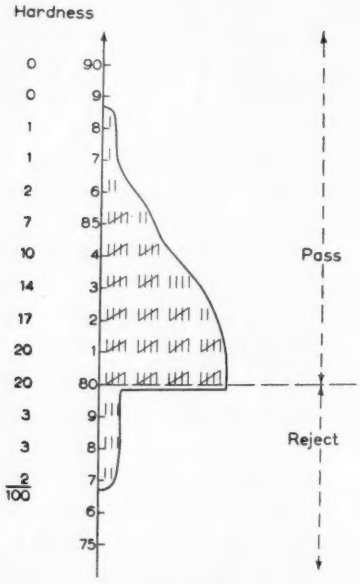

FIGURE 1.

secure sufficient information from tabular information, rearranged as distribution curves, to make subsequent mathematical analysis unnecessary.

As far as time-studies are concerned, one can classify distribution curves as bi-modal, single-mode symmetrical, single-mode non-symmetrical or "skew", and rectangular. The bi-modal distribution indicates that there are two sets of conditions, and suggests that it may be worth while finding out if studies are being made of learners together with fully trained operators. The single-mode symmetrical distribution indicates uniform practice, and the skew distribution represents uniform practice with occasional irregularity, or possibly the presence of a log-normal distribution, frequently found in time-studies in the textile industry.

In this connexion I once found that when machines were the principal factor in times the distributions were normal, whereas when human beings were the principal factor distributions were definitely log-normal, and I

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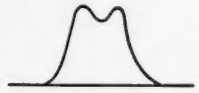
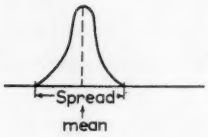
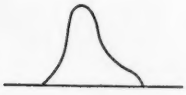
have wondered if this fact may be related to the unproved but interesting theory that human nervous reactions respond to stimuli in a logarithmic fashion. I observe a pithy description in a recent book on the log-normal distribution by Aitchison and Brown, that its genesis is similar to that of



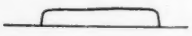
Bi-Modal (two humps) — Two populations; may indicate mixed times from learners and trained operators, etc.



Single Mode, Symmetrical — One population; uniform practice; use mid-point as measure of average and spread as measure of variability.



Single Mode, Skew — One population; may indicate uniform practice and occasional irregularity or log-normal distribution; investigate.



Rectangular — Special type of distribution; may indicate presence of many distributions; investigate.

FIGURE 2. *Distributions found in time-study.*

the normal distribution, small errors being multiplicative instead of additive.

Finally, we may have what tends to horrify the time-study man, the rectangular distribution. This may imply the presence of a highly special set of circumstances which will produce a single rectangular distribution, but it also may mean the presence of a large number of small distributions and, in effect, chaos. In my own work on time-study I have found on more than one occasion, and in connexion with such different processes as rolling steel and crane operation, that a rectangular distribution, for a particular process, could definitely be traced back to chaotic conditions at the process previous to the one being studied.

Tippett has always agreed with the idea that the first step in an enquiry is to set out distributions and examine them visually, and I was interested to find only a few weeks ago in discussion, that a young mathematician

who has made a considerable name for himself in operational research, agreed emphatically with the same procedure.

Developing this theme of simplicity, I would like once again to mention Tippett. I was telling him my ideas about the need to keep simplicity as a permanent aim, and complained a little about the tendency in the papers published in operational research journals, to discuss elaborate and sometimes very abstruse techniques. He said, "Well, 90 per cent of operational research is, in fact, based on quite simple methods, but these are not very interesting to write papers about, so one waits until a specially interesting, and perhaps exciting, job comes along, writes a paper and it is published". I am sure that this is true, and it may also be true that many industrialists, who are anxiously wondering whether operational research is going to be useful to them, obtain a false impression of the difficulty and complexity of operational research from the published papers and from reports, for instance, of conferences such as that which took place at Oxford last September

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Methods used in industrial operational research can perhaps be summarized as follows; although it is a summary the list is reasonably comprehensive.

Methods Used in Industrial Operational Research

Group 1. *Simple statistical techniques of very wide application*
- (a) Visual examination of distribution curves.
- (b) Quality control charts and statistical sampling.
- (c) Significant differences: *t*-test, analysis of variance; χ^2; etc.
- (d) Regression–correlation.

Group 2. *More difficult statistical techniques; very useful but not quite so often applicable to industrial problems*
- (a) Statistically planned experiments, including the Box method of "steepest ascent".
- (b) Queueing theory.
- (c) Monte Carlo simulation.

Group 3. *Mathematical and other methods of potential importance but not yet widely applied*
- (a) Linear programming.
- (b) Decision theory, communications theory, theory of games.
- (c) Symbolic logic.
- (d) Cybernetics.

All of Group 1 and Group 2 are based on probability theory and present-day statistical thinking, and they cover a high proportion of methods in current use. Group 3 tends towards pure mathematics and philosophy. G. W. Thomas, head of the Operational Research Department at the Steel Company of Wales, recently stated in a public lecture that statistics accounted for 90 per cent of operational research work. I believe this to be the case. Queueing theory is definitely based on statistical reasoning, and the Monte Carlo method, which is becoming in my view probably the most powerful tool at present available in operational research, is also based on probability theory.

Let us go back a bit and talk some more about the techniques mentioned in the above list. The quality control chart and its associated method, statistical sampling, are curiously absent from the literature on operational research, and yet Russell Ackoff said at the Oxford Conference that, "ingenuity in the design of optimal control procedure is likely to spring from an intimate knowledge of statistical methods, and a knowledge of the general philosophy and logic of statistical quality control, as well as of its techniques, is very helpful, if not essential, in designing effective control systems".

The statistically planned experiment has had a rather curious career in industry. It was thought in earlier days that it was going to be wonderful, but the fact is that it seldom can be applied in industry, because it nearly always means upsetting established conditions, and production managers will not agree to allow the more extensive variability which the planned experiment must always have. There is hope on the horizon, however, in Dr. Box's "steepest ascent" method, which many of us believe may well be as important a contribution to industrial operational research as any that have been made in the last few years, since it allows a cautious feeling of one's way, which does not upset production, but which is still sound statistically. It is odd, by the way, that we find mention of this method in the statistical journals but not in the operational research journals.

Depth

From simplicity let us pass to depth. The operational research worker, properly trained, has a special ability to probe to the heart of a problem. This faculty is not confined to the operational research worker, but as he must exercise it continually, he may attain special ability in this particular direction. An operational research job, which is now approaching completion at Courtaulds, illustrates this point. The operational research

department was asked to investigate the savings that would be possible if the number of twists in yarn were rationalized at one of the mills. An explanation of the problem is as follows. Rayon yarn, when made, is rewound from the original "package", as it is called, on to other packages, such as "cones" and "cheeses", and a twist is introduced to make the yarn stronger. The number of twists per inch may vary from $1\frac{1}{2}$ to about 40, and, at present, a very large number of different twists is available to the customer. The mill management felt that if this list were shortened the customer would still be as well off as before, provided the intervals were not too great, and the improvement to mill capacity would be considerable. When the operational research department had this request, it realized that an actual experiment with any system of twists, other than that in force at present, would be impossible, and that this would therefore be an ideal case for a simulation technique. A Monte Carlo model has therefore been constructed, based on a year's actual orders, representing five years (a) under the present system of twists, and (b) under a proposed system of rationalized twists. In this way, the benefits that should result from rationalization can be assessed. This, "the biggest aspidistra in the world", or at any rate one of the largest industrial Monte Carlo simulations to date, is being carried out, not surprisingly, on an electronic computer and uses the normal Monte Carlo techniques. The statistical and mathematical part is, however, the simplest part of the problem, and I do not propose to go into it here. The really difficult part has been to read the minds of the managers, and to put down on paper the rules by which they work, so that when the sequence of orders for a given week has been found by the Monte Carlo statistical techniques, they can be in simulation "put on the mill" precisely as would be done at present. This meant a patient cross-examination of the mill executives, leading to the tabulation of a highly complex set of rules, some of which had existed on paper before, but most of which had only been "intuitive", built up in the minds of those concerned over many years of experience. Although the operational research workers on this job had only been in the rayon industry for a few years, this was done to the surprised satisfaction of the mill management. An indication of the variety of conditions which have to be met is, for instance, the "geographical" requirement that two orders for white yarn must not be placed on adjacent spindles, whereas two orders for coloured yarns may be on adjacent spindles. This has been written-in to the computer programme. A short extract from a part of a computer sub-programme illustrates the complexity of the situations analysed.

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Part of Sub-programme for Computer

The machine word shows:

A = 1. Order is for white yarn

Denier (weight in grammes of 9,000 metres of yarn) = Code 3

Number of filaments = Code 4

(Note: yarn is specified as, say 150/40, e.g. 150 denier/40 filaments)

Type of yarn = Code 2

T = Running time of average package (e.g. cone) = 20 hours

Y = Machine compatibility = Code 8

Type of bobbin = 88

TM = 96, e.g. this week the machine is occupied until the 96th hour.

Breadth

Let us now pass on to breadth in connexion with operational research. I have urged that we should not neglect the simple tools which are available to us, and, above all, we should not look down on them. This does not mean, however, that we should neglect the more advanced methods which are becoming available, and there are, certainly, exciting possibilities. Linear programming seems to be in a rather curious intermediate stage as far as application to industry is concerned. It has been much written about, and yet actual applications to industry are still far from numerous, and difficulties of application are very severe. I have it on good authority that the linear programming model is often too simple a representation of the industrial problem one is trying to solve.

Decision theory, communications theory and the theory of games are still in the development stage. Communications theory has been reduced to a mathematical form, and a number of interesting experiments has been made in regard to the efficiency of transmission of information through different arrangements of a group of people. It is at least possible that this type of theory may have highly practical applications in the design of control schemes, checking, for example, the "span of control" conception that no executive should have more than five people working directly for him. This is an interesting potential field for operational research.

The theory of games seems to be much more difficult to apply to industry because of the complexity of the human emotions involved. It will be interesting to see how Ackoff succeeds in his experiment, in which he is applying the theory of games to an actual industrial situation. The psychological aspect of operational research, the relationship between the

operational research group and those among whom it works, is one which has to be studied by each working group, whether it wishes to do so or not. Research, or at least exchange of ideas on a wider basis, is needed.

Cybernetics is in some quarters dismissed as airy-fairy nonsense. Let us not be so unwise. Whether or not the analogue of the human nervous system to industrial systems of control is exact remains to be proved—analogy can be dangerous if pushed too far—but the principles of feedback, which play their part in cybernetics thinking, are becoming more and more important in automatic control of plant, and may well be developed in connexion with overall managerial control. Automatic management may be just round the corner, a rather frightening thought.

I have mentioned symbolic logic. Anyone who has to do with computers, and computers are coming to play an increasingly important part in operational research, has to learn at least the rudiments of the logic of computer design, and he is not harmed if he goes on to learn something of what is called Boolean Algebra, to which there is a first-class, quick introduction in *Faster than Thought*.[1] Logic, in its more usually accepted sense as the basis of argument, is not to be despised as auxiliary reading for the operational research worker. He is very much concerned with making his reports clear and self-evident. A magnificent example of the self-evident statement is the couplet by the Scottish poet, William McGonagall:

> "On yonder hill there stands a coo,
> If she's awa she's no there the noo".

A valuable book I would like to commend as a stimulating sidelight to operational research is *A Preface to Logic* by Morris R. Cohen.[2] The title is not really correct, as the book is a very interesting series of essays mainly on the effect of probability theory, and the statistical view of nature, on formal logic, which instead of "Yes–No" is becoming"Yes–No–Perhaps". An earlier writer on symbolic logic was the Reverend C. L. Dodgson, who also wrote a book on determinants with which this audience is doubtless familiar. One of my valued possessions is a set of exercises in logic set by Mr. Dodgson. These problems give a series of premises and one is asked to find the proper conclusion. These are real enough problems in logic and quite difficult, but the solutions are typical of the author in his other personality. Number 41 is as follows:

Problem

(1) All the Eton men in this College play cricket.

(2) None but the Scholars dines at the high table.

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(3) None of the cricketers rows.
(4) My friends in this College all come from Eton.
(5) All the Scholars are rowing-men.

Conclusion

None of my friends dines at the high table. (A graphical solution is shown in Figure 3.)

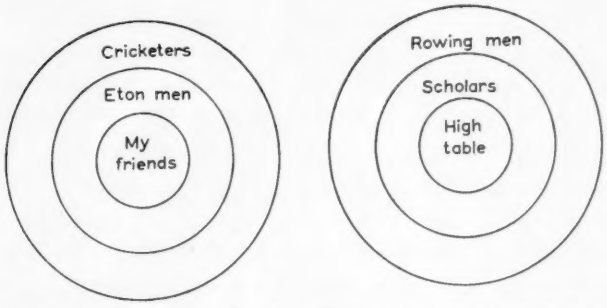

FIGURE 3. *Euler diagram solution to problem in logic* (*Lewis Carroll No.* 41).[3]

The above may be regarded as a slightly frivolous digression, but the purpose is serious. The operational research worker must be clear not only in his thinking, but in the expression of his thoughts. He has to present clear reports, and in this respect I would very much like the Society to publish Barclay Hankin's very interesting paper, which was presented to the Oxford Conference and served as a basis for one of the panel discussions, "The Communication of the Results of Operational Research to the Makers of Policy".

I have from time to time said in papers on organization of operational research, that reports must be written in plain English without mathematical jargon, the mathematics being left to appendices. Reports should start with a summary in the form of object, results, and recommendations. In this way, the works or office executive who has asked for help is given the answers to his questions at the beginning of the report. Ancillary information on methods adopted in the investigation should follow on. Hankin goes much further. He says, "the aim must be to get the results and conclusions studied, respected, and understood. The success of operational research must in the end be judged by the influence that it has upon the making of policy". He proceeds to discuss the problem of communication between operational research and management in some

detail, advocating the circulation of a draft report so that "the user department is warned in advance, and misunderstandings or wrong emphasis can be cleared up". Many operational research departments have adopted this practice for years, including Courtaulds, but it still needs to be said, and said emphatically. Hankin follows with useful suggestions about the final report, with suggestions on methods of reproduction, use of photographs and diagrams, etc. He makes the interesting suggestion that successful operational research must contain an essential backbone of science but must also contain a considerable measure of the arts. I can confirm this dictum from my own experience. Graphs may play an important part in operational research reports, and a knowledge of the special types of graph paper such as logarithmic, probability, reciprocal, etc. and of the rules for size of lettering, etc. are valuable assets to an operational research staff. Just to illustrate the last point, most draughtsmen, but not most operational research mathematicians, physicists, etc. are taught that if a drawing is made on a large sheet of paper to be reduced photographically for ultimate reproduction as, say, foolscap size, the lettering must be drawn so that it appears to be disproportionately large on the original, so that it will reduce to a size convenient for reading on the smaller reproduction.

Another aspect of breadth of view concerns what is, and what is not, an operational research job. My own view is that there is unnecessary fuss about this matter of definition. An operational research department is there to help on any problem which bothers management. I have said earlier in this talk that we should not look down our noses at the simpler techniques, neither should we be haughty about simple jobs. Every operational research department worth its salt receives requests, usually by telephone or by personal call, to solve relatively minor problems. It does them, and does them willingly, and they are still operational research because they are being done by operational research workers who willy-nilly bring to bear their special training and point of view on any problem. So long as one is not actually overlapping the work done by another existing department, any job should be tackled, however small or large it may be, and whether or not it seems to be the type of problem suitable for mathematical analysis, which is normally considered to be operational research.

I have worked to this point of view for several years with very interesting, and I hope, useful results. Until the past few months there was at Courtaulds no O. and M. department. Various investigations of an

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O. and M. type, therefore, came our way and we tackled them without worrying about the fact that they were not necessarily of the mathematical type. The analytical sense, which the operational research worker must have, has been applied successfully to simplification of laboratory and works procedures.

Now that there is an O. and M. department, the operational research department will contribute its own special point of view in connexion with production planning and stock control. This will be done on the sensible basis of a joint panel set up for the purpose.

Industrial operational research seems to be tending more and more to the economic type of problem, such as the relationship of sales, production and stocks; optimum length of run for machines; etc., where the problem is usually to find the best balance among various types of cost. Here the operational research workers are associated with economists, if there are economists, as such, in the organization, or, if not, they read econometrics and provide the knowledge from inside the department. There is always a cost accountants' department, and the operational research department should not attempt to overlap but form joint working parties with the accountants on specific problems.

When we come to work-study, the relationship to operational research is not so easy to define. At Courtaulds there has been no difficulty as the work-study department exists essentially to put in and maintain a bonus incentive system among a number of employees running into thousands, and this is a task that keeps it so fully occupied that it has little time for fundamental research in its own field. There has been, and is, a happy and sensible arrangement whereby work-study call in operational research to help in studies such as machine interference, etc. However, some work-study departments elsewhere hold rather different views about their functions, and their relationship to operational research is not so simple to assess. Is the work-study point of view in general, however, really progressive; does it sufficiently take into account the arrival of new techniques for the analysis of data?

I should be happy to be corrected, but my personal impression is that the thinking on time-study, at any rate, appears to be somewhat static. Very few work-study practitioners either in this country, or in America, think of time-study in terms of variability of the job, of the operator, or of the time-study man's rating estimate, and there is, therefore, little investigation of a statistical type. They seem anxious to link everything to fixed standards; standard element times, standard conditions, standard

ratings by the time-study man. It seems to be significant that the people who are not so happy about all this exactness, and who have applied statistical thinking, are all operational research workers or industrial statisticians, such as Winston Rodgers and Desmond.

The Steel Company of Wales has no work-study department, and when the operational research department has time-studies to do, it puts its own assistants on the job. At a recent lecture, the head of that department was asked if he did not think an experienced work-study man could teach his department a thing or two. His reply was that it could be the other way round. Perhaps it might.

It is odd that there should be a freezing of knowledge in the field of work-study, once called production engineering, since the pioneers, Taylor, Gantt and Gilbreth, as Norton has often told us, were among the real pioneers of operational research. Norton is an admirer of Taylor, and without decrying Taylor's work, I hope I may be allowed a slight digression about Gantt, and a small bit of autobiography. My first job as a young graduate engineer was to assist a former pupil of Gantt in installing the Gantt system of bonus payment and plant management at the works of the Canadian Ingersoll-Rand Company. This involved time-study, use of the Gantt chart to control production, and a variety of other jobs, including the laying out of a new drill-shop, and the design of a special slide-rule to cut down the calculation of times for a tread-milling machine. Later I applied the same principles as production engineer in a works at Dagenham, and was interested to find that the main obstacle to installing graphical control of production through the Gantt chart, which is to this day a useful device, was the British insistence on the shop-chaser system, a pernicious system since it relies on individuals and not on central thinking. On one occasion I was asked to design a simple device to be used by the clerks on bonus payment allocation, to cut out the mental work in an arithmetical calculation which they made many times a day. The first solution produced was a chart in the form of a family of curves, but this was not accepted on the sound basis that while it would do for a skilled person, it would not do for the unskilled, and I was told to produce something else. The final and accepted solution was a simple, special slide-rule, the main features of which were that the answer could only appear at one place under a clearly marked pointer, and this answer was in sterling so that the calculation was self-contained and complete.

Subsequently I worked out a nomograph for the same calculation, but my view was then, and still is, that the special slide-rule is a better device

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than the corresponding nomograph when intended for non-skilled use. A special slide-rule can be made almost error-free by arranging to have the answers appear under specific, fiducial printed pointers. In the operation of a multiple nomograph one is always liable to make a slight error, and this may build up disastrously if there are several stages.

There are several books about nomographs but very little has been written about special slide-rules. A partial filling of this gap is an excellent series of papers which have appeared recently in *The Mechanical World*, January, February and March, 1958, by R. K. Allan.

It might seem from the above that I am about to say that operational research is so wide in its scope that it includes every aspect of modern managerial control. This is not the case. Operational research can help the architect in layout, the engineer in design, the physicist and chemist in research, the accountant and the manager in control, but the operative word is "help". The operational research worker is essentially an investigator; he has much in common with the pure research worker, and this research temperament seems to differ from that of the industrial executive who has to manage, and go on managing, day in day out. This is a generalization, and there is a definition that all generalizations are false including this one. I have known at least one striking exception to my own generalization; one of my former assistants, a first-class statistician and operational research worker, is now a successful industrial magnate. The statement is still, however, true, subject to the occasional exception.

Recapitulation

It is not possible, in a single talk, to cover every aspect of operational research as it is today and as it will be in the future, and I do not propose to try. I will recapitulate my main points. First, I have made a strong plea for simplicity in the general approach to operational research problems, for the use of simple methods as a first step prior to the use of more complex methods, and for a willingness to undertake simple problems, not rejecting these as unworthy of operational research. I then mentioned, briefly, the matter of depth of thinking, and suggested that this has received little formal examination as a part of the operational research field and should be explored. The processes of deep thinking are not well understood and deserve study. In regard to breadth of vision, I suggested that the range of techniques is continuously expanding, and that we must, while not neglecting the simple method, be ready to accept

the presently fantastic as the future practical. I suggested finally that there is little limit to the possibilities of operational research as a consulting aid to management, but that it is, essentially, a consulting aid and not management itself.

ACKNOWLEDGEMENT

The author wishes to thank Messrs. Courtaulds Limited for permission to publish the information given in this paper relating to the activities of that company.

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The Communication of the Results of Operational Research to the Makers of Policy*

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MICHAEL Faraday said that there were three steps in useful research; the first to begin it, the second to end it and the third to publish it. In operational research I suggest that to publish it is not nearly enough. In the broadest possible sense of the word we have to "*communicate*" it. The aim must be to get the results and conclusions studied, respected, understood and fully considered in the formulation of policy. I suggest that the success of operational research must in the end be judged by the influence that it has upon the making of sound policy. If this is true it is very important indeed because it means that the duty of all operational research workers is not only to use scientific methods to analyse a situation or to deduce a most suitable line of policy. In subtle ways they must also use every part of their work to influence the making of policy without treading on the toes of those responsible for it. Some scientists may regard this as unethical, but what is the use of their work unless they take all permissible steps to make it effective? In this approach lies the philosophy behind this paper. It is one of the reasons why operational research is so fascinating and is the main reason why I consider operational research to be quite as much an art as it is a science.

Factors Affecting the Acceptance of Operational Research Results

The table which follows analyses the principal factors which I consider may contribute to the acceptance of the results of operational research at management level.

From the analysis contained in Table 1 I draw the important conclusion that the acceptance of the results of a piece of operational research does not begin at the report stage. The process starts at the beginning of the project and is greatly affected by all previous work.

* Paper used to open a discussion at the International Conference on Operational Research at Oxford in September, 1957.

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TABLE 1

PRINCIPAL FACTORS AFFECTING THE ACCEPTANCE OF OPERATIONAL RESEARCH RESULTS

Principal Factors affecting Acceptance by Management		Supporting Details	Lessons for Operational Research Section
Reputation of the operational research section		Sincerity Impartiality No desire for personal kudos Square, honest co-operative dealing Thoroughness Reliability Accuracy Usefulness	Choice of operational research personnel of suitable character and quality at all levels. They all contribute to the reputation produced by past and present work
Confidence in the work and methods used in a particular project		A conviction is necessary that the methods used are likely to have produced reliable results and are going to add something useful to the knowledge or understanding already available	Nothing less than the most enthusiastic, well planned and thorough examination possible with the time and staff available. Visits by appropriate members of customer dept. to observe research in action. Co-operation in the project by the department involved who will feed back impressions of how the work is undertaken
Some prior knowledge of results of the particular project		New information is often more readily accepted and understood if it is fed in gradually	Participation in the work by the user department where possible. Feeding in and discussion of results and drafts prior to final report
Methods of presenting results	Any information	Simplicity Clarity Well argued and logical Capable of speedy absorption Attractively presented	An important part of the work of all but the most junior operational research personnel. Both arts and science are equally involved.
	Unpleasant information	Tact Recipient should be helped to avoid losing face or appearing foolish Gradual and early information to avoid shock and to give opportunity to put it right	

Effect of Stages leading up to the Report

I think it will be readily accepted that the reputation of an operational research section is based on past work and that this certainly has a bearing on the acceptance of the results of any new work. My contention is that nearly every detail in the execution of any new project also contributes to the ease with which the results can be communicated and to the chance that they will be accepted. I suggest that the following are important.

(a) *Terms of Reference*

These need very careful study and drafting, usually by operational research personnel in conjunction with management. Well chosen terms of reference help to get a good job done in a reasonable time. They help to steer the operational research approach and to prevent management saying at a later stage, "You have not done what I wanted, your results are of little interest".

(b) *Agreement on the Time Available*

It is important both to management and to the operational research section, to reach mutual agreement on the time scale in mind before a project starts. Most work can take years to complete if it is done extremely thoroughly or if it has only a small effort allocated to it. On the other hand, management may be better satisfied by something a little less complete, carried out much more quickly. The speed with which a project can be completed, and the meeting of an agreed date has an important effect on the interest with which management will study the results and on the chances of their being accepted.

(c) *Building up Confidence*

Every stage of the work must be directed towards building up confidence in the accuracy, impartiality and value of the results which eventually come out. In my own organization certain projects are tackled by committees whose chairman and secretary are operational research men, and whose members are from the departments who are affected by the particular project. The bulk of the work must be done by research staff assisted by suitable observers who come under their supervision. The departmental members are useful in making arrangements for research within their departments, for expressing departmental viewpoints and for conveying information and confidence to their departments as the work proceeds. This system may slow down the work slightly, but it can be very valuable in getting the results speedily accepted.

There are many other ways of assisting in building up confidence. I will mention only a few:

(i) Enthusiasm, efficiency and thoroughness on the part of all concerned in the research.

(ii) Preparedness to "get dirty", to really understand and share the lives and hardships of human beings who may be involved in aspects of the study. (To study soldiers you must live with them. To study bus conductors it is best to begin by doing their job, if only for a relatively short time.)

(iii) Reaching the stage where it is accepted that research staff know more about many aspects of the problem than anyone else.

(d) *Planning for Intelligibility and Acceptance*

The operational researcher is constantly faced with decisions as to how he should plan his data collection or his analysis. How should he classify something? What measures or standards should he adopt? He should constantly bear in mind that his methods must be practical, realistic and as simple and intelligible as possible to those who will study his report. In this way the planning and execution of his work can assist in getting it accepted. I would not rate his skill by the length of his formulae or the scientific jargon he displays. What counts is that he should get results which really matter and see that they are understood.

(e) *Wooing those in Responsible Positions*

The responsibility for accepting the results or recommendations of a report is often in the hands of a small number of people and their advisers. Throughout a piece of operational research it is clearly wise to study the mentality, views and reactions of these individuals, with the object of securing their interest and co-operation, and thus preparing them to accept the results of the work. If any part of the data collection phase is of interest to them, it is sometimes useful to invite them to pay a visit to see it being carried out.

(f) *Circulation of Draft Report*

The circulation and discussion of a draft report is a most valuable way to assist in "communicating" it and getting it accepted. The user department is warned of the results in advance and can start to act on them if it wishes. The operational research section should make the draft as near perfect as possible, but if there is misunderstanding, or if there is wrong emphasis, it can be discussed and cleared up. The conclusions and recommendations can be debated and challenged without either side losing face.

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EXAMPLES OF THE USE OF PHOTOGRAPHS IN OPERATIONAL RESEARCH REPORTS

Points to Remember

(a) A photograph can convey much information in a quick and interesting manner.
(b) A photograph can reduce the length of a report and save the time of the reader.
(c) A photograph may convince, where a paragraph of text may not.
(d) A photograph can suggest things which it may not be possible to mention specifically in the text.

Figure 1. *From a report on "The Economics of Telephone Services". Descriptive of automatic apparatus. Assists in emphasizing that skilled maintenance is a necessity, and must be allowed for in considering the advantages of the automatic systems.*

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FIGURE 2. *From a report on "The Economics of Telephone Services". Descriptive of manual system. Human interest for report. Suggests a large number of operators (whose salaries form a large part of annual running costs).*

FIGURE 3. *From a report on "Matters Affecting Escalators". Captures a moving situation for subsequent study. Adds support to statement in text that passengers all look down when getting on escalators and hence the need for adequate lighting.*

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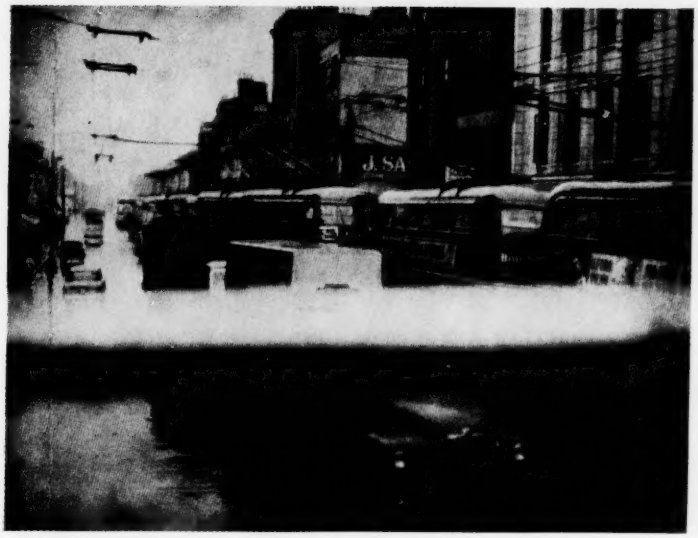

FIGURE 4. *From a report on "The Study of the Regularity of a Bus Service". Congestion in Seven Sisters Road, London, taken from a No. 14 bus. Provides striking visual evidence in support of factual measurements, thus giving reality and meaning to figures.*

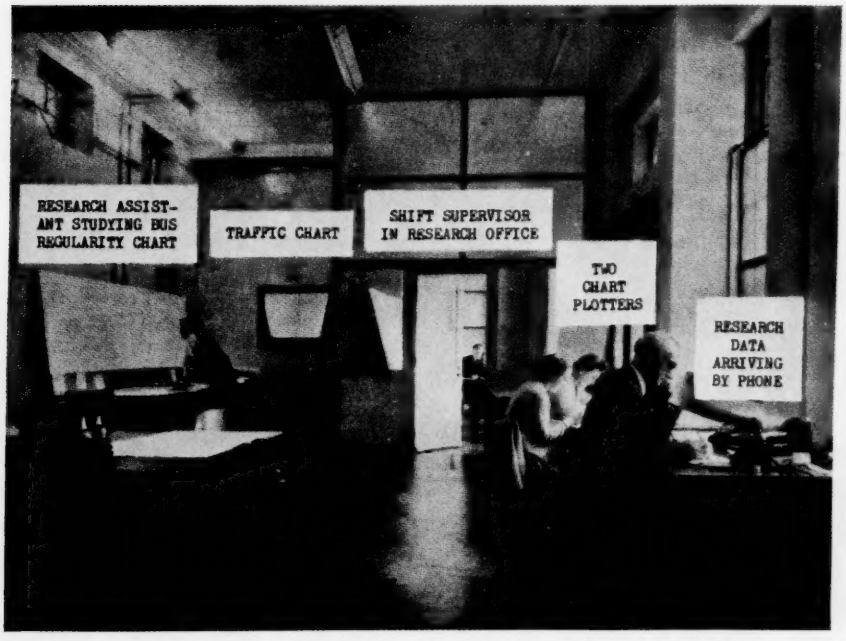

FIGURE 5. *From a report on "The Study of the Regularity of a Bus Service". Annotated photograph is descriptive in a visual manner. Reduces necessity for long descriptions in the text and saves time of the reader.*

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Use of a Report for "Communicating" Results

I have purposely given considerable emphasis to the stages prior to a report, because I think their importance from the point of view of "communication" is not well understood by some operational researchers. The presentation of a good report is also a very important part of any operational research.* It requires much imagination and skill and far too little attention is given to it by many workers.

Deciding on the Objectives

Before writing an operational research report the author should clarify his objects, which are frequently as follows:

(a) To "communicate" the results of work to policy makers and to stimulate action if recommendations have been included.
(b) To inform others who may be interested.
(c) To provide a tidy record of scientific data and method for easy reference.

Every report should be considered on its merits which should be adjusted to suit the particular objects in view. Many authors would improve their reports out of all recognition if they would apply the following dictum:

> "The material in the main part of an operational research report should be severely restricted. Without failing in its objects, it should minimize by all available means the time and mental effort demanded from the policy maker. Everything else which may be necessary should be relegated ruthlessly to appendices."

Factors Affecting Good Communication by Means of a Report

If the time and mental effort demanded from the reader of a report is to be reduced to a minimum, it is worth examining the factors which affect them. They depend on a complex interaction of many effects which include the following:

(a) Pre-knowledge of the subject by the reader.
(b) The intelligence, quickness and freshness of the reader.
(c) The quantity of information to be communicated.

* It is of interest to note that some operational research workers are experimenting with methods of presenting results to management in a "presentation session", thus reducing the labours of report writing and report reading.

(d) The ability of the report to foster or maintain the interest of the reader.
(e) The method or combination of methods chosen for communicating each part of the information (e.g., prose, table, diagram, photograph, etc.).
(f) The skill with which the chosen methods are used (layout and visual appeal, style, selection of material, clarity, etc.).

It can be assumed that at this stage the author cannot do very much about (a) or (b), and that in his mind he has already reduced (c) to the minimum. He is then left to do his best with (d), (e) and (f). In tackling these three aspects of his problem he must draw heavily on his imagination and artistic experience, and rather less on his scientific training. It is also apparent that the imaginative and artistic skills required are similar to many of those commonly used by authors, journalists, advertisers, schoolmasters and artists. Operational researchers should therefore study these professions and borrow from them everything which may be of use to them.

Methods for Communicating Information in a Report

In communicating information in a report the author has four principal methods to choose from:

(a) Words in the form of prose. (Or slightly simplified versions, such as notes.)
(b) Tabulated information.
(c) Diagrams and charts.
(d) Photographs.

It is common experience that information can frequently be communicated more efficiently (i.e. understood in a shorter time and with less effort on the part of the reader), by some form of visual presentation rather than by the use of words.

It is also considered by many people that information acquired from a visual presentation is more easily remembered than when it is received through the medium of the printed word. These statements suggest the need to exploit visual aids to the full in the presentation of our reports.

Skilful Use of the Chosen Methods

It would be quite impracticable to attempt to describe all the skills connected with the presentation of material. I will mention only a few:

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(a) *Style of Writing*

Good English is a foundation for any good writing, but I venture to suggest that an operational research report needs a style very different from that usually demanded in a work of high literary merit. The former will probably be "skimmed" by a very busy man in an office, whilst the latter is intended for pleasure in an armchair at home. I think that we have much to learn from the concise, crisp style of writing taught to military Staff Officers. The technique is based on the demands and experience of battle where it is important to save time and words, to be absolutely clear, and to communicate with very busy people who may be extremely tired.

(b) *Layout, Emphasis and Visual Appeal*

Aesthetic satisfaction in the visual sense is produced by a subtle interrelationship between the shapes, sizes, colours and textures of the component parts forming a whole. Emphasis can be produced by suitable arrangements of shape and colour in relation to the background and adjoining areas. Examples of this can be seen in the "appointments vacant" pages in newspapers and periodicals; the modern tendency is to buy more advertisement space than is necessary, in order to attract attention by leaving some of it empty. Good layout and good visual appeal have an important effect on the communication of information.

(c) *Paper and Print*

Some operational research sections are forced to try to save a pound or two per report by using cheap paper. This may be a false economy by virtue of the bad effect it produces on the readers. The texture, quality and colour of paper and the spacing and variety of the print can assist in "readability" and in variation of emphasis. I recommend any operational research section who has not yet done so to give consideration to the advantages of machines such as the Vari-typer which is a composing typewriter which gives a variety of changeable type sizes and styles of type with square "justified" margins.

(d) *Use of Photographs*

There are many ways of using suitable photographs to illustrate and add interest to a report. A few examples are given at the end of this paper.

(e) *Methods of Reproduction*

There are now excellent methods for quickly and cheaply reproducing typewritten material, diagrams, photographs, maps, etc. They include very satisfactory systems of enlarging or reducing. A working knowledge of

what can be achieved is most helpful in planning the report from the point of view of "communication".

The Balance Between Science and the Arts in Operational Research Personnel

I hope it has become clear that in my opinion scientific qualifications alone will not carry a man far in operational research. Even if work is of excellent scientific quality, it is of very limited value if it fails to "communicate". I have tried to indicate some of the ways in which the chances of successful "communication" can be improved, and to show that nearly all of them depend on qualities other than science. It depends in the end on the people we employ, on the broadness of their education and experience and on their ability to draw upon the arts as well as upon science. In my view this has a considerable bearing on the selection and training of operational research staffs.

Conclusions

(a) Operational researchers should regard it as their duty to use *every* part of their work to take all permissible steps to influence the making of policy. (They should of course be extremely careful to avoid treading on the toes of those responsible for it.)

(b) The process of communication of the results of operational research should start at the beginning of a project. The ease with which it can be set in motion will be affected by all previous work.

(c) The principal factors affecting the success with which communication is achieved are:

(i) The character and quality of operational research staff at all levels.

(ii) The reputation produced by their personal contacts and by their past work.

(iii) The way a particular project has been handled (in order to prepare the way for acceptance).

(iv) The way the results are presented.

(d) The preparation of a report which "communicates" well is an imaginative and artistic exercise requiring considerable skill and experience. Operational researchers can learn much from authors, journalists, advertisers, schoolmasters and artists.

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(e) Successful operational research as a whole must contain an essential backbone of science but must also contain a considerable measure of the arts. This has an important bearing on the selection and training of operational research staffs.

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WHICH MAY BE USEFUL ON THE COMMUNICATION OF RESULTS

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Road Research

THE following eight summaries refer to a selection of the work undertaken by the Road Research Laboratory. They are reproduced by permission of the Director of Road Research, Road Research Laboratory, Harmondsworth, Middlesex, to whom anyone interested should refer for further information.

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Speeds of Private Cars before and after the Installation of Street Lighting on the Bath Road (A.4) near London Airport, Middlesex
RN/2945/AEW.SAH

Vehicle speeds were measured before and after the installation of street lighting on the London–Bath–Bristol trunk road (A.4) near London Airport in the Urban District of Yiewsley and West Drayton (Middlesex). The lighting is a modern Group A aero-screened high-pressure mercury-vapour installation. Simultaneous speed measurements were made on the same road at a control site where there is no street lighting.

At the test site the mean speed of eastbound private cars increased from 41·0 mile/hr before lighting to 44·8 mile/hr after lighting and for westbound private cars the increase was from 39·6 mile/hr to 41·8 mile/hr. At the control site the mean speeds of private cars before and after the lighting was installed at the test site were 43·5 and 44·1 mile/hr for eastbound cars and 43·0 and 43·2 mile/hr for westbound cars. The difference between the changes at the two sites is statistically significant both for westbound and for eastbound cars. It is concluded, therefore, that there was a real increase in the mean speed of private cars due to the improvement in lighting. The actual magnitude of the increase cannot be estimated with precision but appears to have been of the order of 2 to 3 mile/hr.

At the test site, the percentage of eastbound cars travelling at speeds greater than 50 mile/hr increased from 18 to 32 per cent, and of westbound cars from 12 to 20 per cent. At the control site, roughly 23 per cent of private cars travelled at speeds greater than 50 mile/hr on both occasions.

Before the lighting was installed at the test site 95 per cent of drivers used headlamps or passlamps, compared with 15 per cent afterwards. At the control site 98 per cent of drivers used headlamps or passlamps.

A Comparison of the Accident Rates of Pedestrians on and within 50 yards of Zebra Crossings
RN/2952/SJO

In an attempt to investigate whether zebra crossings increase pedestrian safety, counts have been made of the numbers of pedestrians crossing the road on 25 crossings in the London area and also of the numbers crossing within 50 yd of them. These figures have been compared with the numbers of reported pedestrian accidents in these situations. It was found that 67 per cent of the pedestrians who crossed the road within 50 yd of a crossing did so at the crossing itself. Only 53 per cent of the accidents, however, occurred at the crossing itself. It is therefore concluded that the risk to a pedestrian who uses a crossing is only about 50 per cent of the risk he runs off the crossing but within 50 yd of it.

The relative numbers of accidents on and within 50 yd of all the crossings investigated are similar to those elsewhere in the Metropolitan Police District.

Accidents before and after the Provision of Roundabouts and the Staggering of Crossroads
RN/2976/JCT

The note describes an analysis of accident frequencies before and after the staggering of eleven crossroads and the provision of roundabouts at eleven others.

A previous investigation of this kind showed that both staggering and the provision of roundabouts tended to reduce very considerably the frequency of personal-injury accidents. The results from these further sites show the same tendency. The following conclusions are, as far as possible, based on all the data from both investigations:

The staggering of crossroads reduces injury accidents by about 60 per cent on the average. The reduction is greater for the more serious accidents: fatal and serious were 90 per cent fewer, while fatal accidents decreased from 8 before to none after. Damage accidents, and also slight accidents by themselves, decreased only by about 30 per cent, and it is possible for changes of this size to have arisen by chance.

The average cost of the staggering schemes was about £5,000. The money cost of the accidents which these schemes saved each year represents a return of about 15 per cent on this capital expenditure.

The roundabouts that were provided have been classified into those in which a circular island of at least 100-ft diameter was provided, and the rest, where the site either did not permit or did not require use of the standard designs. There was no evidence of any change in the severity of accidents, the overall reductions being about 70 per cent for the first group and 40 per cent for the second.

The average capital cost of the first group of roundabouts was £15,000, while that of the second group was about £6,000. The annual rate of return represented by the saving on accident costs was 6 per cent for the first group and 12 per cent for the second.

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The Effect of a Police Radar Check on Vehicle Speeds in Slough
RN/3070/DHB

In connexion with one of the periodic speed checks on the London–Bath–Bristol trunk road (A.4), east of Slough, which are made by the Slough police using a radar speedmeter, vehicle speeds were measured about half-a-mile before and after the police check point. Speeds were also measured when warning boards were displayed but there were, in fact, no police present, and again when conditions were normal.

While the police speed check was in operation there was a reduction of about 1½ mile/hr in the average speed of vehicles after they had passed the check point and also a reduction in the number of vehicles exceeding the speed limit. The display of warning boards alone on the day following the police check had much the same effect as the police check itself.

Accidents before and after Street Lighting Improvements
RN/3099/JCT

A study has been made of accident frequencies before and after the provision or improvement of street lighting at five roundabouts and on 52 lengths of road, or groups of roads. The following conclusions are based on this information, combined where possible with similar data for other roads given in earlier notes.

(i) There were too few accidents at the roundabouts to show whether the lighting was effective in reducing their frequency, although the indications were favourable.

(ii) On the lengths of road, it is estimated that the frequency of injury accidents in darkness was reduced on the average by 30 per cent.

(iii) There is fairly strong evidence that the reduction was greater for pedestrian accidents than for other types of injury accident.

(iv) There is no firm evidence that the reductions were different for accidents of different severities; the apparent reductions, however, were greater for the more serious accidents.

(v) There were no significant differences between the accident reductions for fluorescent, mercury and sodium installations.

Driver Behaviour at Roundabouts.* (1) *Western Circus, East Acton (Middlesex)
RN*/3122/*JAR

The behaviour of drivers using the roundabout at Western Circus, East Acton (Middlesex), has been studied to find what code of precedence is used by drivers. The following conclusions were reached for conditions at this roundabout:

(1) Vehicles approaching by the less important road were much more likely to give way to vehicles on the roundabout than were vehicles using the trunk road.

(2) There was a marked tendency among both private and commercial vehicles for smaller vehicles to give way to larger vehicles.

(3) There appeared to be no relationship between the speed at which a driver approached a roundabout and his subsequent behaviour at the roundabout.

The Effect of Road Conditions on Vehicle Running Costs: A Pilot Test between London and Nuneaton
RN*/3146/*GJR

To assess some of the economic benefits to be derived from new roads, measurements were made from which certain running costs of vehicles were estimated. With the aid of new apparatus, very detailed information was obtained of the time spent and the fuel used along a 91-mile stretch of the London–Holyhead trunk road (A.5) between London and Nuneaton. Three drivers and two vehicles were employed: one a six-cylinder and the other a four-cylinder car. Comparisons were made between different sections of this route, and also between the route as a whole and a test track, where unimpeded runs at constant speeds were made.

The results showed that on A.5 the average speed with the large car was about 38 mile/hr and with the small car about 36 mile/hr. At these

speeds fuel consumptions were equivalent to those at 60 mile/hr and 50 mile/hr, respectively, on the test track. The faster drivers used more petrol, an additional half gallon being consumed for every ten minutes saved.

In country areas, gradients up to 5 per cent had little effect on the factors studied. Journey time, brake and clutch applications all increased markedly on 6 per cent gradients and, in the case of the six-cylinder car, fuel consumption also increased on these gradients.

Although journey speeds were appreciably lower, and movements of brake and clutch much more frequent in the town than in the country sections of A.5, fuel consumptions in these sections differed little, being the same for the large car and a little less in town for the small car.

The Frequency of Faults found in Private Cars of Different Ages at the Vehicle Testing Centre of the Ministry of Transport and Civil Aviation RN/3166/JA

The note deals with data of cars of nine of the principal makes which were tested at the Vehicle Testing Station at Hendon during the nine months January to September (inclusive) 1956. The number of faults per car has been plotted against its year of manufacture to show trends with age. The proportions of cars with particular faults are also given.

There is a general trend for the number of faults to increase with age, but there are some marked exceptions.

(i) Pre-war cars have fewer faults than would be expected from the post-war trend. Reasons for this may be the fact that the laying-up of cars during the war resulted in generally less wear than if they had been on the road.

(ii) A similar effect was observed for cars made immediately after the war.

(iii) Some changes in trends of lighting faults can be identified with changes in design.

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Abstracts and Reviews

Models in Physics and Psychology.
Z. M. T. TARKOWSKI.
Times Science Rev. (20), 17–18 (Summer 1956); *Engineering*, **185**, 305–306; 429–430; 561–562 (1958).

The nature of models and physical analogies is discussed in general terms, quoting Maxwell, Kelvin, Hooke and others. Lorenz's hydro-dynamic model of action-specific energy release is discussed, and an automatic control model for human behaviour is then drawn up, with two levels of control; three sub-routines within emergency control are postulated.

The following six abstracts are provided by ORbit Ltd., 23Bedford Square, London W.C.1.

The Maximisation of a Linear Function Subject to Linear Restrictions.
R. W. RUTLEDGE.
Statistical Soc. of New S. Wales Bulletin, (16), 14–16 (1956).

Step-by-step rules for Simplex Method, based on Orden (in SCOOP Publication No. 10, 1952).

Les Programmes Linéaires Stochastiques (in French).
G. TINTNER.
Revue d'Économie Politique, March–April, 1957.

Summary of American work on stochastic linear programming with bibliography.

An Investigation into the Costs of Distribution in the Grocery Retail Trade in the Netherlands (in English).
S. C. BAKKENIST and D. E. BEUTICK.
Productivity Measurement Review.

Full (though said to be abridged) report of an E.P.A. study into grocery shop operating costs, based on a sample of 50 shops in Holland. Costs are separately detailed for 96 product groups. The very full tables include profit on each commodity, sales per man-hour for each shop, stock: turnover ratios for each commodity, etc. etc.

The Choice of Farm Enterprises, using Linear Programming.
F. G. JARRETT.
The Economic Record. (*J. Econ. Soc. of Australia & N.Z.*), **33** (65), 234–246 (1957).

Simplex method simply explained by means of a hypothetical but realistic farming example.

The "activities" are the numbers of crossbred ewes, fine wool ewes, dairy cows and amount of hay.

The "constraints" are the labour available each month and the pasture available each quarter. Results are, presumably, corrected to the nearest whole animal.

Étude du Choix des Plans d'Échantillonage Simples d'après leurs Conséquences Pécuniaires (in French).
M. COURTILLOT (Société des Automobiles Peugot).
Revue de Statistique Appliquée, **5** (1), 43–56 (1957).
The author, criticizing the practice of designing sampling schemes by *a priori* risk levels, proposes a total cost criterion. Tables and graphs are given for simple (number defective) schemes.

Some Observations on Input-Output Analysis.
O. LANGE (School of Planning and Statistics, Warsaw).
Sankhya (*The Indian J. of Statistics*), **17** (4), 305–336 (1957).
Mathematical models of a (socialist) economy, based on Leontieff and Marxian economic theory.

Readability.
J. VAN DER PUTTEN.
Tebo, **13**, 25–26 (January 1958) (In Dutch).
The importance of making industrial reports and communications easy and interesting to read is stressed. With reference to various American publications the technique of achieving this objective is discussed. Reference is made to papers by Flesch and to the use of the Dale–Chall formula.

The Influence of the Euromarket on the Extent of Research.
J. M. VAN DE WETERING.
Tebo, **13**, 7–11 (January 1958) (In Dutch).
It is pointed out that the removal of trade barriers between countries in a European market does not eliminate competition between the countries,

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but merely displaces it to a higher level, requiring more efficient organization. The example of inter-state competition in U.S.A. is studied. The special problems and opportunities of the Netherlands are then considered, the need for market and operational research emphasized, and the lines on which this should develop indicated.

Operational Research and Investments in Rolling Stock. Determination of the Optimum Organization of Railway Traffic for Exploitation of the Iron Ore from Fort-Gouraud.

E. VENTURA.

Annales des Mines, 226–230 (April 1958).

This article follows, and refers to, an account of automatic signalling in underground workings.

The work was carried out jointly by railway experts and an operational research team before inviting tenders for construction with the object of organizing traffic to and from Port-Etienne in Mauretania. Two mining centres, one producing quarter of the total, were concerned in open-cast locations and 640 km of railway joined these to the port. The problems were to organize the traffic under the best conditions, to decide what storage accommodation would be necessary and whether or not the working at the port should go on at night and whether it should work a 6-or 7-day week and what should be the tonnage handled per hour. Diesel-electric locomotives were used with 77·5 ton waggons, about 30 to each locomotive, an average daily load of 20,000 tons, a fixed time-table and 7-day operation for 10 months in the year, the mines closing on Sundays. The model system is then defined and minimized for costs. The best conditions proved to be: 2 trains a day (12 hr), each of 130 waggons with 4 locomotives, 6 hr for charge and discharge, locomotives on a 36 hr cycle and wagons on a 48 hr cycle. Sidings to hold 540 waggons. With 30 waggons the cost would be 26 per cent higher, with 60, 8·5 per cent and with 90, 1·3 per cent higher.

Operational Research and Industry.

JEAN STENGEL.

Arts et Manufactures, 22–24 (April 1958).

This is a popular and somewhat pretentious account of the impact and significance of operational research, with an historical account of the science of making decisions quoting Descartes, Bernouilli, Karl Pearson, Spearman and Galois. The development of operational research in Great Britain for military purposes in 1938, and after, is then mentioned. Notes

are added on the work and organization of an operational research group in very general terms.

Reducing the Queue.
B. D. DAGNALL.
Automation Progress, **3**, 194–196 (May 1958).
An introduction to queueing theory is given and the use of constant or random distributions is explained. Simulation by computer and the generation of random and pseudo-random numbers are considered in outline and the method of antithetic variates is explained in principle (10 references).

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The Use of a Computer in a Rolling Mill Office.
M. G. MASSEY.
Process Control and Automation, **5**, 190–195 (May 1958).
A study by the Operational Research Department of the British Iron and Steel Research Association is reported. The general flow of information is itemized and the type of computer and ancillary equipment required is postulated and its method of use then examined. Auxiliary storage was found to be needed and magnetic tape or film was chosen for reasons given. The internal stores in the computer still required to be large, and with present available equipment, punched cards or tape must be used to feed the magnetic stores. The system for preparing tapes is shown diagrammatically making two independent records to ensure accuracy. The proposed system is then outlined and times for computing and printing and staff requirements (7 senior and 16 junior) are made. Costs of equipment are indicated and the Tallimarker system of collecting information is described. The selection of bars for sawing is then outlined and future work is very briefly considered.

Calculation of Process Controllability Using the Error-Squared Criterion.
R. JACKSON.
Trans. Soc. Instrum. Tech., **10**, 68–75 (June 1958).
It is shown that using the integral or the average of the square of the error as a criterion of control quality, the controllability of a loop with a given type of controller may be defined as the optimum quality which can be obtained by adjusting the controller settings. An illustration is given by considering a plant with three exponential transfer stages with a proportional-plus-integral controller of conventional type. Contour

charts are used to display the results and comparisons are made with those obtained from other criteria. In an appendix the calculations for computer programmes are developed.

International Works Transport Conference, Liège (June 1958).
Translations published by the British Iron and Steel Research Association, 11 *Park Lane, London, W.*1.
These nine reports include much data susceptible to operational research method.

(i) Procedures for the Maintenance of Works Railway Tracks.
A. MÜLLER.
Comparisons of points/km, gaugers, foremen, linesmen and pointsmen are made with figures for the Federal Railways. Calculations of permissible loading capacities are considered, sources of ballast, and type of rail and type of fixing are discussed.

(ii) A Comparison of the Performance, Lay-out and Cost of Different Works Railway Systems of German Foundries Supplemented by Mathematical Statistics.
H. MIETH.
As there is no competition between systems and each is adapted to the works a cost basis is difficult to establish. Characteristic factors are, therefore, established for track, rolling stock and for personnel. Statistical studies showed considerable deviations so that either variables have not been taken into account or errors of definition have occurred.

(iii) Technical and Economic Aspects of Track Maintenance in a Steelworks.
R. FORTERRE.

(iv) Handling of Lime and Scrap in Steelworks.
A. F. DE LAFON AND G. GABRIELLE.
Various installations are described and an attempt at comparison on a cost basis is made.

(v) Handling of Lime and Iron Scrap at Cockerill—Ougrée.
L. HUBIN.
Old and new handling systems at Seraing and Ougrée are described with the advantages and disadvantages of the changes. Scrap handling and unloading methods are then outlined.

(vi) Operating Costs of the Internal Transport Services of Iron and Steelworks.
N. MÉVIS.
Choice of a standard for charges and determining factors in the cost are considered and comparative costs (for Belgium) 1948–54 are indicated.

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(vii) The Main Element of the Cost of Internal Transport: Charging.
R. FORTERRE.
Costs of the rail service and cost per hour of the locomotives of a particular works are shown. Loads transported are analysed, and the former method of charging is given. A study of the operation of the locomotives is then reported and it is shown that a utilization of 80 per cent can be regarded as perfect and below 50 per cent efforts at replacement should be made. Method study is then applied and charging in respect of fictitious tonnage and cost per fictitious ton is then explained and comparisons with actual results are made. Simplification of accountancy is attained.

(viii) Transport of Lime and Scrap in an Open-hearth Furnace Steelworks.
W. GESELL.
A fundamental study in terms of material flow is made. Diagrams are drawn up and used to help the understanding of transport conditions. Lime transport has a multiplicity of solutions and no attempt to find the best is made, only exact cost analysis could provide the solution. The scrap flow problem is treated more generally and a basis for discussion for designing is aimed at.

(ix) Steelworks Railway Maintenance: New Methods and Mechanisation.
P. WILLEM.
Costs of laying, maintenance and inspection under a number of conditions are discussed.

Applications of Linear Programming in the Coal and Allied Industries.
W. D. RAY.
Bull. Brit. Coal Utilisation Research Assoc., **22**, 281–286 (July 1958).
The transportation model of Hitchcock and Koopman was applied to the transport of coking coal, using 154 sources and 65 destinations and the optimum solution showed a saving of 10 per cent over the existing practice. Complications to this model are considered arising from the non-homogeneity of the coal, preferential requirements of coke-oven

installations, and the local organization of coal distribution. In this study the ton-mileage instead of cost of transport was minimized. Data of amount of coal received per month from a list of mines, pithead price and delivered price were collected. Instead of attempting to find the shortest rail distance between 10,010 pairs of points, the point distances (from mine to oven) were themselves used in a way which minimized transport costs and also gave the shortest rail distance of all routes used in the solution. Transport to power stations, movement of coal in the U.S., the blending of aviation spirits, the distribution of production in multiple plant operations and alloy making to specification are also briefly reviewed as applications of linear programming technique.

The Application of Statistical Procedures in Industrial Investigations.
M. KNAYER.
E. Q. Bull., The Hague, 24–27 (1958) (In English).

Operational research is only possible in large enterprises but statistical methods of a simplified kind have many applications in industry and data of some sort are often available in more or less private notes in different departments such as stores; the tally chart is a simple device for such locations. Examples are given of average size of order in a garment factory where order slips were sampled and found to account for delay in cutting; of shoe factory production and quilting department delay where the work tickets gave the necessary information on sampling 250 at random from 3,000; of a wholesale firm which obtained the rare U-form of histogram for the number of accounts sent before payment was received; and of production statistics needed by small and medium sized firms, of which the examples of a spinning mill and a weaving mill are given for study of production variations, and a study of the causes of breakdown of knitting machines, also by means of a tally sheet.

A Simulation of Melting Shop Operation by Means of a Computer.
R. NEATE and W. J. DACEY.
Process Control and Automation, **5**, 264–272 (July 1958).

The problem is too complicated for queueing theory but, as frequency distributions for operational times proved to be relatively narrow, Monte Carlo methods can be used. The accuracy becomes greater for longer times simulated and a complex situation would require weeks of plotting. However, use of a computer makes the method practicable. The problem considered was the effect of oxygen enrichment on operation times of an

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open-hearth steel furnace. Tests had shown the optimum degree of enrichment (5·5 per cent) in one case. Sets of conditions were drawn up and production rate for these was to be estimated. Many of the factors are interdependent so that the shop must be considered as a unit. Charts could be drawn up to represent the operations showing positional locations against time and a portion of such a chart is shown. The use of the Monte Carlo method is shown by an example of refining time in one furnace. It is now necessary to write a suitable programme for a computer and the activities of the shop are analysed for this purpose and the output information is given in the form of number of casts and delays to furnaces (5 headings); queueing for the cranes is particularly stressed. The furnace cycle is resolved into 7 components and while the human operator could see the situation the computer requires some order of working of the lines; it is best to choose an interval in which all lines are worked once. The method of drawing up the programme is outlined and a sample programme is given.

Operational Research in Spain.
Trabajos de Estadistica (2) 141–142 (1958–9).
Reference list of 36 papers published in Spanish on Operational Research.

Digital Calculating Machines.
G. A. MONTGOMERIE.
Blackie and Son Ltd., Glasgow, 1956. 262 *pp.* 30*s.*
We are, at present, in the midst of a spate of books on electronic digital computers. Casual reference to the above title might lead one to suppose that here was yet another. In fact, the scope of this particular book is wider, dealing as it does with digital machines of all kinds, from the Chinese abacus to the E.N.I.A.C. electronic computer.

The author classifies digital machines into four main groups. The classification is based on the ability to store firstly, numerical data and secondly, instructions for operations on that data. Group I consists of machines without extended storage capacity for either numbers or instructions. These make up a large and important class which is necessarily further sub-divided. It includes devices up to the level of the ordinary electric desk calculating or adding machines. Group II includes those machines which provide extended storage for numbers but not for instructions. Almost the only representative of this class is the punched card accounting system. In Group III are those machines with extended storage

for instructions but not numbers. These are generally special purpose machines. Finally, Group IV contains the electronic computers which have extended storage for both numbers and instructions.

Within each of these four classifications, a general description of the nature and function of the machines is given, illustrated by reference to a selection of existing machines. A discussion is also given of the types of computing problems the machines in the particular groups are best suited to solve, together with some useful examples. The main emphasis is on Group I machines which occupy 133 out of a total of 262 pages. The other three sections are necessarily treated in a less detailed fashion in view of the limited scope of the book.

The book is claimed to be written for scientific and engineering workers and it is likely to prove a useful short introduction to digital machines for such an audience. Far too often in science and engineering too little thought is given to the choice of the most suitable machinery for special computing problems of medium size. There is overmuch emphasis on the familiar electric desk calculator, which although generally useful in this sort of work, is not necessarily always the best choice. The clear definitions of machine functions given here will serve as an introduction to the wide range of devices available in the increasingly important field of computing.

R. F.

The Rationalization of Industrial Decision Processes.

WILLIAM T. MORRIS.

Ohio State University Studies, Engineering Series, March 1957, **24** (1). 117 *pp.* $2.

This monograph contains sixty-four references to the literature of decision-making which it is useful to have collected together, although there are some notable omissions. The author has concentrated on making a survey rather than novel contributions. The reader should not be led by the title into expecting any particular emphasis on industrial activities since the treatment is substantially a general one.

Incentive Payment Systems.

R. MARRIOTT.

Staples Press Ltd. 21*s.*

Since about 40 per cent of the wage-earners in the manufacturing industries are now paid by some form of incentive scheme, any new book on the

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subject deserves careful consideration. Mr. Marriott is Assistant Director of the Industrial Psychology Research Group, Medical Research Council, and his book, of which the sub-title is "A Review of Research and Opinion", undoubtedly contributes to the already extensive literature on incentive schemes. Just as one would not recommend a book of scientific abstracts as an elementary text book, so one would not suggest this book for the reader with little previous knowledge of the problems of introducing incentive schemes into industry. In no sense does it detract from the value of the book to say that it is not one for the operational research man "who wants to know a bit about incentives".

Although this book quotes extensively from the writings of authors of many countries, it is not a book of abstracts, for these many quotations on incentive schemes and on the inseparable subject of human relations are linked together by informative and critical comment which serves to bring the subject-matter into proper perspective. The international field covered by the author is both so extensive and so comprehensive that the more experience the reader already has of the subject, the more interest will he derive from the book, even though, as is inevitable with such a controversial subject, he may not agree with all the author's conclusions.

The book opens with an interesting historical background dating from the Babylonians of nearly 4,000 years ago, and leads on to some of the current problems of research on incentives and motivation. There follows a broad classification of incentive payment systems, and a discussion on their advantages and disadvantages. The author then reviews the setting of time or work standards which he follows with detailed consideration of successes and failures of incentive schemes: he quotes some interesting case examples. The first of the two concluding chapters widens the horizon by considering the opinions not only of those who have installed and administered incentive schemes, but also of trade unionists and of social scientists. The final chapter is a general appraisal of incentive payment schemes and a description of present trends. These last two chapters form a well-balanced conclusion to the earlier parts of the book.

As appendices, there is a comprehensive bibliography of well over two hundred references, and an interesting table of the basic requirements of incentive schemes as propounded by different authorities.

One of the most significant comments made by the author follows an extract from *Manchester Guardian Survey*, 1956: ". . . A growing proportion of the workers, and some of the unions, are interested in raising output and reducing costs—if they have confidence in the management". On this,

the author comments: "There is little doubt that the last seven words are vital".

For those who want to broaden their view and increase their knowledge of incentive schemes, this book is a sound investment.

J. C. B.

Proceedings of the Fourth Annual Computer Applications Symposium. October 24–25, 1957.

Sponsored by: Armour Research Foundation of Illinois Institute of Technology. 126 *pp.* $3.00.

This is a collection of 14 papers given at this Symposium, and a transcript of the final discussion.

Of the 14 papers, four are concerned with data processing, three with computer applications in operational research, three with mathematical problems, and four with computer techniques and future developments.

As in all collections of papers, the quality varies. Perhaps the greatest disadvantage of this sort of collection is that a number of interesting points are touched upon, but not sufficiently elaborated, probably due to the time restrictions imposed on speakers. Nevertheless, they make interesting reading.

A special mention must be made of the D. H. Heiser and Dorothy P. Armstrong contribution entitled "Data-Processing Tasks for the 1960 Census", which gives an excellent exposition of how the U.S. Bureau of Census set about choosing their computer and organizing the department for this future task.

System Engineering.

HARRY H. GOODE and ROBERT E. MACHOL.

McGraw-Hill, New York, Toronto and London, 1957. 551 *pp.* 77*s.* 6*d.*

This is an ambitious book. Messrs. Goode and Machol have come to the conclusion that the urgent need in present-day industry and in military tactics is to deal satisfactorily with complex systems as a whole, and their purpose is to give an introduction to what they consider to be the principal methods by which this aim can be successfully achieved. Operational research is taken to be part of system engineering.

Part I of the book sets out the growth in complexity of systems up to the present day, leading to the proposition that systems should be examined as a whole rather than bit by bit.

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Parts 2 and 3 have the titles "Probability" and "Exterior System Design". They form, in fact, a continuous section, developing probability and statistical theory, and their application to system analysis. This part of the book provides an admirable introduction to statistical thinking and practice, useful in itself, and with an excellent bibliography, in which the British reader will observe the names of Professors Aitken, Bartlett and M. G. Kendall, and Sir Ronald Fisher. There is a particularly interesting chapter on mathematical models, with a distinction between "rigid" and "probability" models.

Part 4 breaks the flow in regard to theory, being concerned with digital and analogue computers, on the basis that computers are essential to what the authors call "interior system design". This is rather a curious point to make since a computer is just as useful for Monte Carlo applications which the authors reserve for so-called "exterior system design". This is perhaps the weakest section in the book, suffering from brevity, particularly in the introductions to machine logic and machine "hardware". The explanation of the "and", "or", "not", and delay computer "gates", and how these are used for various computer operations, might to advantage be longer and more complete, and does not compare favourably with the corresponding section in Bowden's *Faster than Thought*.

There is not quite as much original thinking in this section as in the rest of the book, but there is one paragraph which is arresting and thought provoking, whether one agrees with it or not. "The human being, at least in his input and output, is inherently analogue, and so the machine with which he deals must also be analogue in most cases. It is possible for an operator to have a yes–no (digital) relationship with a machine, but the operator's bandwith in this situation is very small".

There follows a useful chapter on how to lay out system block diagrams of various types, and some general principles in regard to tackling large problems. The following quotations are taken from this section:

"Optimising a component of a part of a system does not necessarily improve system performance, nor does optimising all elements ensure obtaining an over-all optimum. System design is the process of obtaining a group optimum."

" The fundamental principle of system design may be stated as maximising, in some sense, the expected value".

"The essence of competitive design is randomisation of response".

Part 5 deals with what the authors call "the single thread system", on the basis that, "it is possible in every large-scale system, no matter how

complex, to isolate a single thread, that is, the manner in which the system operates for one input of each different type". It is in reality, however, a continuation of Parts 2 and 3 developing still further, logical, statistical and mathematical theory and their application. It includes a chapter on "High Traffic—Queueing Theory", with a useful bibliography which includes papers by D. G. Kendall, Lindley and T. C. Fry, followed by several interesting chapters dealing with games theory, linear programming, group dynamics (topology), cybernetics, communications—information theory and servo-mechanism theory.

Finally, there are chapters on human engineering, mainly discussing the limitations of human sight and hearing and their effect on the visual and aural presentation of information, and on testing and evaluation.

It will be seen from the above that system engineering concerns itself with a great many different techniques, and it is remarkable how successful the authors have been in providing a series of what are, in effect, "digest" instruction books in these techniques. Much care has been taken to be as simple as possible, as brief as possible consistent with providing adequate information, and as up-to-date as possible. The authors make no claims to originality but there is, in fact, much original and valuable thought in the book, which is one that can be warmly recommended.

A. W. S.

Metrika. (Journal for theoretical and applied statistics.)
O. ANDERSON, Munich and W. WINKLER, Vienna (Editors).
Physica Verlag, Wurzburg, Germany.

This is a new journal, which takes the place of the German language publications "Mitteilungsblatt fur mathematische Statistik" and "Statistische Vierteljahresschrift". Both these former journals were founded shortly after the end of the last war, with the aim to disseminate in the German speaking countries the new developments in the theory and applications of statistics, which were made during the last 25 years, especially in Great Britain and the U.S.A.

The merger of the two publications is the result of the common effort of the professors and lecturers in statistics at the Universities of Berlin—Charlottenburg, Berlin—Dahlem, Berne, Geneva, Hamburg, Munich, Vienna, and the director of the German Federal Statistical Office, Wiesbaden.

The aims of the new journal are given as:

(i) To consider topics in the whole range of modern statistics, theoretical as well as applied. Special regard will be paid to econometrics, and statistical methods in sociology, psychology, and medicine, as there do not exist German statistical publications in these fields.

(ii) To treat statistics from the mathematical viewpoint.

(iii) To devote some space to papers on problems and developments in related fields of mathematics and logic.

(iv) To keep in contact with statistical work outside the German speaking countries. Papers in the English and French language will, therefore, also be published.

The first number of the journal contains four papers; two theoretical ones, on discrete autoregressive schemes and on the probability distribution of numbers of objects over a number of places, and two on applications, one of which describes a linear programming model for production planning in a fluctuating market and the other dealing with approximation methods for estimating variances. One paper in each of the groups is in English.

The journal also contains some book reviews and notes on meetings of interest to German statisticians.

To judge by this first number it is a journal of interest to operational research scientists.

E. K.

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Feedback Theory and its Applications.

P. H. HAMMOND.

English Universities Press, London, 1958. 348 *pp.* 35*s.*

The author has produced a very interesting book, although it will be more valuable to the electronic engineer than to the designer of power type servomechanisms. The commencement follows the now orthodox line of dealing with the elements of the Laplace transform as applied to linear systems and the opening chapters contain much of value on basic concepts, although it is surprising that the final and initial value theorems are not mentioned. In particular the explanation leading up to the convolution integral and Duhamel's theorem is very attractive.

The book is, like most, better in some respects than in others. The chapter on cathode followers, long-tailed pairs and virtual earth amplifiers, and those on on–off controlled servos, the phase plane and non-linearities are very good indeed, but the treatment of servomechanisms and their stabilization is sketchy and could be expanded with benefit. The approach

to the full Nyquist stability criterion, too, will not readily be understood by the student, since the practical determination of the number of poles in the positive half of the p-plane is not dealt with adequately. A plot of the Nyquist diagram will only give the difference between the number of zeros and number of poles in the positive half of the p-plane and in order to find the number of zeros the poles must be determined by a separate analysis. When, as is often the case, the transfer function is not known except as a plotted transfer locus, determination of the number of poles can be difficult in the case of non-minimum phase systems.

The author has followed Nyquist in drawing his transfer loci around the $+1,j0$ point and this will be criticized by servo engineers whose other text books all follow the now accepted convention of using the $-1,j0$ point. This is particularly confusing when the transfer loci of lag and other networks are drawn in their accepted positions.

Apart from the above, and a few other minor faults, the book has much to commend it and offers a great deal of valuable information which has not appeared in text book form before. It is particularly welcome as a clear exposition of the subject of feedback theory and application from a British author. At 35s. it is by far the best value the writer has encountered on the subject.

L. N. B.

Productivity and Social Organization—The Ahmedabad Experiment.
A. K. RICE.
Tavistock Publications Ltd., London, 1958. xiii + 298 *pp.* 35*s.*

This book describes the reorganization of an Indian textile mill, by a team of sociologists working within the factory. There is a clear exposition of both the social background of the factory and the team's conceptual tools, which had been evolved in similar studies elsewhere. In the light of their preliminary work, the team assisted the factory staff (including the operatives where they were affected) to devise and introduce a form of organization that seemed likely to be both more efficient, and more satisfying to the staff concerned. Continuous records of quality and output were kept. Detailed analysis of these (well described in the book) showed an initial improvement in efficiency, followed by a marked fall; by further study in the factory, the sources of the fall were identified and efficiency pulled up.

Rice's account is of value to operational research workers first, for the use of sociological principles in dealing with the workings of a factory in operation; and secondly, for the detailed account of the analysis of current

productivity data. [Readers interested only in the latter aspect will find the essential information in the same author's "Productivity and Social Organization in an Indian Weaving Mill" (*Human Relations*, 1953, **6**, p. 297 and 1955, **8**, p. 399).] No use is made of sophisticated mathematical techniques; but it should be noted that the application of linear programming described by Morton in *Operat. Res. Quart.* 1958, **9**, *p.* 198 was associated with the reorganization.

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What I expect from O. & M.
G. H. GIBSON.
Public Administration, Summer 1958, **36**, *p.* 169.
Though Dr. Gibson, a County Medical Officer of Health, is describing his ideal of an O. & M. officer, most of what he says applies, almost without modification, to an operational research worker. This is an admirable statement of a user's point of view.

Curso de Aplicaciones Industriales de la Estadistica. (In Spanish.)
F. J. ANSCOMBE, trans. F. AZORÍN and J. ROMANÍ.
Instituto de Investigaciones Estadísticas. Consejo Superior de Investigaciones Científicas, Madrid. Monografias de Ciencia Moderna. No. 57, 81 *pp.*
This is the Spanish translation of a short course of lectures on Industrial Applications of Statistics, given by the author at the Instituto de Investigaciones Estadísticas of the Consejo Superior de Investigaciones Científicas in Madrid, March–April 1953. The main topic is sampling inspection—control charts, Dodge-Romig plans, Wald sequential plans, rectifying sequential plans, continuous sampling plans, minimization of total costs. There is also some material on sequential estimation, and, in an introductory lecture, some remarks on consumer research, operational research, and design of experiments. All material presented was published, or in course of publication, at the time of the lectures, except for an investigation of optimum plans for rectifying inspection of lots, related to unpublished work by D. G. Champernowne, which has since been revised and published separately in *Trabajos de Estadística*, 1954, p. 5.

Experimental Designs in Industry.
V. CHEW (Editor).
John Wiley & Sons, New York, Chapman & Hall, London, 1958. 252 *pp.* 48*s.*
This book is a selection of ten papers presented at a Symposium held at North Carolina State College, five on experimental designs and five

on applications. It suffers from some drawbacks which would be expected from its origin. The first five papers summarize various classes of experimental design, but most of the material already exists in one or other of the available text books, an exception being a paper by Box and Hunter, which is a brief up-to-date description of the designs associated with Dr. Box himself; some of this, although it has appeared in journals, is too recent to have reached the text-books. Most of this part of the book is unnecessary for those who have a reasonable knowledge of the subject, and too concise and difficult to read for those who have not, except perhaps statisticians who have not particularly studied design. There are however several useful references to various difficulties, such as multiple comparisons, inadequacy of the mathematical model, to the close relationship between factorial experiments on continuous variables and regression analysis, and so on. A very full bibliography is included.

The papers on industrial applications may seem at first sight disappointing, since they are in some cases very brief, and few of the examples would be considered typical industrial applications. They are, however, more useful than they appear, since they introduce many practical difficulties which involve departures from the standard text book designs, and show clearly that a superficial knowledge of experimental design is of very limited use in industry. It is a pity that more papers of this type are not published. They need not describe novel or complicated designs. They would be valuable in showing the experimenter that he must be prepared to compromise between the ideal design and the practical difficulties and to make do with non-standard designs. This can be learned only from experience, and better if possible from someone else's experience. One criticism is that there is a tendency to concentrate on an elaborate analysis of the data and tests of significance, and to give awkwardly expressed sets of conclusions. The first step in handling data should be to look at them carefully and intelligently, draw some simple graphs, and so on. This will often give most of the important information, and is the basis of a means of explaining the main conclusions to management. A practical paper should also give practical conclusions, for example what action was taken, or what further experiments were indicated—for experimentation is essentially a sequential operation, seldom reaching a final conclusion after one campaign, however good the design.

Workers in operations research are not likely to find this book of great value. Where some at least of the variable factors in a problem can be controlled at will, experiments designed on statistical principles

should be used, since they lead to economy of effort and precision in drawing conclusions. This subject is more or less well treated in several text books. But the practical difficulties are not sufficiently emphasized, nor are the means for overcoming them. The book does something to remedy this defect, but the moral is that this aspect of the subject deserves more attention, which can come only from the publication and discussion of actual cases—a suitable subject for some later Symposium, and one which should interest operational research workers as well as industrial statisticians.

F. R. H.

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Queues, Inventories and Maintenance.
P. M. MORSE.
John Wiley & Sons, New York, 1958. 202 *pp.* $6.50.

This volume, the first in the series "Publications in Operations Research" sets a fine example and gives promise of a comprehensive literature of the basic concepts and practical applications of general use in the field. The series is jointly sponsored by the Operations Research Society of America and John Wiley and Sons. This volume grew from a study, supported by the Army Office of Ordnance Research, the object of which was to produce queueing tables of general interest. The author found that a short preface to the tables would be inadequate to define the subject and outline the fields of application. Professor Morse has produced an excellent introduction which, while intended to be expository, will allow the reader to make use of the techniques. The earlier chapters introduce probability concepts, the description of arrivals and service distributions and the single exponential channel. Here a somewhat new notation is introduced without adding to the confusion of "newcomers" or "old hands" to queue theory. The book includes discussions of multiple channels, priorities, transient solutions, maintenance problems and inventory queue models. Throughout the work Professor Morse avoids straying from the practical path; sample problems are related to realistic problems in industry. Cost considerations, as well as service levels, are used to define optimal conditions. While the book is largely concerned with the analytical aspects of queue theory, simulation is not ignored; Chapter 5 is entitled "Simulation of Non-Exponential Distributions". There has been a need for a definitive volume on queue theory, if only to organize ideas and serve to standardize notation. This volume does even more; it develops the basic concepts in a clear and concise manner.

This is not a book for the mathematically immature; some background in statistics and calculus is essential, as it should be for a comprehensive review of an important subject for operations research workers. A glossary of symbols is provided which allows the volume to be used for reference purposes. The book also includes seven tables of great value to those using queue theory in real problems. The tables include "Factorials of Integers", "The Erlang Density Function", and "Poisson Functions for Erlang Channels".

E. K.

Book Received

Studies in the Economics of Transportation. (Second printing.)
MARTIN BECKMAN, C. B. MCGUIRE and CHRISTOPHER B. WINSTEN. Introduction by TJALLING C. KOOPMANS.
Published for Cowles Foundation for Research in Economics, Newhaven, 1956 (Yale University Press); London, Geoffrey Cumberlege (Oxford University Press). xix+232 *pp.*: illus., 32*s.*
Previously reviewed in *Operat. Res. Quart.* **7**, No. 4, December 1956.

News and Notes

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Movements of Members

THE following movements of members have come to our attention in the course of the last few months:

Dr. E. Lee (D.O.R. Admiralty), to Deputy Director, National Physical Laboratory, Teddington, Middlesex (March 1958).

H. C. Calpine (D.O.R. Admiralty) in succession to Dr. E. Lee.

R. R. P. Jackson, formerly of Imperial Chemical Industries Ltd., has joined the staff of Shell Mex-B.P. Limited.

P. Hutson joined the staff of The United Steel Companies Ltd., in December 1957.

A. W. Swan retired from Courtaulds Limited on 1 May, 1958 to undertake consulting work. His address is: 6, Vicarage Road, Lillington, Leamington Spa.

Linear Programming

THE Brunel College of Technology, London, W.3, announces a course of ten evening lectures commencing 13 January, 1959. They will include a series by Dr. S. Vajda, Admiralty Research Laboratory, on basic theory, and practical applications described by D. Caplin, Shell Petroleum Co. Ltd., J. Stringer, Central Electricity Authority, Miss A. Doig, London School of Economics, and N. Williams, Unilever Ltd.

Notice

THE Education Sub-Committee of the Society (Chairman, Mr. R. S. Gander) would be interested to receive information on education in operational research in the United Kingdom. In particular they would welcome details and syllabuses of any intended courses.

Such information should please be sent to the Society's Secretary—Miss G. Heselton, Third Floor, 20 Albert Embankment, London, S.E.11.

The Rand Corporation

THE Rand Corporation will hold a Symposium on Mathematical Programming from 16 to 20 March 1959 under the general direction of George Dantzig. Among the topics to be discussed will be problem formulation; solution of large linear programmes; nonlinear-programming methods; discrete programming; network flow and distribution problems; programming under uncertainty.

The symposium will be unclassified and there will be no charge for attendance; inquiries to: Dr. Philip Wolfe, The Rand Corporation, 1,700 Main Street, Santa Monica, California.

Operational Research Society 1959 Conference at Harrogate

THE Second Annual Conference of the Society will be held at the Old Swan Hotel, Harrogate, on 10, 11, and 12 June, 1959.

The object of this Conference will be to give members of the Society and others concerned an opportunity to explore and discuss possible directions for the future development of operational research, particularly in the field of communications and managerial decision making. As a basis for such exploration the Conference will include the results of a survey of the present position of operational research and a symposium of papers on how decisions are reached either as a routine or at a higher level in business and public administration.

Copies of the detailed programme and application form should be requested from the Secretary, The Operational Research Society, 64 Cannon Street, London, E.C.4. They will be ready to send out at the beginning of April.

International Federation of Operational Research Societies

THE International Federation of Operational Research Societies came into existence on 1 January with as its objects "the development of operational research as a unified science and its advancement in all nations of the world". The initial membership of IFORS consists of the Operations Research Society of America, the Operational Research Society (United Kingdom) and the Société Française de Recherche Opérationnelle. Membership is open to other national societies whose primary object is

the advancement of operational research and whose membership includes qualified scientists working in the field.

The Federation is governed by a Board of Representatives, one representative coming from each member society. According to the statutes the voting power of each representative is proportional to the square root of the number of members in the society he represents—a formula designed to give the right weight to size.

One of the first activities of IFORS will be to sponsor the second international conference on operational research following the successful first conference held in Oxford in 1957. The second conference is provisionally planned to take place in Aix-en-Provence in southern France in early September 1960.

Sir Charles Goodeve has agreed to act as the first Secretary of IFORS and Mr. Donald Hicks as Treasurer. The address of the Federation in the first instance will be 11 Park Lane, London W.1.

Further information from Sir Charles Goodeve: Telephone—Grosvenor 4751.

Operational Research Quarterly

THE Editors are very conscious that they owe their readers an apology for the lateness of the last and the present issue. In order to provide a better service the publication of the Quarterly is now being undertaken by Messrs. Pergamon Press Ltd. This change has inevitably brought with it a need for an extended time schedule for the preparation of the journal and it has not been possible to achieve this in the initial stages. It is hoped that publication will be on time in future.

ADVERTISEMENTS

ENGINEERS AND PHYSICISTS

A limited number of vacancies exist in one of the country's leading Operational Research Groups. The work involves the application and development of mathematical and engineering techniques to a wide variety of problems, and includes the utilisation of Linear and Dynamic Programming together with Games Theory, Inventory Control, Queueing and Replacement. The work is by no means easy but is extremely interesting and satisfying. Candidates must hold a good class honours degree and be under 35 years of age. The initial salary will be in the scales £690–£1200 male, and £674–£1147 female, and there are outstanding opportunities for further advancement. Interviews will be held in London and informal confidential interviews can be held in the evenings if desired.
Further information may be obtained from the Head of the Group, Mr. B. H. P. Rivett, Reliance 8151, Ext. 359.

Application forms may be obtained from Staff Department (X.1368 H), National Coal Board, Hobart House, London, S.W.1.

G.K.N. GROUP RESEARCH

The **G.K.N. Group Research Laboratory** is forming a new unit to introduce and develop operational research methods within Group companies in South Wales. Applications to join this unit are invited from all who already have experience in operational research in any field, or who are interested in gaining such experience as a member of a team with a future.
Experience, in any field, of the application of statistical methods to industrial control and management or time and motion study would be an advantage. For some of the vacancies the possession of a degree in a mathematical or engineering discipline would be helpful.

Applications should be addressed to: The Recruitment Officer, G.K.N. Group Research Laboratory, Birmingham New Road, Lanesfield, Wolverhampton, quoting reference SW/OR/1. Interviews may be arranged at locations to suit applicants.

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